

Examen de mécanique (Corrigé)

mécanique du point

Barèmes		
(21)	ou	(21)
(7)	ou	(8)

1-1-a $m \vec{a}_{M/R} = q \vec{v} \wedge \vec{B} = q \begin{vmatrix} \vec{v}_x \\ \vec{v}_y \\ \vec{v}_z \end{vmatrix} \wedge \begin{vmatrix} 0 \\ 0 \\ B \end{vmatrix}$ 0,5

$\ddot{x} = \frac{qB}{m} v_y \quad \ddot{y} = -\frac{qB}{m} v_x \quad \ddot{z} = 0$ 3 x 0,5

1-b $\begin{cases} \dot{x} = A \omega \sin \omega t \\ \dot{y} = A \omega \cos \omega t \\ \dot{z} = 0 \end{cases} \Rightarrow \begin{cases} A \omega \sin \omega t = \omega A \sin \omega t \\ A \omega \cos \omega t = -\omega (-A \cos \omega t) \end{cases}$ 0,5
0,5

1-c $\dot{a}t = 0 \quad v_x = -v_0 \quad v_y = 0 \quad v_z = 0$ 1 1 ou 1,5
 $-v_0 = -A \quad 0 = 0 \quad 0 = 0$

1-d $\delta W_B = 0$ car $\vec{F}_B \perp d\vec{OM}$ 0,5
 $d\vec{E}_{B/R} = \delta W \Rightarrow \vec{E}_B = c\vec{v}$ 0,5
 $\vec{E}_B = d\vec{v} = \vec{E}_B(t=0) = \frac{1}{2} m v_0^2 \Rightarrow \|\vec{v}\| = v_0$

1-2-a $x = -\frac{A}{\omega} \sin \omega t + cte \quad v = 0 + cte \quad x = -\frac{v_0}{\omega} \sin \omega t$ 0,5
 $y = -\frac{A}{\omega} \cos \omega t + cte \quad v = -\frac{A}{\omega} + cte \quad y = \frac{v_0}{\omega} (1 - \cos \omega t)$ 0,5

2-b $\frac{x^2}{(v_0/\omega)^2} + \frac{(y - \frac{v_0}{\omega})^2}{(v_0/\omega)^2} = \cos^2 \omega t + \sin^2 \omega t = 1$
 $\frac{x^2}{a^2} + \frac{(y - y_0)^2}{a^2} = 1$ rayon a 1

2-1-a $\ddot{x}_1 = \frac{qB}{m} v_{y1} + \frac{q}{m} E \quad \ddot{y}_1 = -\frac{qB}{m} v_{x1} \quad \ddot{z}_1 = 0$ 1

2-1-b i. $v_{1x}(t=0) = v_{1y}(t=0) = v_{1z}(t=0) = 0$
 $0 = -A_1 + E/B \quad A_1 = E/B$ 0,5
 $0 = 0$
 $0 = D_1$ 0,5

ii. Repère en translation (pas de rotation) 0,5
 $v_x = v_{1x} - v_0 \quad -v_0 \cos \omega t = -\frac{E}{B} \cos \omega t + \frac{E}{B}$
 $\Rightarrow v_0 = E/B$ 0,5

2-2-a $x_1(t) = -a \sin \omega t + a \omega t + cte \quad v = 0 + 0 + cte$
 $y_1(t) = -a \cos \omega t + cte \quad v = -a + cte$
 $v_x(t) = -a \sin \omega t + a \omega t$ 0,5
 $y_1(t) = a(1 - \cos \omega t)$ 0,5

2-b $\|\vec{v}_1\| = \{v_x^2 + v_y^2 + v_z^2\}^{1/2} = a\omega \{(1 - \cos \omega t)^2 + \sin^2 \omega t\}^{1/2}$
 $= a\omega \{4 \sin^4 \omega t/2 + 4 \sin^2 \omega t/2 \cos^2 \omega t/2\}^{1/2}$
 $= a\omega \{4 \sin^2 \omega t/2 (\sin^2 \omega t/2 + \cos^2 \omega t/2)\}^{1/2}$
 $= 2a\omega |\sin \omega t/2| = 2a\omega |\sin \frac{\varphi}{2}|$ 1

mais pour $\varphi/2 = k\pi \quad (\varphi = 2k\pi)$

S'annule pour $\varphi/2 = \frac{1}{2}\pi$ $\varphi = 2\frac{1}{2}\pi$

2-2-c $a_{x1} = a\omega^2 \sin \omega t$ $a_{y1} = a\omega^2 \cos \omega t$ $a_{z1} = 0$

$\|\vec{a}_1\| = a\omega^2$ 0,5

2-2-d $\vec{a}_1 = \vec{a}_T + \vec{a}_n = \frac{d\|\vec{v}_1\|}{dt} \vec{e}_T + \frac{v_1^2}{\rho_c} \vec{e}_n$ 0,5

2-2-e $a_T = a\omega^2 \cos \frac{\varphi}{2}$ ($\dot{\varphi} = \omega$) 0,5

$a^2 \omega^4 = a^2 \omega^4 \cos^2 \frac{\varphi}{2} + a_n^2$

$a_n = a\omega^2 \sin \frac{\varphi}{2}$ 0,5

$\frac{v^2}{\rho_c} = \frac{4a^2 \omega^4 \sin^2 \frac{\omega t}{2}}{\rho_c} = 4a\omega^2 \sin^2 \frac{\omega t}{2}$

$\rho_c = 4a \sin \omega t / 2 = 4a \sin \varphi / 2$

$\rho_c = 0$ ($\varphi = 0$); $4a$ ($\varphi = \pi$); 0 ($\varphi = 2\pi$)

1,5

Mécanique du solide.

⑦ ou ⑧

1-a- $\vec{\Omega}_{D/R} = \dot{\varphi} \vec{e}_z = \omega \vec{e}_z$ Rem: (ω algébrique)

b. $\vec{U}_{A/R} = \vec{U}_{B/R} + \vec{\Omega} \wedge \vec{BA}$ (ou $\vec{AB} \wedge \vec{\Omega}$)

c- $\vec{U}_{J/R} = \vec{U}_{C/R} + \vec{\Omega} \wedge \vec{CJ} = \dot{x} \vec{e}_x + \omega \vec{e}_z \wedge (-a \vec{e}_y)$
 $= (\dot{x} + a\omega) \vec{e}_x$

d- $\vec{U}_{J/R} = \vec{U}_{J/R} = (\dot{x} + a\omega) \vec{e}_x = \vec{0}$ $|\dot{x} + a\omega = 0|$

0,5

2- $I_{xz} = I_{yz} = 0$ car $z = 0$ $I_{xy} = 0$ par symétrie

$I_z = \int_0^a x^2 dm = \int_0^a x^2 \frac{m}{\pi a^2} 2\pi x dx = \frac{2m}{a^2} \int_0^a x^3 dx = \frac{ma^2}{2}$

$I_x = I_y = I_D/2 = \frac{ma^2}{4}$

$\underline{I}_C = \begin{pmatrix} \frac{ma^2}{4} & 0 & 0 \\ 0 & \frac{ma^2}{4} & 0 \\ 0 & 0 & \frac{ma^2}{2} \end{pmatrix}$

3-a- axes // centre de R origine C

b- $\vec{L}^* = \underline{I}_C \vec{\Omega} = \frac{ma^2}{2} \omega \vec{e}_z$

c- $\mathcal{E}_R^* = \frac{1}{2} \vec{L}^* \cdot \vec{\Omega} = \frac{1}{2} \frac{ma^2}{2} \omega^2 = \frac{1}{4} ma^2 \omega^2$

d- $\mathcal{E}_{R/R} = \mathcal{E}_R^* + \frac{1}{2} m v_C^2$
 $= \frac{1}{4} ma^2 \omega^2 + \frac{1}{2} m \dot{x}^2$

$= \frac{3}{4} ma^2 \omega^2$ si seulement sans glissement.

0,5

0,5

0,5

0,5

1

1,5