

Exercice I
 1) a) $\vec{OA} = r \vec{e}_r = r \sin \theta_0 \vec{e}_{x'} + r \cos \theta_0 \vec{e}_z$

$$\vec{v}(A/R') = \left[\frac{d\vec{OA}}{dt} \right]_{R'} = \dot{r} \sin \theta_0 \vec{e}_{x'} + \dot{r} \cos \theta_0 \vec{e}_z \Rightarrow \vec{a}(A/R') = \ddot{r} (\sin \theta_0 \vec{e}_{x'} + \cos \theta_0 \vec{e}_z)$$

b) $\vec{v}(O' \in R' / R) = \vec{0} \Rightarrow$ pas de translation. $\left. \begin{array}{l} \vec{\Omega}(R' / R) = \omega_0 \vec{e}_z \end{array} \right\} \Rightarrow$ Rot. de rot. pure de R' / R .

c) $\vec{v}_e(A, R' / R) = \vec{v}(O' \in R' / R) + \vec{\Omega}(R' / R) \times \vec{OA}$
 $= \omega_0 \vec{e}_z \times (r \sin \theta_0 \vec{e}_{x'} + r \cos \theta_0 \vec{e}_z) = r \omega_0 \sin \theta_0 \vec{e}_{y'}$

$$\vec{a}_e(A, R' / R) = \vec{a}(O' \in R' / R) + \underbrace{\left[\frac{d\vec{\Omega}}{dt} \right]_R}_{=\vec{0}} \times \vec{OA} + \vec{\Omega} \times (\vec{\Omega} \times \vec{OA})$$

$$= \omega_0^2 \vec{e}_z \times (\vec{e}_z \times r \sin \theta_0 \vec{e}_{x'}) = -r \omega_0^2 \sin \theta_0 \vec{e}_{x'}$$

$$\vec{a}_c(A, R' / R) = 2 \vec{\Omega}(R' / R) \times \vec{v}(A / R')$$

 $= 2 \omega_0 \vec{e}_z \times (r \sin \theta_0 \vec{e}_{y'}) = 2 \omega_0 r \sin \theta_0 \vec{e}_{x'}$

1) $\vec{v}(A/R) = \vec{v}(A/R') + \vec{v}_e(A, R' / R)$
 $= (\dot{r} \sin \theta_0 \vec{e}_{x'} + \dot{r} \cos \theta_0 \vec{e}_z) + r \omega_0 \sin \theta_0 \vec{e}_{y'}$

$$\vec{a}(A/R) = \vec{a}(A/R') + \vec{a}_e(A, R' / R) + \vec{a}_c(A, R' / R)$$

 $= \ddot{r} (\sin \theta_0 \vec{e}_{x'} + \cos \theta_0 \vec{e}_z) - r \omega_0^2 \sin \theta_0 \vec{e}_{x'} + 2 \omega_0 r \sin \theta_0 \vec{e}_{x'}$

Directement:

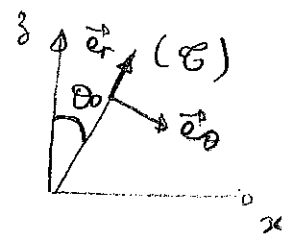
$$\vec{v}(A/R) = \left[\frac{d\vec{OA}}{dt} \right]_R = \dot{r} \sin \theta_0 \vec{e}_{x'} + r \sin \theta_0 \underbrace{\left[\frac{d\vec{e}_{x'}}{dt} \right]_R}_{\omega_0 \vec{e}_{y'}} + \dot{r} \cos \theta_0 \vec{e}_z \quad \text{CQFD.}$$

$$\vec{a}(A/R) = \left[\frac{d\vec{v}(A/R)}{dt} \right]_R = \ddot{r} \sin \theta_0 \vec{e}_{x'} + \dot{r} \sin \theta_0 \underbrace{\left[\frac{d\vec{e}_{x'}}{dt} \right]_R}_{\omega_0 \vec{e}_{y'}} + \dot{r} \omega_0 \sin \theta_0 \vec{e}_{y'} + r \omega_0 \sin \theta_0 \underbrace{\left[\frac{d\vec{e}_{y'}}{dt} \right]_R}_{-\omega_0 \vec{e}_{x'}} + \ddot{r} \cos \theta_0 \vec{e}_z$$

CQFD.

$$\vec{\Omega}(R'/R) = \omega_0 \vec{e}_3$$

II) Étude du mt dans R' , non galiléen $\Rightarrow \vec{F}_i^0$



a) $\vec{P} = -mg\vec{e}_z$

$\vec{T}$, $\perp$ tige car pas de frottements $\Rightarrow \vec{T} = T_x \vec{e}_x + T_y \vec{e}_y$

$\vec{F}_{ic} = -2m\vec{\Omega}(R'/R) \times \vec{v}(M/R) = -2m\omega_0 \vec{e}_z \times \dot{r} \vec{e}_r = -2m\omega_0 \dot{r} \sin\theta_0 \vec{e}_y$

$\vec{F}_{ie} = -m \left[\underbrace{\vec{a}(O' \in R'/R)}_{O \equiv O' \text{ fixe}} + \underbrace{\left[\frac{d\vec{\Omega}(R'/R)}{dt} \right]_R}_{=0, \text{ car } \omega_0 = \text{cte}} \times \vec{OM} + \vec{\Omega}(R'/R) \times [\vec{\Omega}(R'/R) \times \vec{OM}] \right]$

$\Rightarrow \vec{F}_{ie} = -m\omega_0^2 r \vec{e}_z \times (\vec{e}_z \times \vec{e}_r) = m\omega_0^2 r \sin\theta_0 \vec{e}_{x'}$

b) $d\mathcal{E}_{P_{ie}} = -\delta W_{ie} = -\vec{F}_{ie} \cdot d\vec{l} = -(m\omega_0^2 \sin\theta_0) r dx'$ $x' = r \sin\theta_0 \Rightarrow dx' = \sin\theta_0 dr$

$\Rightarrow \mathcal{E}_{P_{ie}}(r) = (m\omega_0^2 \sin^2\theta_0) \frac{r^2}{2} + \text{cte} \Rightarrow \mathcal{E}_{P_{ie}}(r) = -\frac{m\omega_0^2 \sin^2\theta_0}{2} r^2$

c) $\vec{T} \perp$ tige $\Rightarrow \mathcal{E}_p(\vec{T}) = 0$

$\vec{F}_{ic} \perp$ vitesse $\Rightarrow \vec{F}_{ic}$ ne travaille pas.

Donc $\mathcal{E}_p(\Sigma \vec{F}) = \mathcal{E}_p(\vec{P} + \vec{F}_{ie}) = +mg r \cos\theta_0 - \frac{m\omega_0^2 \sin^2\theta_0}{2} r^2$

d) $E_q \Rightarrow \left(\frac{d\mathcal{E}_p}{dr} \right)_{re} = 0 = mg \cos\theta_0 - m\omega_0^2 \sin^2\theta_0 r_e$

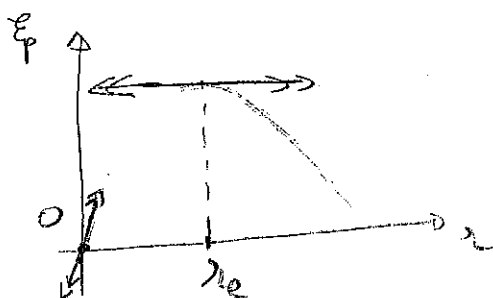
$\Rightarrow r_e = \frac{g \cos\theta_0}{\omega_0^2 \sin^2\theta_0}$

Eq stable $\Leftrightarrow \left(\frac{d^2\mathcal{E}_p}{dr^2} \right)_{re} > 0$

or, $\left(\frac{d^2\mathcal{E}_p}{dr^2} \right)_{re} = -\left(\frac{m\omega_0^2 \sin^2\theta_0}{\text{cte}} \right) < 0$

$\Rightarrow r_e$ est une position d'équilibre instable.

e)



$\left(\frac{d\mathcal{E}_p}{dr} \right)_{r=0} = mg \cos\theta_0 > 0$

Donc, si $r_0 < r_e$, la masse m retombe en O.

si $r_0 > r_e$, la masse m remonte la tige, jusqu'à sortir.

(de manière à tendre vers 1 énergie minimale).