

Ex 5

$$H = \frac{H_0}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$Q=50, \quad f_0 = 500 \text{ Hz}$$

$$3) f = 500 \text{ Hz} = f_0$$
$$\Delta f = \frac{f_0}{Q} = \frac{500}{50}$$

$$f_{c1} = 500 - 5 = 495 \text{ Hz}$$

$$f_{c2} = 500 + 5 = 505 \text{ Hz}$$

$$3 \text{ dB passband } [f_{c1}, f_{c2}] \Rightarrow |H| = \frac{2E \times 1 \cos(\omega t + \phi)}{\pi}$$

$$4) \frac{S_1}{S_3} \leq 10\% = \frac{10}{100} \Rightarrow \frac{S_3}{S_1} \geq 10.$$

$$S_1 = \frac{2E}{\pi} G(f)$$

$$S_3 = \frac{2E}{3\pi} G(3f)$$

$$f_0 = 3 \cdot f$$

$$G(nf) = \frac{1}{\sqrt{1 + Q^2 \left(\frac{n f}{f_0} - \frac{f_0}{n f} \right)^2}}$$

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$$\Delta f = \frac{h}{Q} = \frac{500}{50}$$

$$3) \text{ et } 5 \text{ f} \notin [f_{c1}, f_{c2}] \Rightarrow s(t) = \frac{2E \times 1 \cos(\omega t + \phi)}{\pi}$$

$$4) \frac{S_1}{S_3} \leq 10\% = \frac{10}{100} \Rightarrow \frac{S_3}{S_1} \geq 10.$$

$$S_1 = \frac{2E}{\pi} G(f)$$

$$f_0 = 3 \cdot f$$

$$S_3 = \frac{2E}{3\pi} G(3f)$$

$$G(nf) = \frac{1}{\sqrt{1 + Q^2 \left(\frac{nf}{f_0} - \frac{f_0}{nf} \right)^2}}$$

$$G(3f) = 1$$

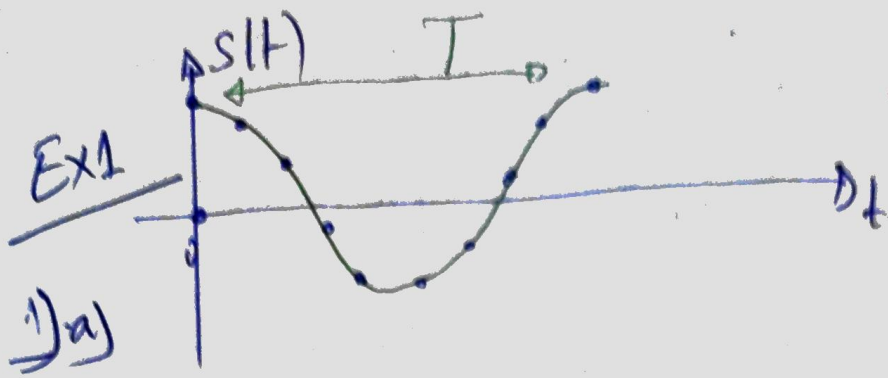
$$G(f) = \frac{1}{\sqrt{1 + Q^2 \left(\frac{1}{3} - 3\right)^2}}$$

$$= \frac{1}{\sqrt{1 + Q^2 \frac{64}{9}}}$$

$$\frac{S_3}{S_1} = \frac{28}{3R} \times 1 \times \frac{\sqrt{1 + \frac{64}{9} Q^2}}{\frac{28}{R}} \geq 10$$

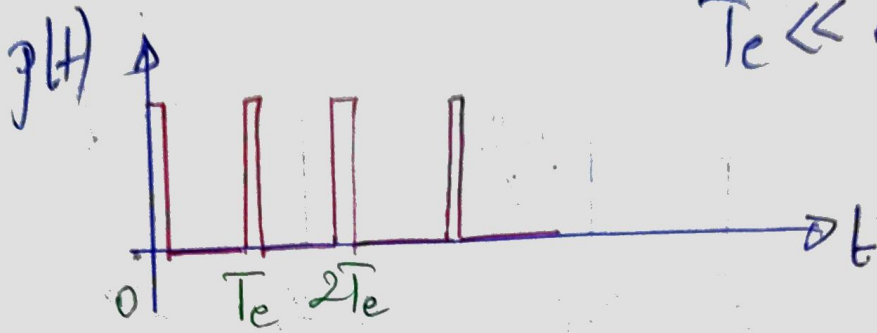
$$= \frac{1}{3} \sqrt{1 + \frac{64}{9} Q^2} \geq 10$$

$$Q \geq 114$$

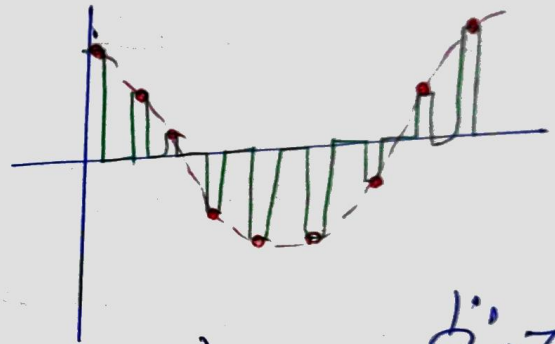


$$s(t) = S_0 \cos(\omega t)$$

$$T_e \ll T$$

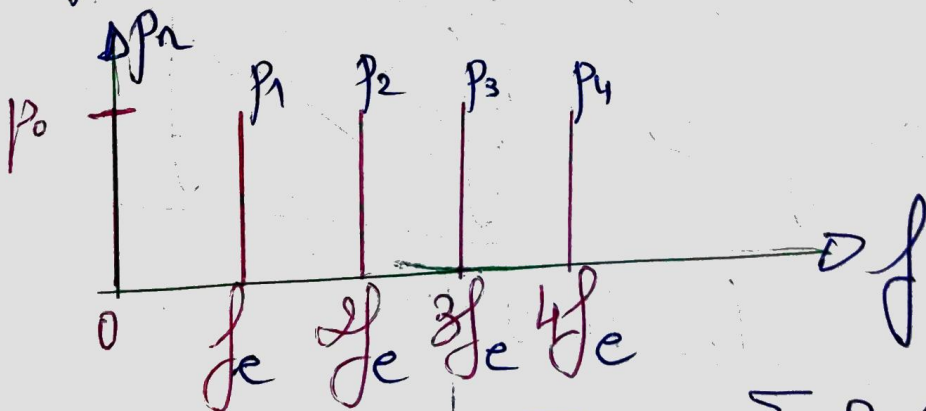


$$s^*(t) = s(t) \cdot p(t)$$



b)

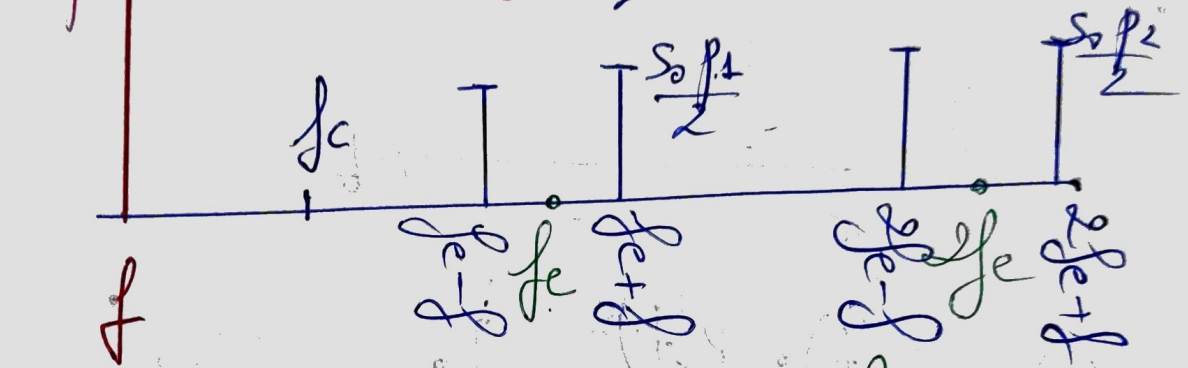
$$p(t) = p_0 + \sum_{n=1}^{\infty} p_n \cos(n\omega_e t)$$



c)

$$s^* = S_0 \cos(\omega t) \left[p_0 + \sum p_n \cos(n\omega_e t) \right]$$

$$\begin{aligned}
 S^* &= S_0 p_0 \cos(\omega t) + \sum p_n S_0 \cos(\omega t) \cos(n\omega_e t) \\
 &= S_0 p_0 \cos(\omega t) + \sum \frac{p_n S_0}{2} \cos(n\omega_e + \omega)t \\
 &\quad + \sum_{n=1}^{\infty} \frac{S_0 p_n}{2} \cos(n\omega_e - \omega)t
 \end{aligned}$$



pour récupérer le signal "f"
 filtre passe-bas (2^{ème} ordre)
 -40dB/décad

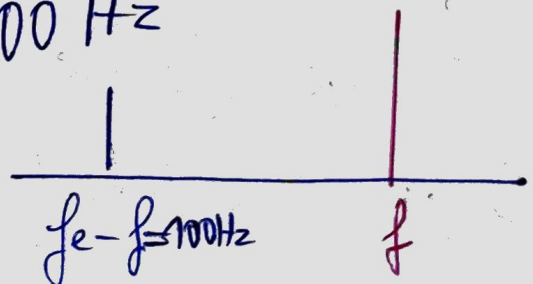
$$f < f_c < f_c - f$$

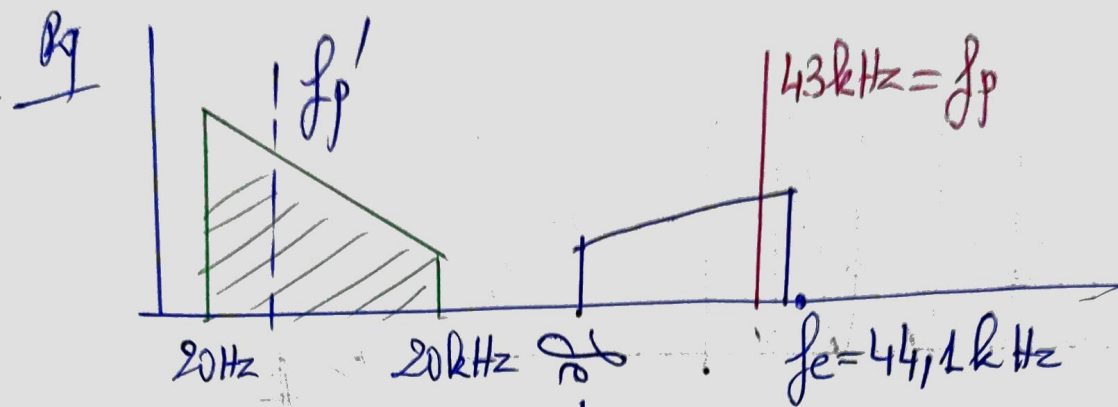
$$f < f_c - f \Rightarrow$$

$$\boxed{f_c > 2f}$$

$$f = 300 \text{ Hz}, \quad f_c = 1000 \text{ Hz}$$

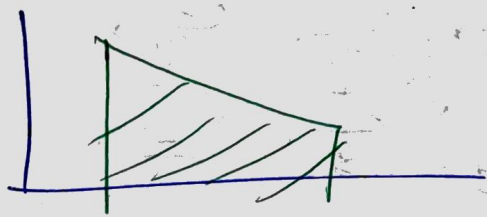
$$f_c - f = 100 \text{ Hz}$$





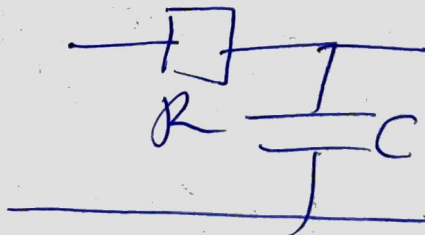
$$f_p' = f_c - f_{\text{max}} = 44.1 - 43 = 1100 \text{ Hz}$$

on utilise un filtre passe bas $f_c \ll f_p$



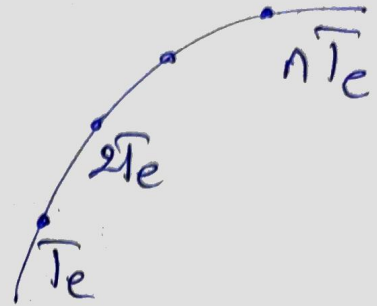
$$\underline{\text{Ex 2}} \quad \underline{H} = \frac{1}{1 + j\omega\tau} ; \quad \tau = \frac{1}{\omega_0} = RC$$

$$\frac{V_s}{V_e} = \frac{Z_c}{Z_c + R}$$



$$\frac{s}{e} = \frac{1}{1+j\omega Z} \Rightarrow$$

$$\boxed{e = s + Z \frac{ds}{dt}}$$



$$e(t) \longrightarrow e(t=nT_e) = e_n$$

$$s(t) \longrightarrow s(t=nT_e) = s_n$$

$$\frac{ds(t)}{dt} = \frac{s(t+dt) - s(t)}{dt}$$

$$= \frac{s(nT_e + T_e) - s(nT_e)}{T_e}$$

$$\boxed{\frac{ds}{dt} \longrightarrow \frac{s_{n+1} - s_n}{T_e}}$$

$$e_n = s_n + Z \left(\frac{s_{n+1} - s_n}{T_e} \right)$$

$$\text{or } s_n = S e^{j\omega n T_e}$$

$$s_{n+1} = s_n e^{j\omega T_e}$$

$$H_{\text{num}} = \frac{s_n}{e_n}$$

$$\underline{e}_n = \underline{s}_n + \frac{\tau}{T_e} (e^{j\omega T_e} - 1) \underline{s}_n$$

$$\frac{\underline{s}_n}{\underline{e}_n} = \frac{1}{1 + \frac{\tau}{T_e} (e^{j\omega T_e} - 1)} = \underline{H}_{rm}$$

$$\underline{H}_{an} = \frac{1}{1 + j\omega\tau}$$

b) si $\omega T_e \rightarrow \infty$ $\omega = 2\pi f$
 $\frac{2\pi f}{f_e} \ll 1$; $f_e \gg 2f$
 $e^{j\omega T_e} \approx 1 + j\omega T_e$

$$\underline{H}_{rm} = \frac{1}{1 + \frac{\tau}{T_e} (1 + j\omega T_e - 1)}$$

$$\approx \frac{1}{1 + j\omega\tau} \quad (6)$$

$$P9 \quad \frac{d^2 s}{dt^2} = \frac{d}{dt} \left(\frac{ds}{dt} \right)$$

$$= \frac{d}{dt} \left(\frac{s_{n+1} - s_n}{T_e} \right)$$

$$= \frac{\frac{s_{n+1} - s_n}{T_e}}{T_e} = \frac{s_{n+1} - s_n}{T_e^2}$$

$$\boxed{\frac{d^2 s}{dt^2} = \frac{s_{n+1} + s_{n-1} - 2s_n}{T_e^2}}$$

$$c) \quad G_{\text{num}} = \frac{1}{1 + \frac{Z}{T_e} (e^{j\omega T_e} - 1)}$$

$$= \frac{1}{1 + Z}$$

$$|Z|^2 = Z \bar{Z}$$

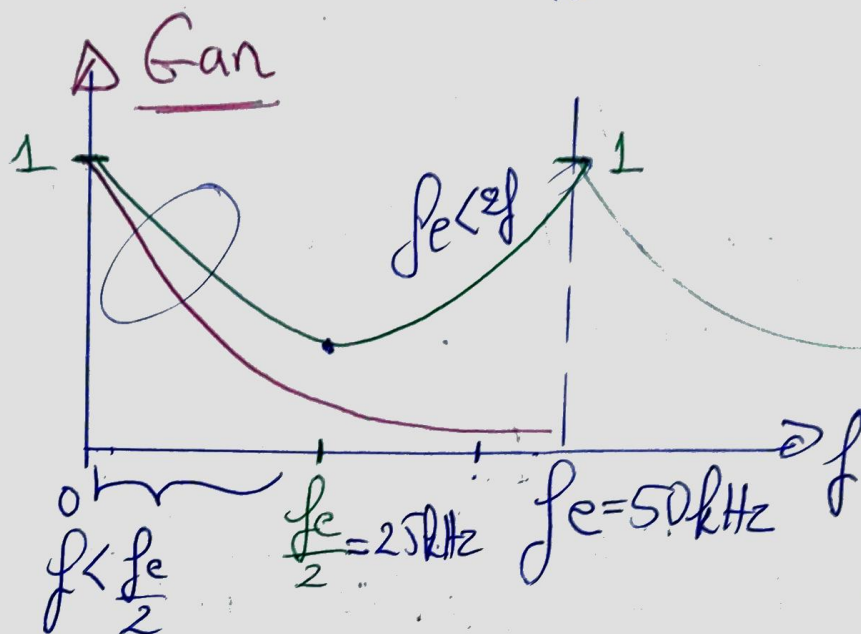
$$= \left(1 + \frac{Z}{T_e} (e^{j\omega T_e} - 1) \right) \left(1 + \frac{\bar{Z}}{T_e} (e^{-j\omega T_e} - 1) \right)$$

$f=0$	$f=\frac{f_c}{2}$	$f=f_c$
$e^{j\omega_c T_c} = 1$	$e^{j\frac{\omega_c T_c}{2}} = e^{j\pi} = -1$	$e^{j\omega_c T_c} = e^{j2\pi} = 1$

$$G_{mm} = 1$$

$$G_m = \frac{1}{|1 - \frac{2Z}{T_c}|}$$

$$G_{mm} = 1$$



$$\begin{aligned} \frac{Z}{T_c} &= \frac{1}{\omega_0} f_c \\ &= \frac{1}{\omega_c} f_c \\ &= \frac{1}{2\pi f_c} f_c \\ &= \frac{50}{2\pi \times 3} \end{aligned}$$

$$G_{mm}\left(\frac{f_c}{2}\right) < 1$$

Ex 3

$$S_1 \longrightarrow f_1 = 1 \text{ kHz} \longrightarrow f_{\min} = 2 \text{ kHz}$$

$$S_2 \longrightarrow f_2 = 10 \text{ Hz} \longrightarrow f_{\min} = 20 \text{ Hz}$$

$$S_3 \longrightarrow (S_1 + S_2) \longrightarrow f_{\min} = 2 \text{ kHz}$$

$$S_4 \longrightarrow S_1 \cdot S_2 \begin{cases} (2020 \text{ Hz}) \leftarrow f_1 + f_2 \\ (1980 \text{ Hz}) \leftarrow f_1 - f_2 \end{cases}$$

$$f_{\text{ech}_{\min}} = 2 \times 1010 = 2020 \text{ kHz}$$

$$2) \quad A_{2k+1} = \frac{8A}{\pi} \frac{1}{(2k+1)^2}$$

$$f_{\max} = f_0 \times (2k+1), \quad f_0 = 1 \text{ kHz}$$

$$k \nearrow f_{\max} \nearrow ; f_{\text{ech}} ??$$

$$\frac{A_{2k+1}}{A_1} \leq 1\% \quad A_1 = \frac{8A}{\pi}$$

$$\Rightarrow \frac{1}{(2k+1)^2} \leq \frac{1}{100}, \quad k \geq 4,5$$

$$s_6(t) = \frac{8A}{\pi} \sum_0^4 \frac{1}{(2k+1)^2} \sin[(2k+1)f_0 2\pi t]$$

$$f_{\max} = (2 \times 4 + 1) f_0 = 9 \text{ kHz}$$

$$f_{\text{ech}} = 18 \text{ kHz}$$

3) $f_0 = 4 \text{ kHz}$; $f_{\text{ech}} = 30 \text{ kHz}$

$$A_{2k+1} = \frac{4A}{\pi} \left(\frac{1}{(2k+1)} \right)$$

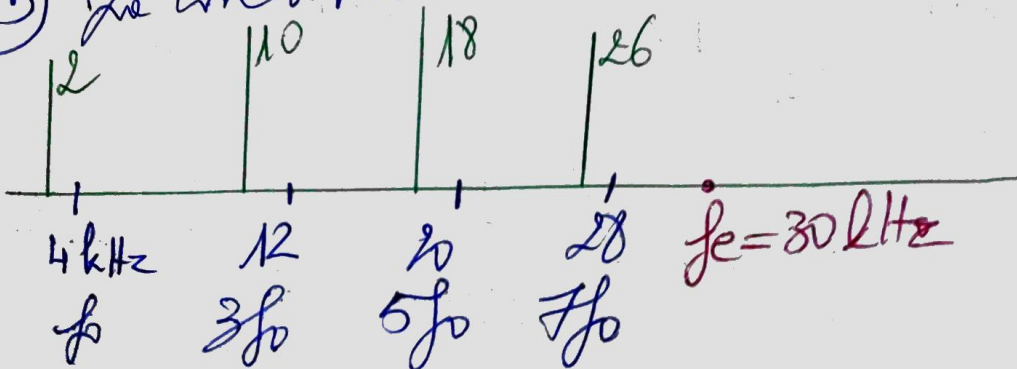


$$\frac{1}{(2k+1)}$$



$$\frac{1}{(2k+1)^2}$$

b) Le critère de Shannon est Non vérifié



$$c) \frac{A_{2k+1}}{A_1} < 1\% \Rightarrow \frac{1}{(2k+1)} \leq \frac{1}{100}$$

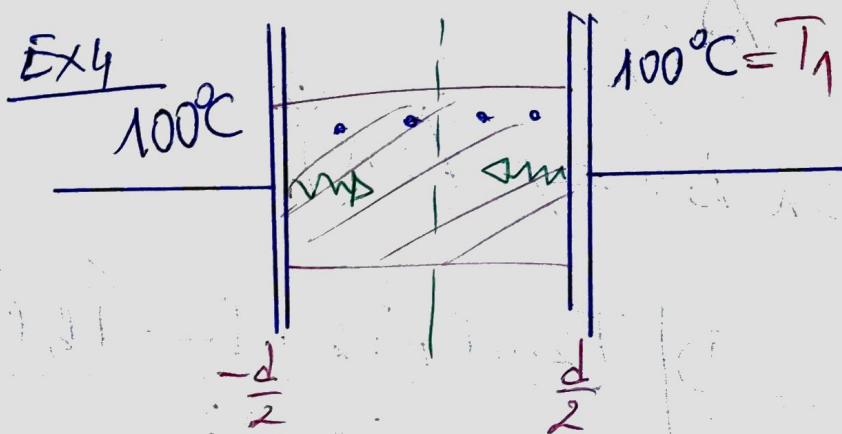
$$(2k+1) \geq 100 \Rightarrow \boxed{k \geq 49,5}$$

$$\sum_0^{49}$$

$$f_{\max} = \cancel{2k} f_0(2k+1)$$

$$= 99 f_0 = 792 \text{ kHz}$$

$$2 \sin(\omega t) + 3 \cos(2\omega t) + 10^{-6} \sin(5\omega t)$$



$$\frac{\partial T(x,t)}{\partial t} = D \frac{\partial^2 T}{\partial x^2}$$

$$T(x,t) \rightarrow T\left(-\frac{d}{2} + k \Delta x; l \Delta t\right)$$

$$S(t) \rightarrow S_n; \quad T(x,t) \rightarrow T_{k,l}$$

$$\frac{ds}{dt} \longrightarrow \frac{S_{n+1} - S_n}{T_e}$$

$$\frac{\partial T}{\partial t} \longrightarrow \frac{T_{k,l+1} - T_{k,l}}{\Delta t}$$

$$\frac{\partial T}{\partial x} \longrightarrow \frac{T_{k+1,l} - T_{k,l}}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} \longrightarrow \frac{T_{k+1,l} + T_{k-1,l} - 2T_{k,l}}{(\Delta x)^2}$$

$$\Delta x^2 = 2x \Delta x \neq (\Delta x)^2$$

$$\frac{T_{k,l+1} - T_{k,l}}{\Delta t} = D \left[\frac{T_{k+1,l} + T_{k-1,l} - 2T_{k,l}}{(\Delta x)^2} \right]$$

on pose $\frac{1}{\Delta t} = \frac{2D}{(\Delta x)^2}$

$$T_{k,l+1} = \frac{T_{k+1,l} + T_{k-1,l}}{2}$$

		R					
l	T_i	1	2	3	4	5	T_a
0	100	20	20	20	20	20	100
1	100	60	20	20	20	60	100
2	100	60	40	20	40	60	100
3	100	70	40	40	40	70	100
4	100	70	55	40	55	70	100
5	100	77	55	55	55	77	100
6	100	77	66	55	66	77	100
7	100	83	66	66	66	83	100

b) D'après le tableau: $\tau = 7 \Delta t$ pour cuire la viande à cœur, Pour un morceau d'un centimètre divisé en 5 tranches d'épaisseur $\Delta x = 2 \text{ mm}$

$$\tau = \frac{7 (\Delta x)^2}{2D} = 140 \text{ s} = 2 \text{ mn } 20 \text{ s.}$$

l	k						T_l
	T_1	1	2	3	4	5	
0	100°	20	20	20	20	20	100°
1	100°	60					100
2							

$$T_{1,1} = \frac{T_{1,0} + T_{0,0}}{2} = \frac{20 + 100}{2} = 60$$

$\downarrow$
 $(l+1)$

$$\text{longueur antenne} \simeq \frac{\lambda}{2} = \frac{cT}{2} = \frac{c}{2f}$$

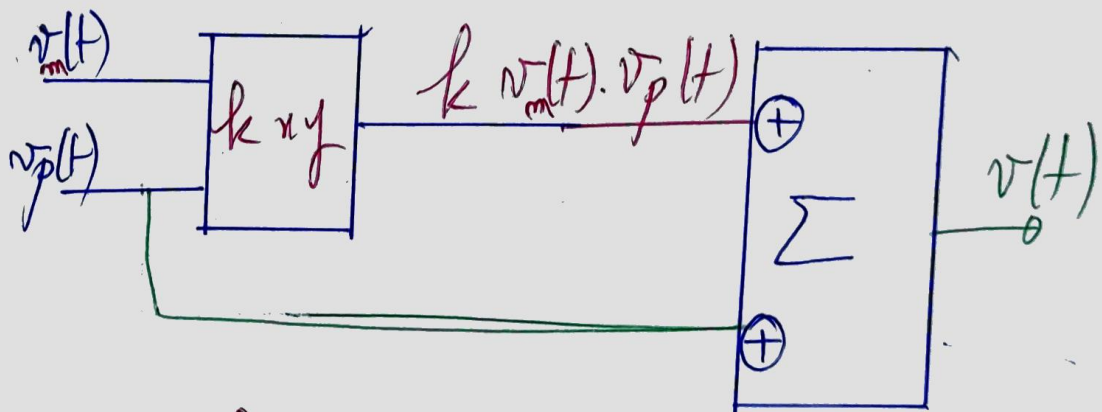
$$f_{\max} = 20 \text{ kHz} \Rightarrow l_{\text{antenne}} = \frac{3 \cdot 10^8}{2 \times 20 \cdot 10^3}$$

$$l_{\text{antenne}} = 7,5 \text{ km}$$

$$v_m(t) = V_m \cos(\omega_m t)$$

$$\omega_p \gg \omega_m$$

$$v_p(t) = V_p \cos(\omega_p t)$$

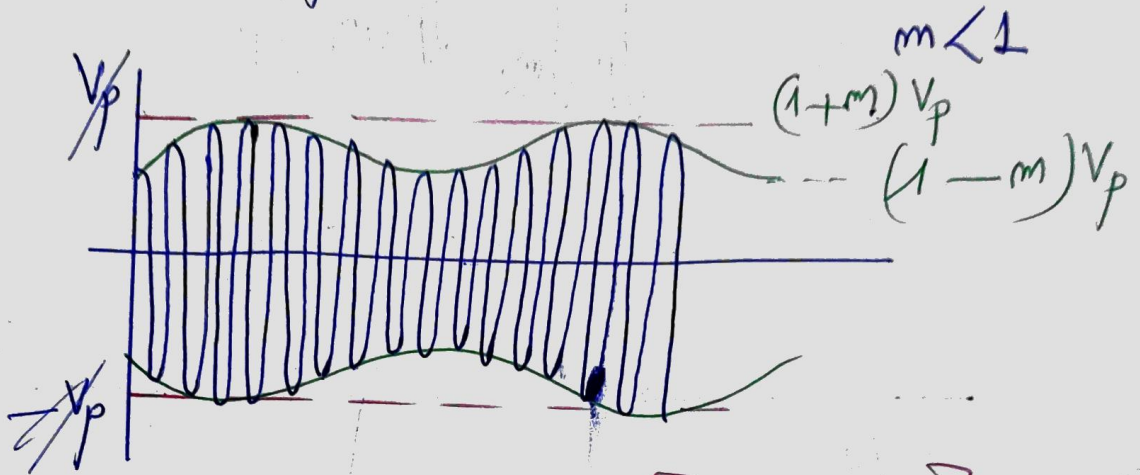


$$v(t) = k v_m(t) v_p(t) + v_p(t)$$

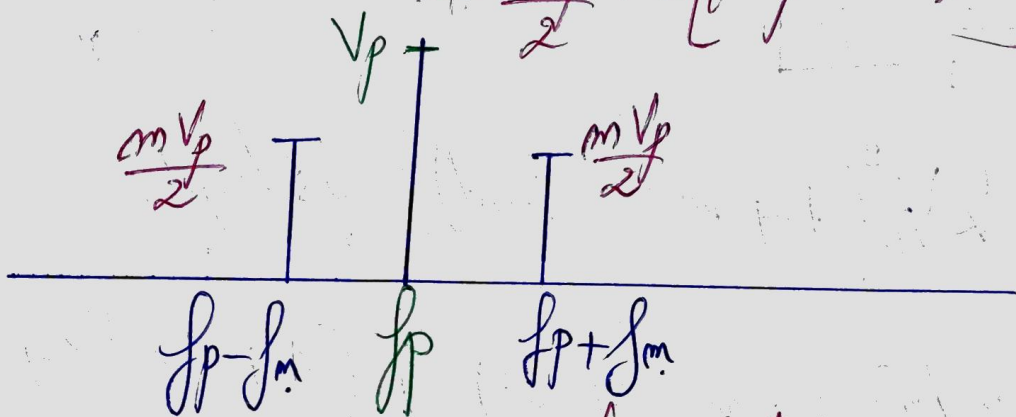
$$= V_p \left(1 + \underbrace{k V_m \cos(\omega_m t)}_m \right) \cos(\omega_p t)$$

$$v(t) = V_p \left[\underbrace{1 + m \cos(\omega_m t)}_{\text{enveloppe}} \right] \cos(\omega_p t)$$

$$f_p \gg f_m \rightarrow \text{periode } T_m \gg T_p$$



$$v(t) = V_p \cos(\omega_p t) + \frac{m V_p}{2} \cos[(\omega_p + \omega_m) t] + \frac{m V_p}{2} \cos[(\omega_p - \omega_m) t]$$

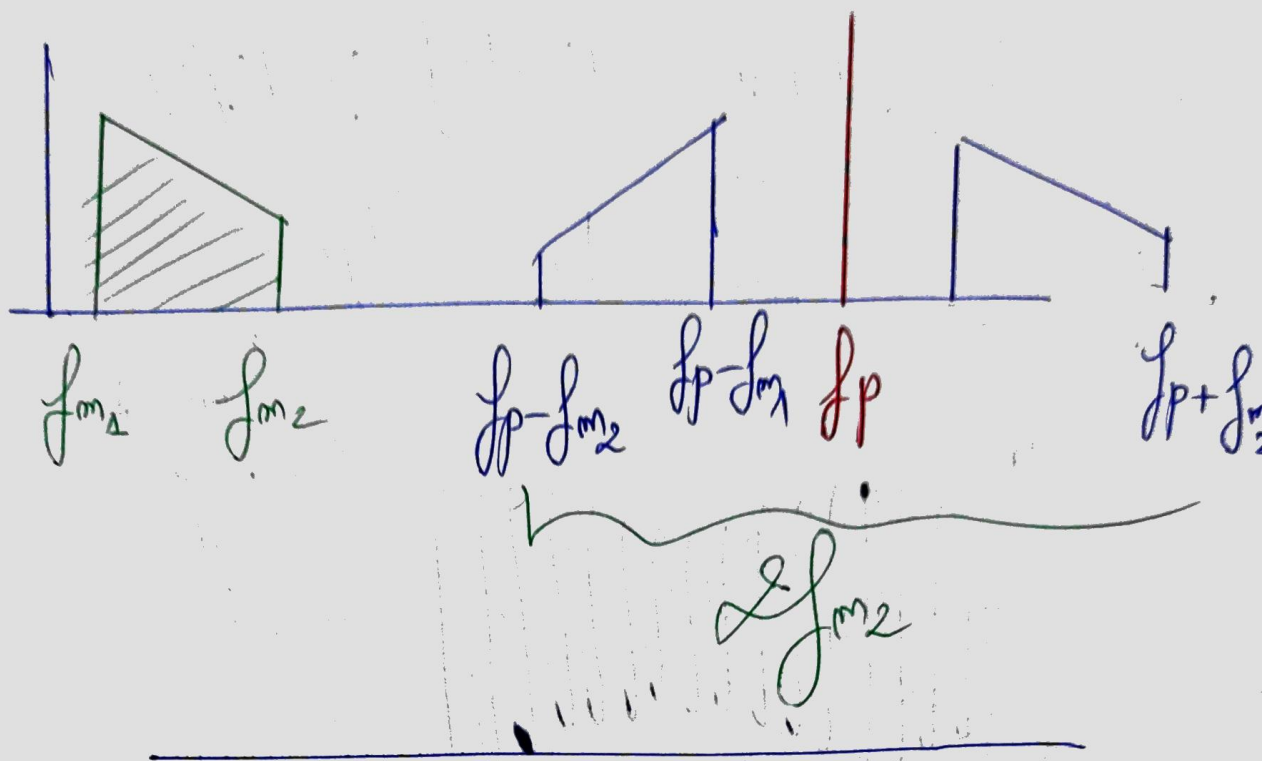


$$(1+m) V_p = V_2 = 18 \text{ V}$$

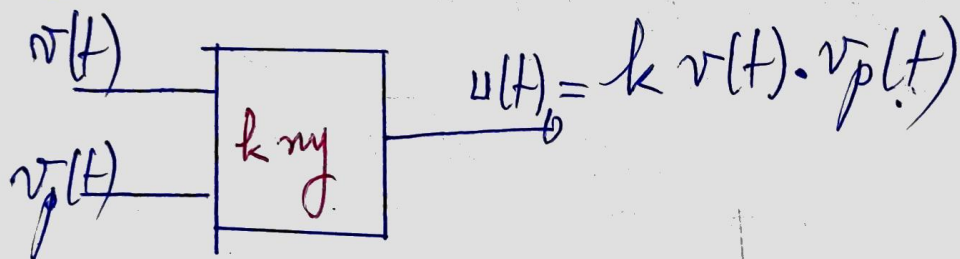
$$(1-m) V_p = V_1 = 2 \text{ V}$$

$$\frac{V_2}{V_1} = \frac{1+m}{1-m}$$

$$m = \frac{V_2 - V_1}{V_1 + V_2} = 0,8$$



Démodulation



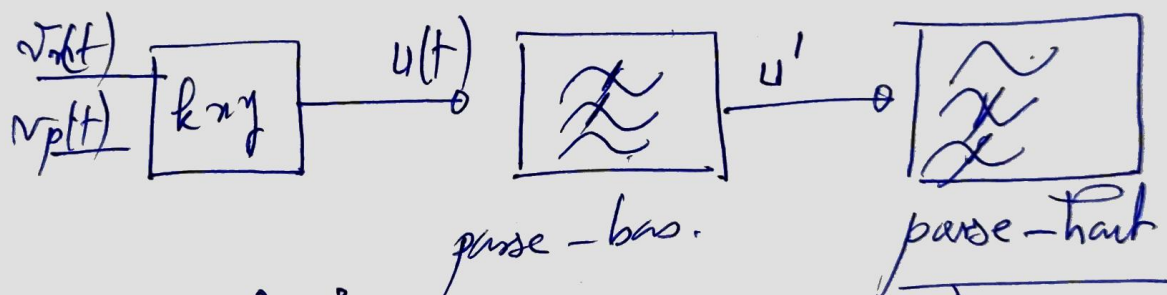
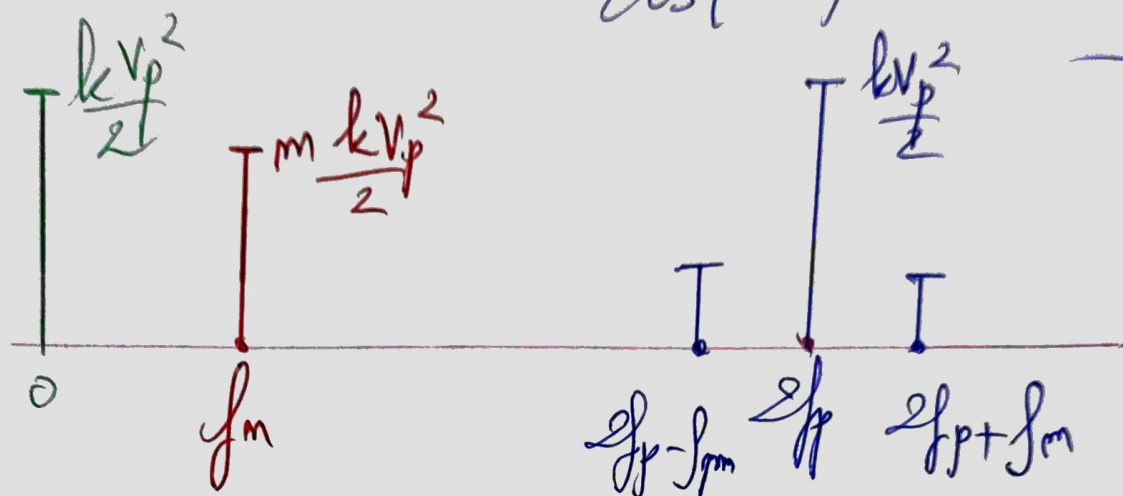
$$u(t) = k V_p^2 \left(1 + m \cos(\omega_m t) \right) \cos^2(\omega_p t)$$

$$= \frac{k V_p^2}{2} \left(1 + m \cos(\omega_m t) \right) (1 + \cos(2\omega_p t))$$

$\Rightarrow$

$$u(t) = \frac{kV_p^2}{2} + \frac{kV_p^2}{2} m \cos(\omega_m t) + \frac{kV_p^2}{2} \cos(2\omega_p t)$$

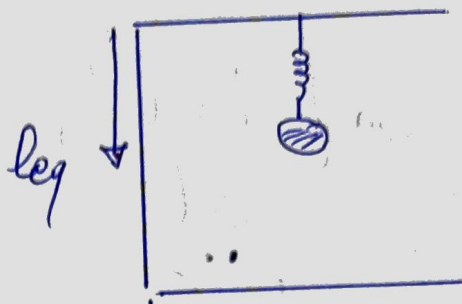
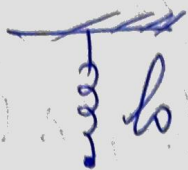
$$+ \frac{kV_p^2}{2} \frac{m}{2} \left[\cos(2\omega_p + \omega_m)t + \cos(2\omega_p - \omega_m)t \right]$$



$$u'(t) = \frac{kV_p^2}{2} (1 + m \cos(\omega_m t))$$

$$u_d(t) \approx \frac{kV_p^2}{2} m \cos(\omega_m t)$$

ET2 2013; ET2 2020

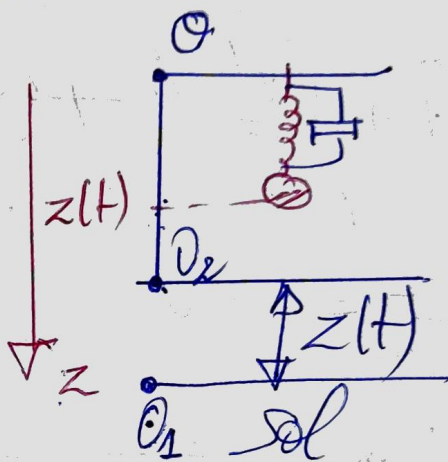


$$mg = k(l_{eq} - l_0)$$

$$l_{eq} = \frac{mg}{k} + l_0 = l_0'$$

$$m\vec{a} = \vec{T} + m\vec{g} + \vec{F}_{vis} + \vec{F}_{ic} + \vec{F}_{ec}$$

$$m\ddot{z} = -k(z(t) - l_0) + mg - h\dot{z} - m\ddot{Z}$$



$$m\ddot{z} = -k(z(t) - l_0) + k(l_{eq} - l_0) - h\dot{z} - m\ddot{Z}$$

$$m\ddot{z} = -k(z(t) - l_{eq}) - h\dot{z} - m\ddot{Z}$$

$$m\ddot{u} = -ku - h\dot{u} - m\ddot{Z}$$

$$\ddot{u} + \frac{h}{m}\dot{u} + \frac{k}{m}u = -\ddot{Z}$$

$$\underline{\alpha = -1}$$

$$\vec{F}_{ic} = -m \vec{a}_c = -m \frac{1}{6} \left(\vec{\omega}_{R'/R} \wedge \vec{r}_R \right)$$

$$\vec{F}_{ie} = -m \left[\frac{d^2 \vec{OO}'}{dt^2} + \vec{\omega}_{R'} \wedge (\vec{\omega}_{R'} \wedge \vec{OO}') + \dots \right]$$

$$= -m \ddot{\vec{Z}}$$

$$\ddot{u} + \frac{\sqrt{2}}{\tau} \dot{u} + \omega_0^2 u = -\ddot{\vec{Z}}$$

$$\sqrt{\frac{k}{m}} = \omega_0$$

$$\frac{\tau}{m} = \frac{\sqrt{2}}{\tau}$$

$$\frac{m}{L} \equiv \frac{1}{C}, \quad \frac{1}{C} \equiv \frac{1}{k}$$

$$\frac{u}{\ddot{\vec{Z}}} = H(j\omega) = \frac{1}{\omega^2 + \frac{\sqrt{2}}{\tau} j\omega + \omega_0^2}$$

$$= -(\ddot{\vec{Z}})$$

filtre passe-haut

$\frac{d}{dt}$