

Relation de négligeabilité

Def: $U_n = o(v_n) \Leftrightarrow$

$$\exists (\varepsilon_n)_{n \in \mathbb{N}} \text{ tq } \begin{cases} U_n = \varepsilon_n v_n \\ \lim_{n \rightarrow +\infty} \varepsilon_n = 0 \end{cases}$$

$$\Leftrightarrow \forall \varepsilon > 0, \exists n_0 \in \mathbb{N} \text{ tq } \forall n \geq n_0, |U_n| \leq \varepsilon |v_n|$$

$\rightarrow$ Si $v_n \neq 0$ à partir d'un certain rang $U_n = o(v_n) \Leftrightarrow \lim_{n \rightarrow +\infty} \frac{U_n}{v_n} = 0$

Propriétés

1) $\begin{cases} U_n = o(v_n) \\ v_n = o(w_n) \end{cases} \Rightarrow U_n = o(w_n)$

2) $\begin{cases} U_n = o(w_n) \\ v_n = o(w_n) \end{cases} \Rightarrow U_n + v_n = o(w_n)$

3) $U_n = o(v_n) \Leftrightarrow \forall \lambda \neq 0, U_n = o(\lambda v_n)$

4) $U_n = o(v_n) \Rightarrow U_n \cdot w_n = o(v_n \cdot w_n)$

5) $U_n = o(1) \Leftrightarrow \lim_{n \rightarrow +\infty} U_n = 0$

6) $\lim_{n \rightarrow +\infty} U_n = \ell \Leftrightarrow U_n = \ell + o(1)$

7) si $\begin{cases} U_n = o(v_n) \\ v_n \sim w_n \end{cases} \Rightarrow U_n = o(w_n)$

Relations de Comparaison

Relation d'équivalence

Def: $U_n \sim v_n \Leftrightarrow U_n - v_n = o(v_n)$

$$\Leftrightarrow \exists (\varepsilon_n)_{n \in \mathbb{N}} \text{ tq } \begin{cases} U_n = \varepsilon_n v_n \\ \lim_{n \rightarrow +\infty} \varepsilon_n = 1 \end{cases}$$

$U_n \sim v_n \Leftrightarrow \forall \varepsilon > 0, \exists n_0 \in \mathbb{N} \text{ tq } \forall n \geq n_0, |U_n - v_n| \leq \varepsilon |v_n|$

- Si $v_n \neq 0$, à partir d'un certain rang $U_n \sim v_n \Leftrightarrow \lim_{n \rightarrow +\infty} \frac{U_n}{v_n} = 1$

Propriétés

1) $U_n \sim v_n \Rightarrow U_n \sim w_n$

2) $U_n \sim v_n \Leftrightarrow \forall \lambda \neq 0, \lambda U_n \sim \lambda v_n$

3) $U_n \sim v_n \Rightarrow U_n \cdot c_n \sim v_n \cdot c_n$

4) $U_n \sim v_n \Rightarrow U_n^\alpha \sim v_n^\alpha$

5) $U_n \sim v_n \Rightarrow \lim_{n \rightarrow +\infty} U_n = \ell$

6) $\lim_{n \rightarrow +\infty} U_n = \ell \neq 0 \Leftrightarrow U_n \sim \ell$

Relation de dominance

Def: $U_n = O(v_n) \Leftrightarrow$

$$\exists \eta > 0 \text{ tq } |U_n| \leq \eta |v_n|, \forall n \geq n_0$$

- Si $v_n \neq 0$, $U_n = O(v_n) \Leftrightarrow$ la suite $\left(\frac{U_n}{v_n}\right)$ est bornée.

Propriétés

1) $U_n = o(v_n) \Rightarrow U_n = O(v_n)$

2) $U_n \sim v_n \Rightarrow U_n = O(v_n)$

3) $\begin{cases} U_n = O(v_n) \\ v_n = O(w_n) \end{cases} \Rightarrow U_n = O(w_n)$

4) $\begin{cases} U_n = O(w_n) \\ v_n = O(w_n) \end{cases} \Rightarrow U_n + v_n = O(w_n)$

5) $U_n = v_n + o(v_n) \Rightarrow U_n = O(v_n)$

$U_n = O(v_n)$