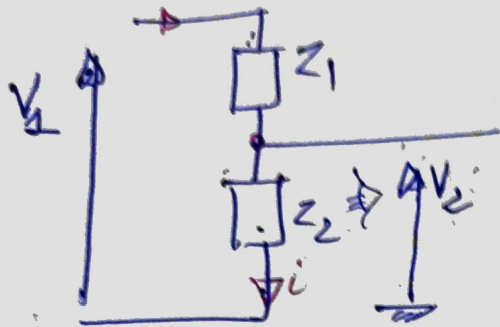


Ex 1

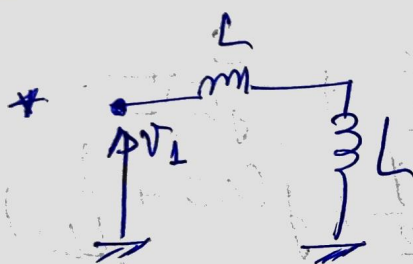


$$V_2 = (Z_1 + Z_2) i$$

$$V_2 = Z_2 i$$

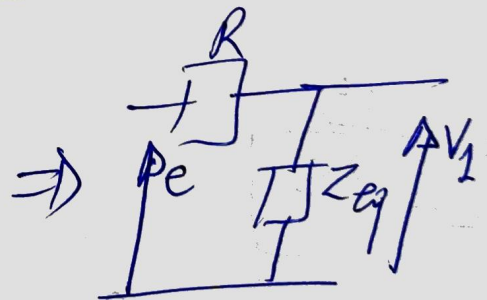
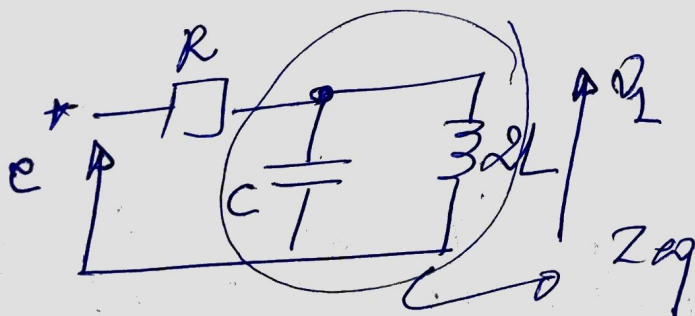
$$\frac{V_2}{V_1} = \frac{Z_2}{Z_1 + Z_2}$$

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_1$$



$$S = \frac{Z_L}{Z_L + Z_L} V_1 = \frac{1}{2} V_1$$

$$\Rightarrow V_1 = 2S$$

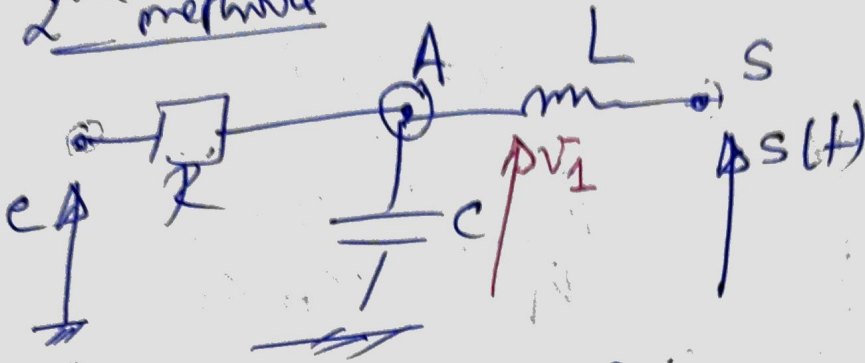


$$V_1 = \frac{Z_{eq}}{Z_{eq} + R} e = \frac{1}{1 + R Y_{eq}} e$$

$$Y_{eq} = \frac{1}{Z_{eq}} = \text{admittance} ; \frac{1}{Z_{eq}} = j\omega C + \frac{1}{2jL\omega}$$

$$\frac{1}{R} = G = \text{conductance (S)} \quad \text{Siemens} \quad \Omega^{-1}$$

2^e méthode



$$V_A = \frac{\frac{e}{R} + \frac{0}{Z_C} + \frac{s}{Z_L}}{\left(\frac{1}{R} + \frac{1}{Z_C} + \frac{1}{Z_L} \right)} \Rightarrow$$

$$V_A - V_{\max} = V_1 = \frac{\frac{e}{R} + \frac{s}{j\omega L}}{\left(\frac{1}{R} + j\omega + \frac{1}{j\omega} \right)}$$

$$\boxed{V_1 = 2s}$$

$$2s \left[\frac{1}{R} + j\omega + \frac{1}{j\omega} \right] = \frac{e}{R} + \frac{s}{j\omega L}$$

$$s \left[\frac{2}{R} + 2j\omega + \frac{1}{j\omega} \right] = \frac{e}{R} \times \left. \begin{matrix} R \\ (2) \end{matrix} \right\}$$

$$s \left[2 + 2j(RC)\omega + \frac{R}{jL\omega} \right] = e$$

$\tau = RC$ $\downarrow$ s^{-1}
 pe. cm

$$\frac{s}{e} = \frac{1}{2 + 2jRC\omega + \frac{R}{jL\omega}}$$

$$= \frac{H_0 \cdot 2m \cdot j\omega_0}{1 - \frac{\omega^2}{\omega_0^2} + 2m j \frac{\omega}{\omega_0} \cdot \frac{jL\omega}{R}}$$

$$= \frac{\frac{2jL\omega}{R} - 2LC\omega^2 + 1}{1}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{2LC}} \quad \left| \quad \left(\frac{2m}{\omega_0} \right) = \frac{2L}{R} \right.$$

$$\left(\frac{2m}{\omega_0} \right) H_0 = \frac{L}{R} \Rightarrow \frac{2L}{R} H_0 = \frac{L}{R}$$

$H_0 = \frac{1}{2}$

$$\boxed{M = \frac{1}{R} \sqrt{\frac{L}{2C}}}$$

$$2) G(\omega) = |H(j\omega)|$$

$$\varphi(\omega) = \arg[H(j\omega)]$$

$$\varphi = \arg(N) - \arg(D)$$

$$= +\frac{R}{2} - \operatorname{arctg}\left[\frac{2\omega/\omega_0}{1 - \omega^2/\omega_0^2}\right]; \omega < \omega_0$$

$$= \frac{R}{2} - \left[\operatorname{arctg}\left(\frac{2\omega/\omega_0}{1 - \omega^2/\omega_0^2}\right) + \pi\right]; \omega > \omega_0$$

$$z = x + jy \Rightarrow \arg z = \begin{cases} \operatorname{arctg}\left(\frac{y}{x}\right); x > 0 \\ \pi + \operatorname{arctg}\left(\frac{y}{x}\right); x < 0 \end{cases}$$

$$G_{dB} = 20 \log G(\omega) \quad \left| \quad P_e = \frac{U^2}{R} \right.$$

$$G_{dB} = 10 \log \left(\frac{P_s}{P_e} \right) \quad (4)$$

$$\underline{\omega \ll \omega_0} \quad H \approx \frac{j \frac{2m H_0 \omega}{\omega_0}}{1} = \underline{\underline{\frac{s}{e}}}$$

$\boxed{\omega \rightarrow \infty: \rho \rightarrow \gamma}$

$$\lim_{x \rightarrow \infty} (1 + j(x) - x^2) \approx 1$$

$$x \rightarrow \infty$$

$$Z_L = 10^{-3} + j 10^4 \approx j 10^4$$

$$\underline{s} = \frac{2m H_0}{\omega_0} (j \omega \underline{e}) \quad \hookrightarrow \frac{d\underline{e}}{dt}$$

$$G(\omega \ll \omega_0) \approx \frac{2m H_0 \omega}{\omega_0}$$

$$G_{dB} = 20 \log(2m H_0) + 20 \left(\log \frac{\omega}{\omega_0} \right)$$

$$x = \frac{\omega}{\omega_0} = \text{pulsation r\u00e9duite}$$

$$\gamma = A + 20X$$

droite de pente + 20 dB/d\u00e9cade

$$* \varphi(\omega \ll \omega_0) \approx +\frac{\pi}{2}$$

$$\underline{\omega \gg \omega_0} \quad \underline{H} \approx \frac{\lim H_0 j\omega/\omega_0}{-\frac{\omega^2}{\omega_0^2}}$$

$$H \approx \frac{\lim H_0 \omega_0}{(j\omega)} = \frac{\underline{S}}{\underline{C}}$$

$$\underline{S} = \lim H_0 \omega_0 \left(\frac{\underline{e}}{j\omega} \right) \rightarrow \int \underline{e} dt$$

$$\underline{e}(t) = E e^{j\omega t}$$

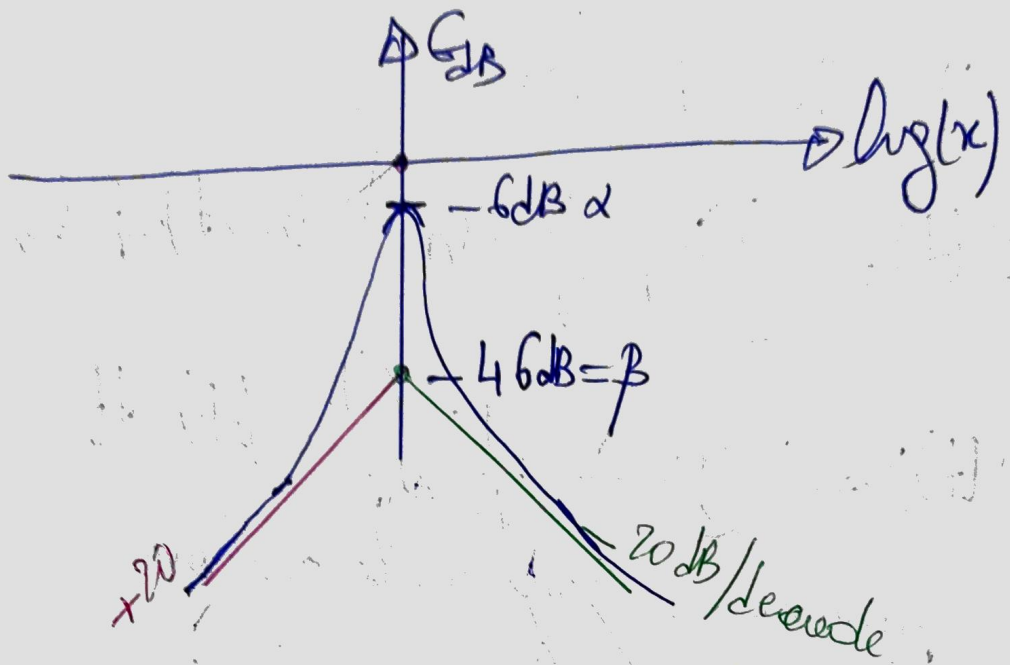
$$\frac{d\underline{e}}{dt} = j\omega \underline{e} \quad \therefore \int \underline{e} dt = \frac{\underline{e}}{j\omega}$$

$$* \frac{\underline{e}}{-\omega^2} = \iint \underline{e} dt = \text{double intégration}$$

$$-\omega^2 \underline{e} = \frac{d^2 \underline{e}}{dt^2} = \text{ " dérivations}$$

$$* \varphi(\omega \gg \omega_0) = -\frac{\pi}{2}$$

$$G_B = 20 \log(\lim H_0) - 20 \log\left(\frac{\omega}{\omega_0}\right)$$



$$(G_{dB})_{\text{asymptot}} = 20 \log(\lim_{\omega \rightarrow \omega_0} H_0); \quad \omega = \omega_0$$

$$= -46 \text{ dB} = \beta$$

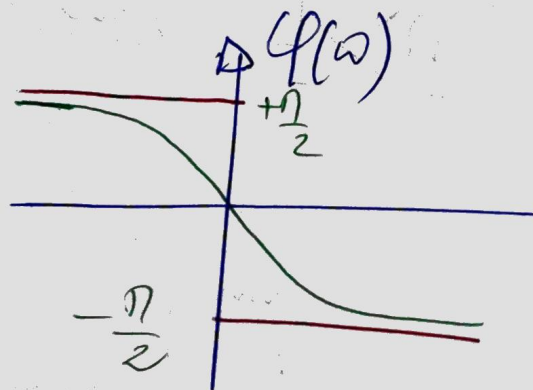
$$\underline{H} = \left(\frac{\lim_{\omega \rightarrow \omega_0} H_0 j}{1 + 2m j - 1} \right) = \underline{H}; \quad \omega \neq \omega_0$$

$$\underline{H} = H_0 = \frac{1}{2}$$

$$\arg \underline{H}(\omega = \omega_0) = 0$$

$$\Delta G_{dB} = -20 \log 2$$

$$= -6 \text{ dB} = \alpha$$



Joseph - Fourier

$$e(t) = \left(\frac{a_0}{2}\right) + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

$$\langle e(t) \rangle = \frac{a_0}{2} = \frac{1}{T} \int_0^T e(t) dt$$

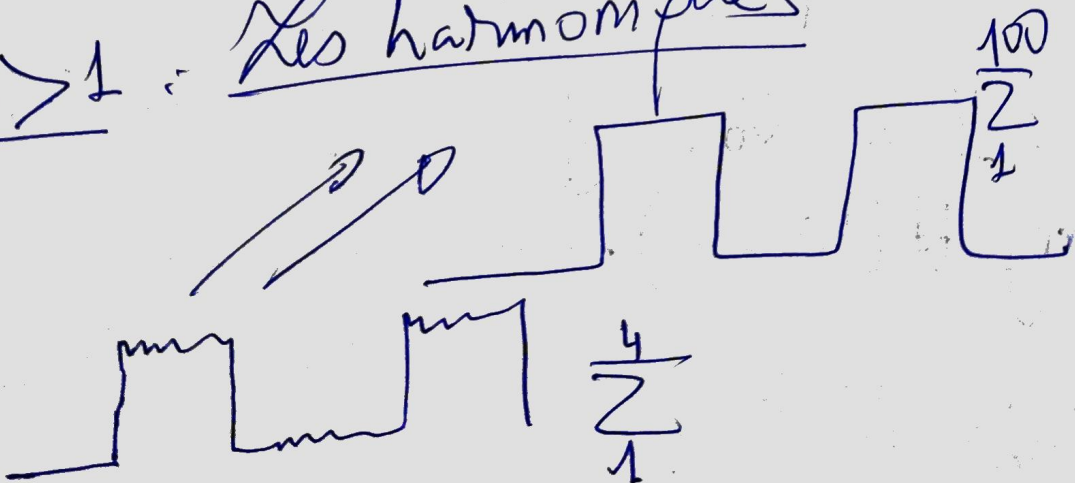
moyenne = composante continue

$$a_n = \frac{2}{T} \int_0^T e(t) \cos(n\omega t) dt$$

$$b_n = \frac{2}{T} \int_0^T e(t) \sin(n\omega t) dt$$

$n=1$: fondamentale $\Rightarrow$ fréquence f

$n > 1$: Les harmoniques



$$\begin{cases} a_n = c_n \cos \varphi_n \\ b_n = -c_n \sin \varphi_n \end{cases} \quad \sqrt{a_n^2 + b_n^2} = c_n$$

$$e(t) = c_0 + \sum c_n \cos(n\omega t + \varphi_n)$$

impair $a_n = 0$, pair $b_n = 0$

$$\underline{e} = c_0 + \sum c_n \exp j(n\omega t + \varphi_n)$$

↓ filtre

$$\underline{s} = c_0 \underline{H}(\omega=0) + \sum c_n \underline{H}(n\omega) \exp j(n\omega t + \varphi_n)$$

$\swarrow \searrow$
 $G(n\omega) \quad e^{j\Phi_n}$

$$\underline{s} = c_0 \underline{H}(0) + \sum c_n G(n\omega) \exp j(n\omega t + \varphi_n + \Phi_n)$$

$$s(t) = c_0 \underline{H}(0) + \sum c_n \underline{G}(n\omega) \cos(n\omega t + \varphi_n + \Phi_n)$$

$$\Phi_n = \arg \underline{H}(jn\omega).$$

$$4) e_2(t) = E_0 + E_1 \cos(\omega_1 t)$$

analyse : un G.B.F + offset

$$5) s_2(t) = E_0 \cdot H(0) + E_1 \cdot G(\omega_1) \cos(\omega_1 t + \Phi_1)$$

$$H(0) = 1$$

$$H_0 = \left(\frac{1}{2}\right)$$

Lo fréquence

$$H_0 \approx \omega_1 / \omega_0$$

$$\text{or } G(\omega_1) = \frac{1}{\sqrt{\left(1 - \frac{\omega_1^2}{\omega_0^2}\right)^2 + \left(\frac{2m\omega_1}{\omega_0}\right)^2}}$$

$$\omega_1 = \omega_0 \Rightarrow H(\omega_1 = \omega_0) = H_0 = \frac{1}{2}$$

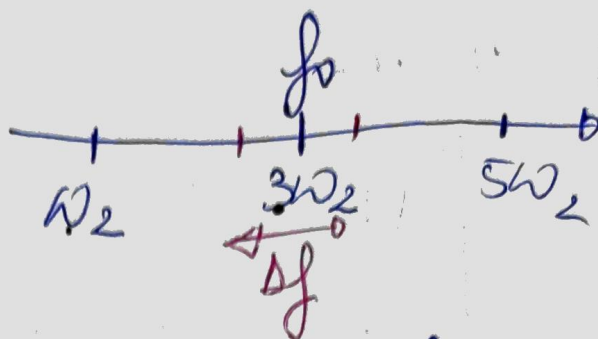
$$G(\omega_1 = \omega_0) = \frac{1}{2}; \quad \Phi(\omega_1 = \omega_0) = 0$$

$$s_2(t) = E_1 \times \frac{1}{2} \cos(\omega_1 t)$$

$$6) T = 6\pi\sqrt{2LC} \Rightarrow \omega_2 = \frac{2\pi}{T}$$

$$\omega_2 = \frac{2\pi}{6\pi\sqrt{2LC}} = \frac{1}{3}\omega_0$$

$$\boxed{\omega_0 = 3\omega_2}$$



$$E_{eff}^2 = \frac{1}{T} \int_0^T E^2(t) dt = \langle E^2(t) \rangle$$

$$= \frac{1}{T} \int_0^{\frac{T}{2}} E_{2\omega}^2 dt \times 2 = E_{2\omega}^2$$

$$\boxed{E_{eff} = E_{2\omega}}$$

$$\underline{Pg}: \sum_0^{\infty} \frac{4E_{2\omega}}{\pi} \frac{\sin((2p+1)\omega_2 t)}{(2p+1)} = e_2(t)$$

$$e_2(t) = C_0 + \sum C_n \cos(n\omega t + \phi_n)$$

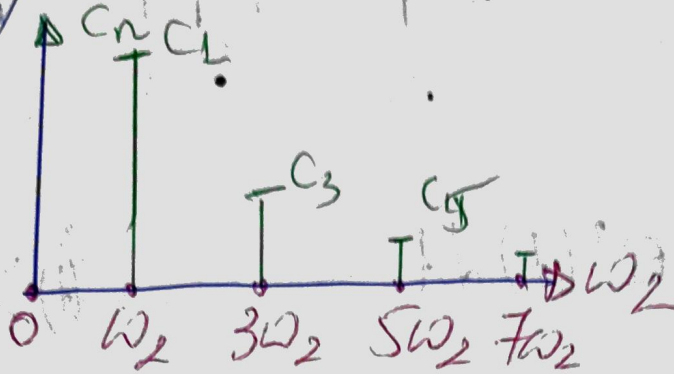
$$E_{eff}^2 = C_0^2 + \frac{\sum C_n^2}{2} \quad \underline{\text{Parseval}}$$

$$= 0 + \frac{1}{2} \sum_0^{\infty} \left(\frac{4E_{2\omega}}{\pi} \right)^2 \frac{1}{(2p+1)^2}$$

$$= \frac{1}{2} \left(\frac{4E_{2\omega}}{\pi} \right)^2 \left(\sum_0^{\infty} \frac{1}{(2p+1)^2} \right)$$

on done $\rightarrow \frac{\pi^2}{8}$

7) spectre



$$C_1 = \frac{4\bar{E}_{20}}{\pi}$$

$$C_3 = \frac{4E_{20}}{3\pi}$$

$$C_3 = \frac{1}{3} C_1$$

$$C_5 = \frac{4E_{20}}{5\pi} = \frac{1}{5} C_1$$

$$\rightarrow C_{2p+1} \sim \frac{1}{2p+1}$$



$$\leadsto C_{2p+1} \leadsto \frac{1}{(2p+1)^2}$$

8)

$$E_1 = \frac{4E_{20}}{\pi}$$

$$E_3 = \frac{4\bar{E}_{20}}{3\pi}$$

$$E_5 = \frac{4E_{20}}{5\pi}$$

$$S_1 = \frac{4E_{20}}{\pi} G(\omega_2)$$

$$S_3 = \frac{4\bar{E}_{20}}{3\pi} G(3\omega_2)$$

$$S_5 = \frac{4E_{20}}{5\pi} G(5\omega_2)$$

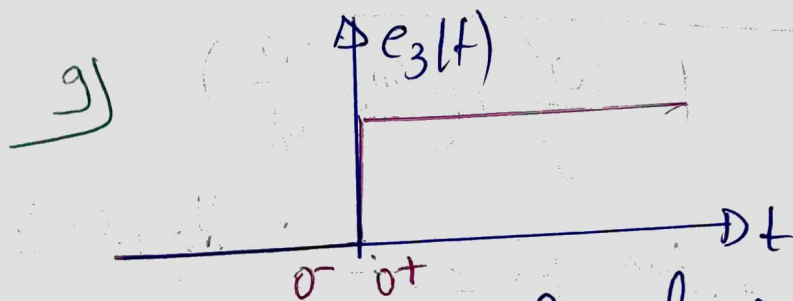
$$\underline{or} \quad \omega_2 = \frac{\omega_0}{3} \Rightarrow \boxed{\omega_0 = 3\omega_2}$$

$$G(3\omega_2 = \omega_0) = H_0 = \frac{1}{2}$$

$$G(\omega_2) = \frac{H_0 \cdot 2m \cdot \frac{1}{3}}{\sqrt{\left(1 - \frac{1}{9}\right)^2 + \left(2m \cdot \frac{1}{3}\right)^2}} \rightarrow 0$$

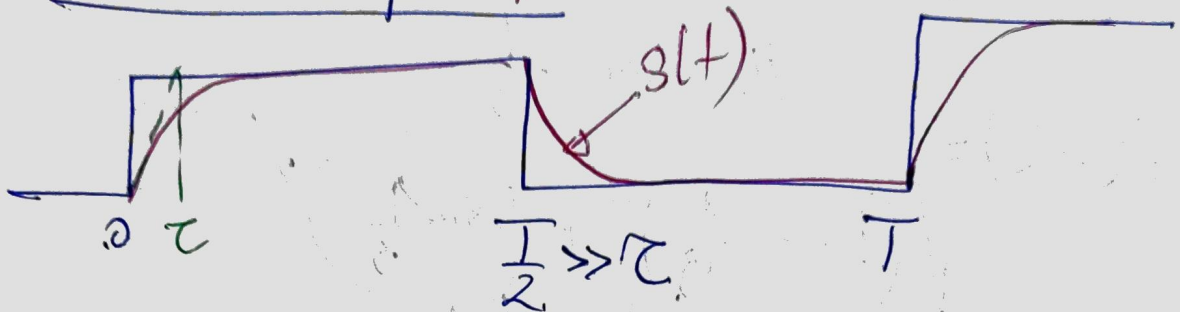
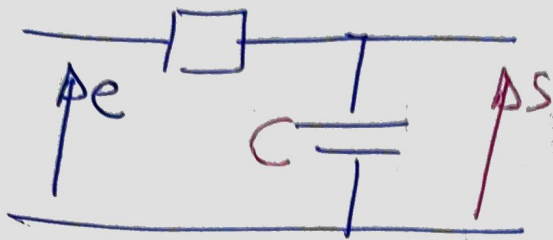
$$G(5\omega_2) = \frac{H_0 \cdot 2m \cdot \frac{5}{3}}{\sqrt{\left(1 - \frac{25}{9}\right)^2 + \left(2m \cdot \frac{5}{3}\right)^2}} \rightarrow 0$$

$$\Rightarrow S_2(t) \approx \frac{4E_{\omega_0}}{3\pi} \times \frac{1}{2} \cos(3\omega_2 t) \quad \text{Sin} \quad \rightarrow \text{tripler}$$



sert à identifier la nature du filtre

en effet



$$t \rightarrow 0, \quad s(t \rightarrow 0) \approx 0$$

$$t \rightarrow \infty, \quad s(t \rightarrow \infty) \approx e(t)$$

filtre passe-bas

$$\Delta t \cdot \Delta \omega = ds$$

$$10) \quad \underline{H} = \frac{s}{e} = \frac{2m \text{ k}\Omega j\omega/\omega_0}{1 - \frac{\omega^2}{\omega_0^2} + (2m j\frac{\omega}{\omega_0})}$$

$$s - \frac{\omega^2 s}{\omega_0^2} + \frac{2m}{\omega_0} (j\omega s) = \frac{2m \text{ k}\Omega}{\omega_0} j\omega e$$

$$s + \frac{1}{\omega_0^2} \frac{d^2 s}{dt^2} + \frac{2m}{\omega_0} \frac{ds}{dt} = \frac{2m \text{ k}\Omega}{\omega} \frac{de}{dt}$$

edhelun

$$\omega_0 = \frac{1}{\sqrt{2LC}} ; \quad \frac{2m}{\omega_0} = \frac{2L}{R}$$

$$(2LC) \frac{d^2 s}{dt^2} + \left(\frac{2L}{R}\right) \frac{ds}{dt} + s = 0$$

$$(2LC) r^2 + \left(\frac{2L}{R}\right) r + 1 = 0$$

$$\Delta = \left(\frac{2L}{R}\right)^2 - 8LC \stackrel{\text{AN}}{\approx} -8LC < 0$$

régime pseudo-périodique

$$r_1 = -\frac{1}{2RC} + j \frac{1}{\sqrt{2LC}} = -\frac{1}{\tau} + j\omega_0$$

$$r_2 = -\frac{1}{2RC} - j \frac{1}{\sqrt{2LC}} = -\frac{1}{\tau} - j\omega_0$$

$$s_3(t) = \exp\left(-\frac{t}{\tau}\right) \left[A_1 \cos(\omega_0 t) + A_2 \sin(\omega_0 t) \right]$$

$$\text{ou} = \exp\left(-\frac{t}{\tau}\right) \left[C_1 \cos(\omega_0 t + \varphi) \right]$$

* $\Delta = 0$: critique ; $r_1 = r_2 = -\frac{1}{\tau}$

$$s_3(t) = (A_1 t + A_2) e^{-t/\tau}$$

* $\Delta > 0$: apériodique

$$r_1 = -\frac{1}{\tau} + \gamma$$

$$r_2 = -\frac{1}{\tau} - \gamma$$

$$s_3(t) = e^{-t/\tau} [A_1 e^{\gamma t} + A_2 e^{-\gamma t}]$$

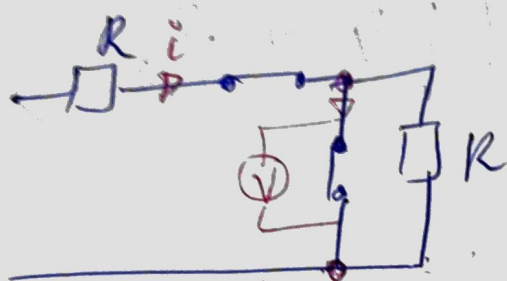
$$= e^{-t/\tau} [c_1 \text{ch}(\gamma t) + c_2 \text{sh}(\gamma t)]$$

Ex 2

$\omega \rightarrow 0$ $Z_C = \frac{1}{j\omega} \rightarrow \infty$; 

$\Rightarrow V_s = 0$ (B.F) $\left\{ \begin{array}{l} \text{pass - band} \\ \text{" bande} \end{array} \right.$

$\omega \rightarrow \infty$; $Z_C \rightarrow 0$ 
 $Z_L = j\omega L \rightarrow \infty$; 



$S = 0$

pass - bande

2) $H_0 = Q = \frac{1}{3}$; $\omega_0 = \frac{1}{RC}$

3) $G(\omega) = \frac{G_{max}}{\sqrt{2}} \Rightarrow \frac{H_0}{\sqrt{1 + Q^2(x - \frac{1}{x})^2}} = \frac{H_0}{\sqrt{2}}$

$Q^2 \left(\frac{x^2 - 1}{x} \right)^2 = 1 \Rightarrow \boxed{x^2 \mp \frac{x}{Q} - 1 = 0}$

$\Delta = \left(\frac{1}{Q^2} + 4 \right) > 0$

$x_{c1} = -\frac{1}{2Q} + \frac{\sqrt{\Delta}}{2}$
 $x_{c2} = -\frac{1}{2Q} - \frac{\sqrt{\Delta}}{2}$

$$x_{C_2} - x_{C_1} = \frac{f_{C_2} - f_{C_1}}{f_0} = \frac{1}{Q}$$

$$Q = \frac{f_0}{\Delta f}$$

donc

$$f_{C_1} = f_0 \left[-\frac{1}{2Q} + \frac{1}{2} \sqrt{4 + \frac{1}{Q^2}} \right]$$

$$f_{C_2} = f_0 \left[+\frac{1}{2Q} + \frac{1}{2} \sqrt{4 + \frac{1}{Q^2}} \right]$$

si $Q \gg 1$

$$f_{C_1} = f_0 \left[1 - \frac{1}{2Q} \right]$$

$$f_{C_2} = f_0 \left[1 + \frac{1}{2Q} \right]$$

$$f_{C_2} - f_0 = f_{C_2} - f_{C_1} = f_0 + \frac{\Delta f}{2}$$

$$f_0 - \frac{\Delta f}{2}$$

$$5) \underline{AN} \quad \omega_0 = \frac{1}{RC} = 2000 \text{ rad/s.}$$

dimension $\left\{ \begin{array}{l} \text{Longueur} \\ \text{Masse} \rightarrow M \end{array} \right.$

L

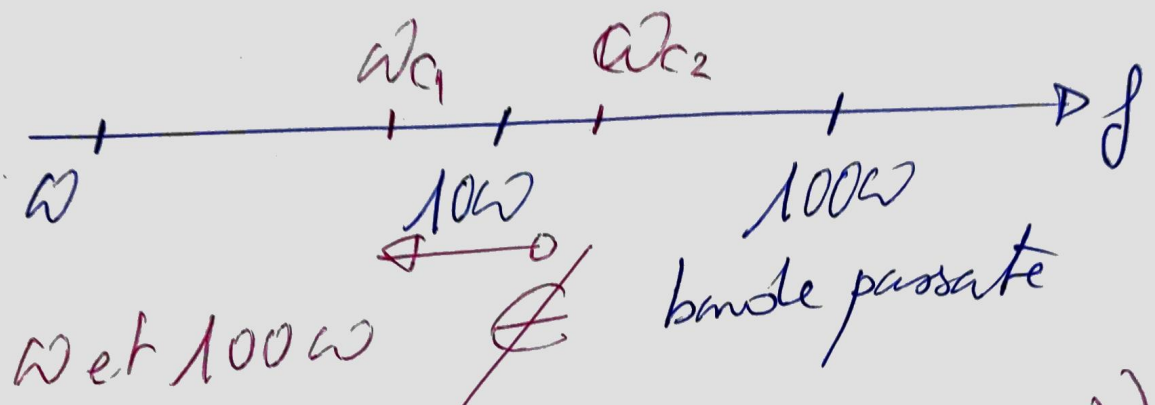
M

Temps $\rightarrow T$

Température $\rightarrow \theta$

$$e(t) = V_0 + V_0 \cos(\omega t) + V_0 \cos(10\omega t) + V_0 \cos(100\omega t)$$

or $\omega_0 = 10\omega$; $\omega = 200 \text{ rad/s}$



$$s(t) = V_0 G(10\omega) \cos(10\omega t + \Phi(10\omega))$$

or $H(j\omega_0) = H(j10\omega) = H(j\alpha=1)$

$$= H_0 = \frac{1}{3} \Rightarrow \Phi(10\omega) = 0$$

$$s(t) = V_0 \frac{1}{3} \cos(10\omega t).$$