Induction

Circuit mobile et $\vec{B}$ **stationnaire** $\frac{\partial}{\partial t} = 0$

$\Rightarrow \Phi = \iint \vec{B}\, d\vec{S}_{circuit} =$ dépend du temps

$\Rightarrow e(t) = -\frac{d\Phi}{dt}$: naissance d'un courant induit

Cas de Lorentz

Circuit fixe: et $\vec{B}(t)$

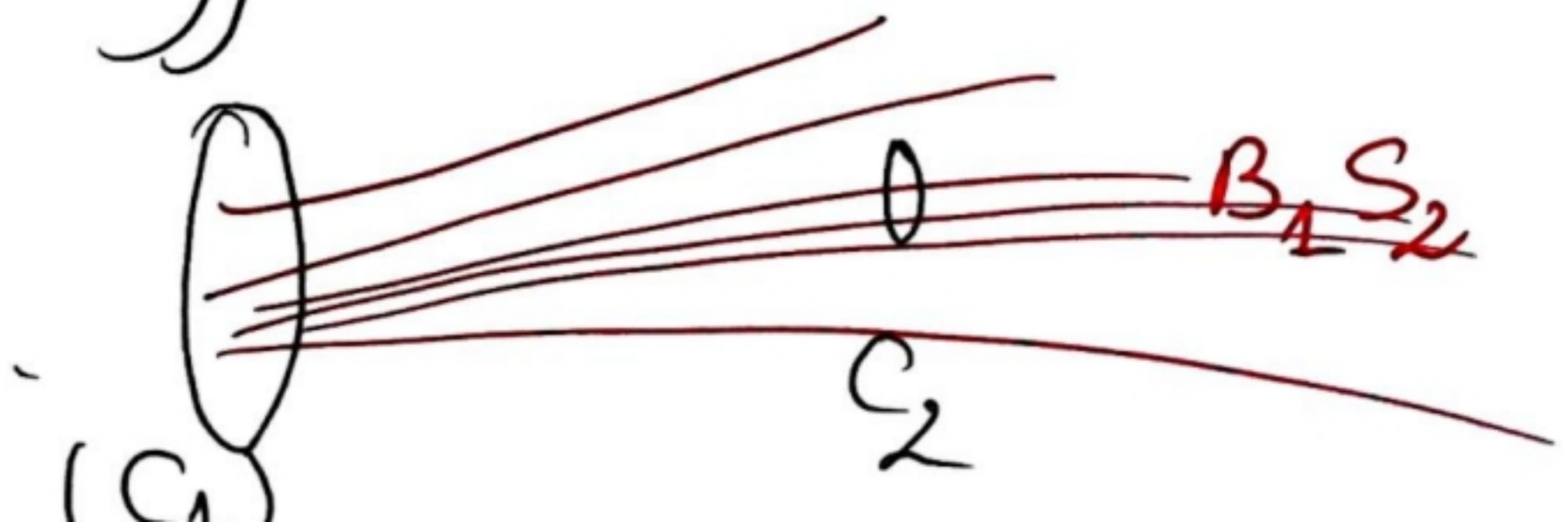
$\Phi = \iint \vec{B}(t) \cdot d\vec{S} \Rightarrow e(t) = -\frac{d\Phi}{dt}$

Cas de Neumann

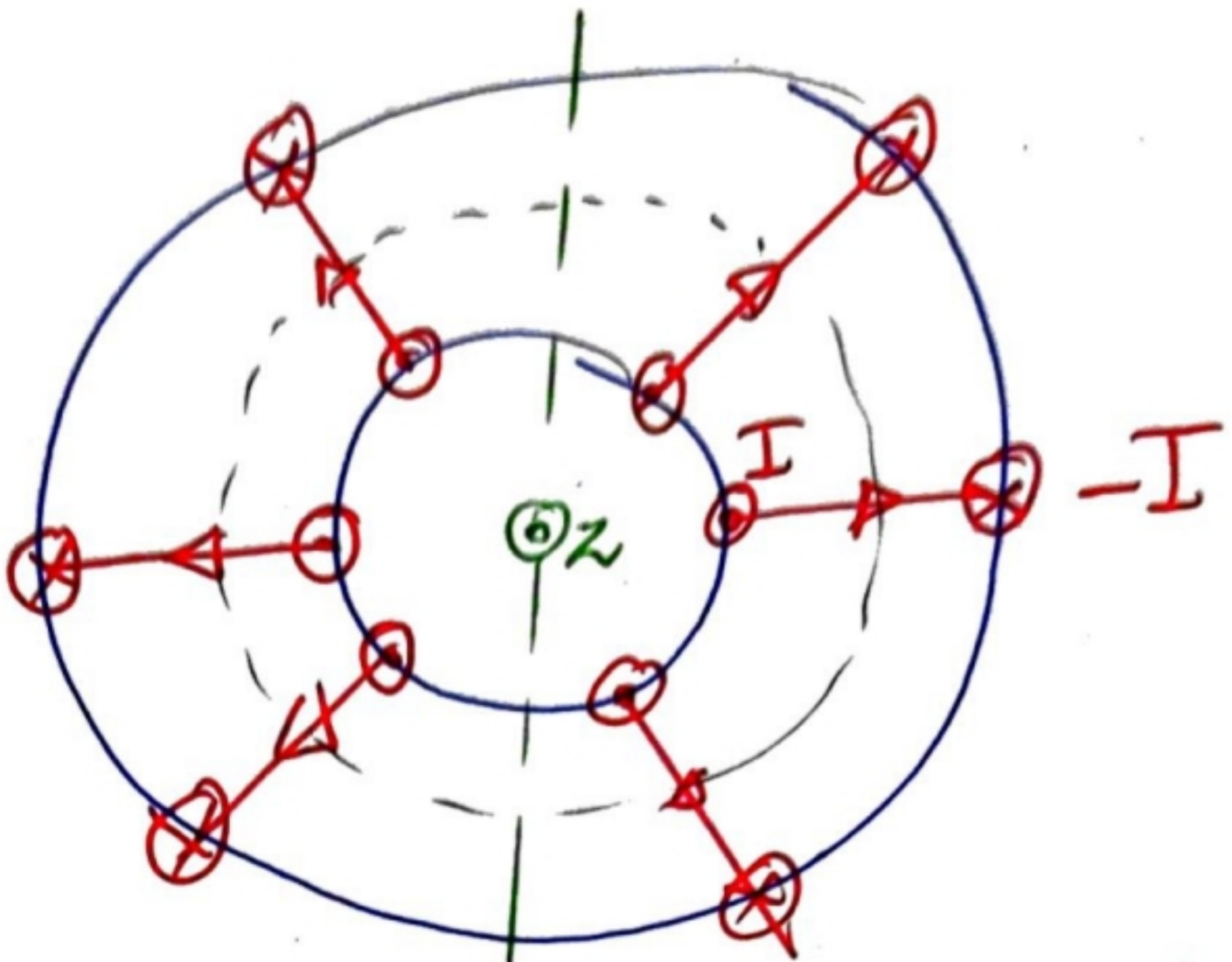
$\Phi_p = \iint \vec{B}_p \cdot d\vec{S}_p = (L)\, i_p$



$\Phi_{mutuelle} = \iint \vec{B}_1 \cdot d\vec{S}_2 = (M)\, i_1$



Exercice



1)

$\Rightarrow \vec{B}^0 \perp \quad \vec{B_\theta}(r,z)$

2) $\oint \vec{B}.\vec{dl} = 2\pi r B_\theta(r,z) = \mu_0 I N$

θ ; cercle

$$\vec{B}_{tore} = \frac{\mu_0 N I}{2\pi r}\vec{e_\theta}$$

$r \in [R-a ; R+a]$; $z \in [-a, a]$

$\iint dr\,dz$

$\vec{B}_{tore} = \vec{0}$; ailleurs

3) $\Phi = \iint \vec{B}_{tore}\,\vec{dS}_{tore} = \int_{R-a}^{R+a} \frac{\mu_0 N I}{2\pi r}\,dr \int_{-a}^{a} dz\,N$

$$\Phi_p = \left[\frac{\mu_0 N^2 \pi a}{2\pi} \ln\left(\frac{R+a}{R-a}\right)\right] I$$

$$L = \frac{\mu_0 N^2 a}{\pi} \ln\left(\frac{1+a/R}{1-a/R}\right)$$

b) $W_{mag} = \frac{1}{2} L I^2 = \iiint \frac{B^2}{2\mu_0} d\tau$

$\frac{B^2}{2\mu_0}$ = densité d'énergie magnétique

$\frac{\varepsilon_0 E^2}{2}$ = " " électrique

$\left(\frac{J}{m^3} = \frac{N \cdot m}{m^3} = \frac{N}{m^2} \rightarrow Pa\right)$

$$W_{mag} = \iiint \frac{1}{2\mu_0} \left(\frac{\mu_0 I N}{2\pi r}\right)^2 (r\, dr\, d\theta\, dz)$$

$$= \frac{1}{2} \left[\quad \right] I^2$$

c) $a \ll R$

$\hookrightarrow dl$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}$$

$$\ln\left(\frac{1+x}{1-x}\right) = 2x + \frac{2}{3}x^3$$

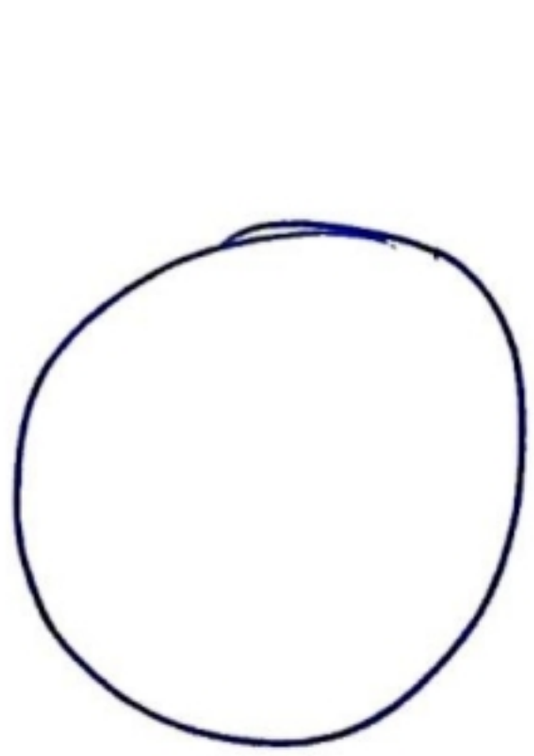
$$= 2x\left(1 + \frac{1}{3}x^2\right), \quad x = \frac{a}{R}$$

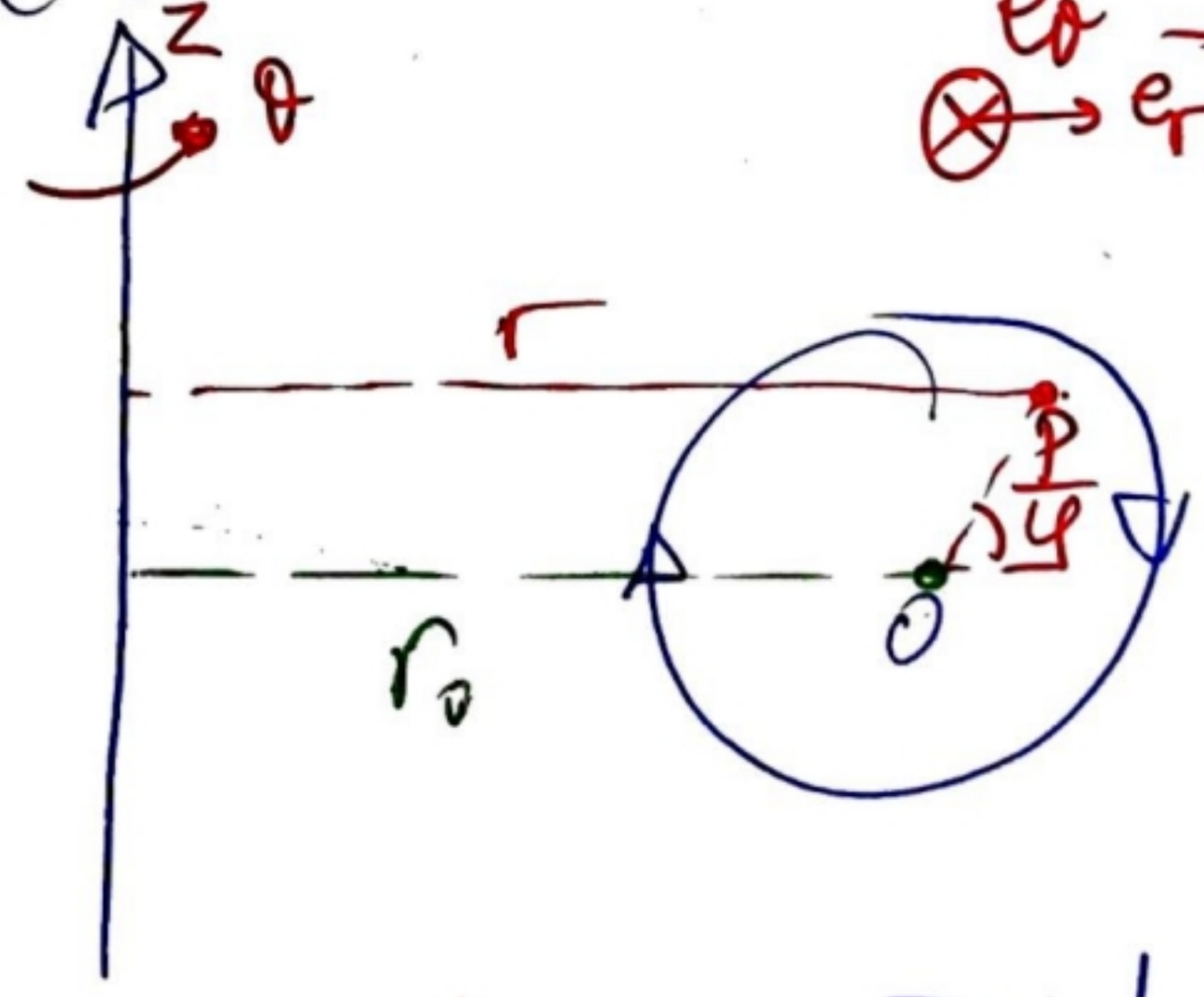
$$L = L_0\left(1 + \frac{1}{3}\left(\frac{a}{R}\right)^2\right)$$

$$\frac{L - L_0}{L_0} = \frac{1}{3}\left(\frac{a}{R}\right)^2 \leq 1\%$$

mp2
2012

B)



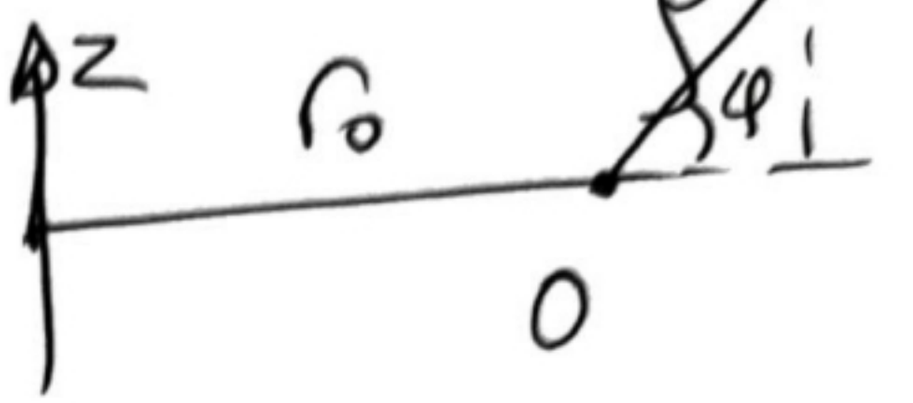


$$\oint \vec{B}\,\vec{dl} = 2\pi r B_\theta(r, \vec{e}_\varphi) = \mu_0 I N$$

$$\vec{B} = \frac{\mu_0 N I}{} \vec{e}_\varphi$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi r_0 \left(1 + \frac{\rho}{r_0}\cos\varphi\right)} \vec{e}_\theta$$

$$r = r_0 + \rho\cos\varphi$$



$$\frac{d.l}{\left(1 + \frac{\rho}{r_0}\cos\varphi\right)} \simeq 1 - \left(\frac{\rho}{r_0}\cos\varphi\right) + \left(\frac{\rho}{r_0}\right)^2 \cos^2\varphi$$

$$\Phi = \iint \vec{B}.\left(\rho\, d\rho\, d\varphi\right)\vec{e}_\theta\, N$$

$$= \frac{\mu_0 N I}{2\pi r_0} \iint \left(1 - \frac{\rho}{r_0}\cos\varphi + \frac{\rho^2}{r_0^2}\cos^2\varphi\right) \rho\, d\rho\, d\varphi$$

$$\Phi_1 = \frac{\mu_0 N^2 I}{2\pi r_0}\, \pi a^2$$

$$\Phi_2 = \left(\quad\right) \int_0^{2\pi} \cos\varphi\, d\varphi = 0$$

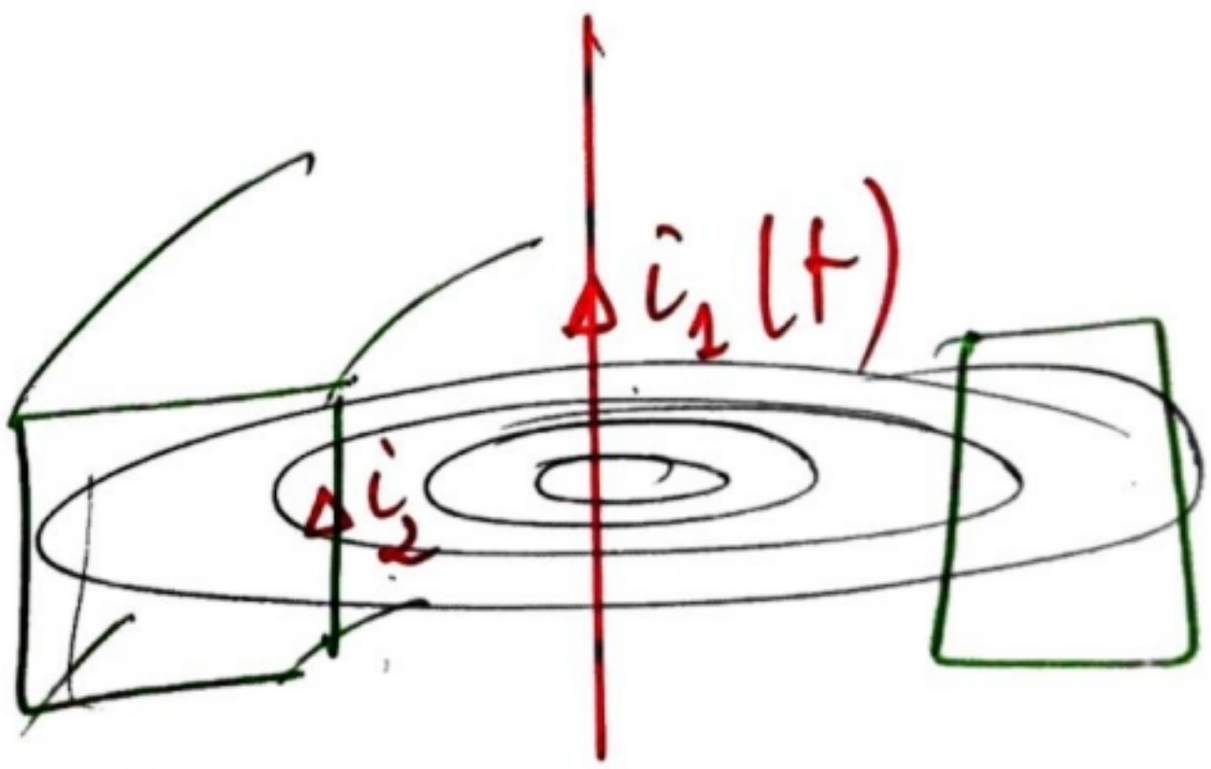
$$\Phi_3 = \frac{\mu_0 N^2 I}{2\pi r_0^3} \int_0^a \rho^3 d\rho \int_0^{2\pi} \cos^2\varphi \, d\varphi$$

$$= \frac{\mu_0 N^2 I}{2\pi r_0^3} \frac{a^4}{4} \pi$$

$$* \frac{1}{2\pi} \int_0^{2\pi} \cos^2\varphi \, d\varphi = \frac{1}{2} .$$

$$L = L_0 \left(1 + \frac{1}{4} \frac{a^2}{r_0^2}\right)$$

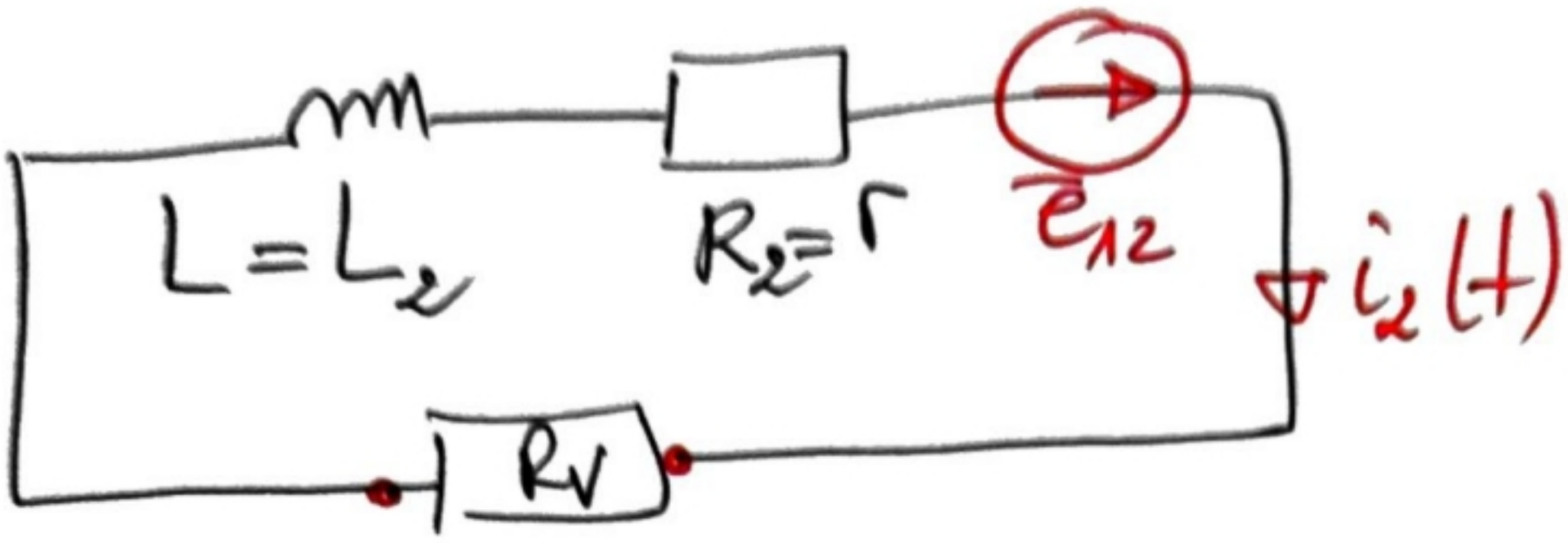




$$\Phi_{fil \to Tore} = \Phi_{12} = \Gamma_{12} \, i_1$$

$$= \iint \frac{\mu_0 \, i_1 \, N}{2\pi r} \left(dr \, dz\right) = \frac{2a \, i_1 \mu_0 N}{2\pi} \ln\frac{(R+a)}{(R-a)}$$

$$M_{12} = \frac{\mu_0 N a}{\pi} \ln\left(\frac{R-a}{R-a}\right) = \frac{L}{N}$$



$$L\frac{di_2}{dt} + (r + R_v)\, i_2(t) = -M\frac{di_1(t)}{dt}$$

si $i_1(t) = I_0 \cos(\omega t)$

$$\left[jL\omega + (r + R_v)\right] \underline{i_2} = -jM\omega \underline{i_1}$$

$$\underline{U}_{vol} = R_v \underline{i_2}$$

$$\underline{U}_{vol} = \frac{-jM\omega R_v}{(r+R_v) + jL\omega} I_0 e^{j\omega t}$$

$I_{eff}\sqrt{2}$

$$|\underline{U}_{vol}| = \frac{M\omega R_v I_0}{\sqrt{(R_v+r)^2 + (L\omega)^2}}$$

$$U_{vol} = \frac{U_{max}}{\sqrt{2}}$$

$$\arg \underline{U_V} = -\frac{\pi}{2} - \arctg \frac{L\omega}{(R_V + r)}$$

$$\underline{H} = \frac{\underline{U}}{\underline{I_1}} = \frac{R_V \underline{I_2}}{\underline{I_1}} = \frac{-jM\omega R_V}{(R_V + r) + jL\omega}$$

$$\underline{H} = \frac{H_{max}\left(-j\omega/\omega_0\right)}{1 + j\omega/\omega_0}$$

$$\omega_0 = \frac{R_V + r}{L} \quad , \quad \frac{H_{max}}{\omega_0} = \frac{-M R_V}{(R_V + r)}$$

$$H_{max} = \frac{-M R_V}{L} = -\frac{R_V}{N}$$

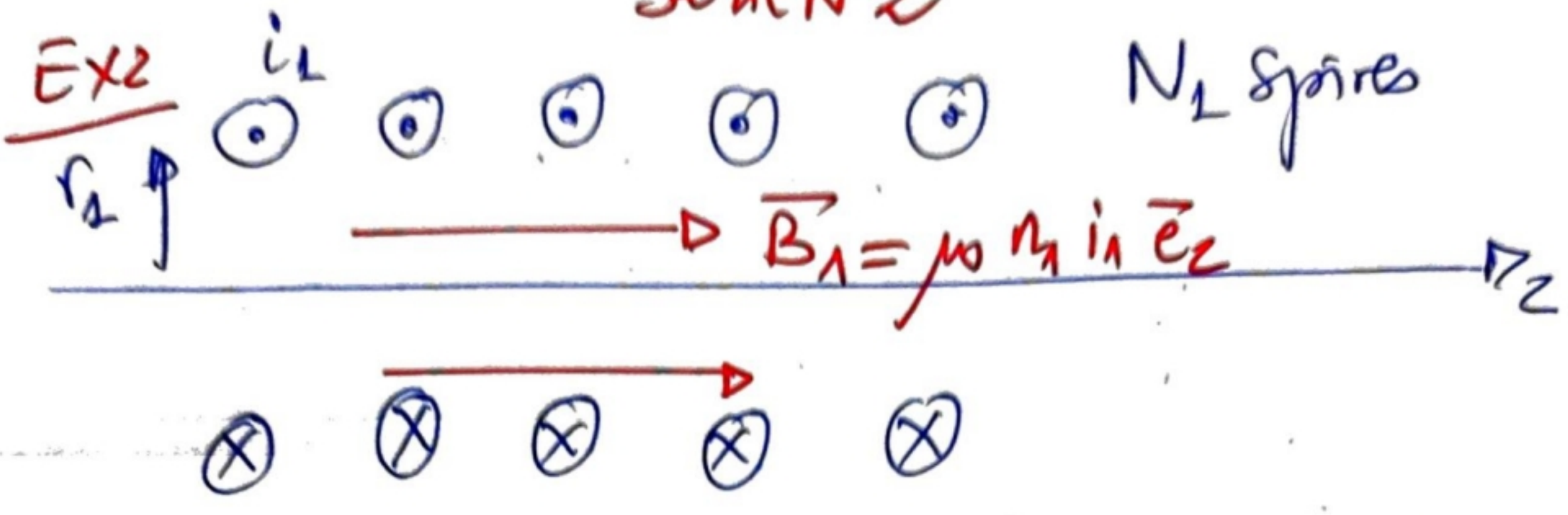
$\omega \to 0 \quad H \to 0$

$\omega \to \infty, \quad H = -H_{max}$

pince ampem mesure
les courts de très H.F

Serie N°2

Ex2



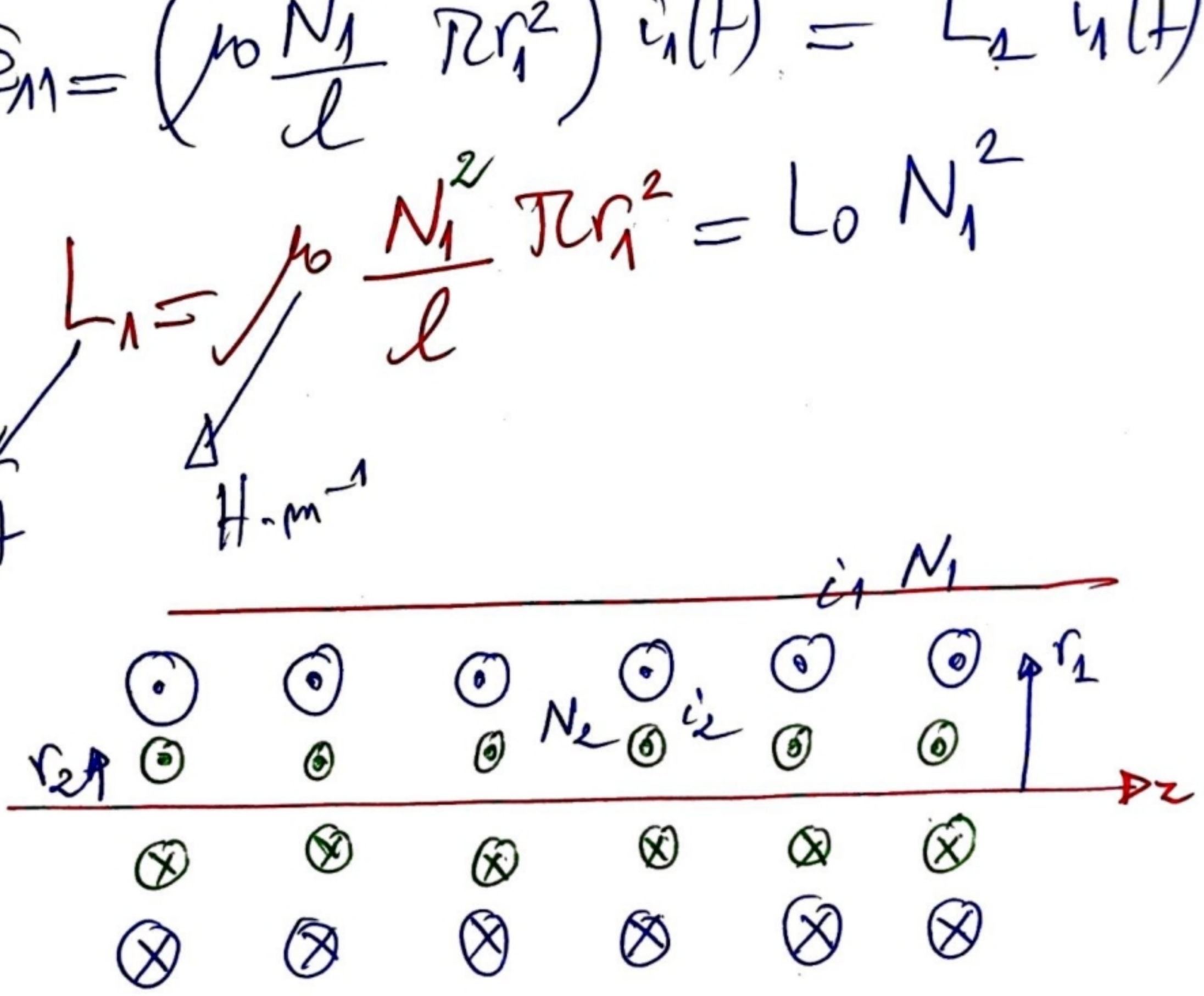
$$\Phi_{11} = \Phi_{propre} = \iint \vec{B}_1 \cdot d\vec{S}_1 \quad (weber)$$

$$\Phi_{11} = \left[\mu_0 \left(\frac{N_1}{l}\right) i_1(t)\right] \pi r_1^2 N_1$$

$$\Phi_{11} = \left(\mu_0 \frac{N_1^2}{l} \pi r_1^2\right) i_1(t) = L_1 i_1(t)$$

$$L_1 = \mu_0 \frac{N_1^2}{l} \pi r_1^2 = L_0 N_1^2$$

L_1 : H , $H \cdot m^{-1}$



$$\Phi_{12} = \iint \vec{B_1} \, \overrightarrow{dS_2}$$

$$= \left(\mu_0 \frac{N_1}{l} i_1(t) \right) \pi r_2^2 N_2$$

$$\Phi_{12} = \left(\mu_0 \frac{N_1 N_2}{l} \pi r_2^2 \right) i_1(t)$$

$$\Phi_{12} = M_{12} \, i_1(t)$$

$$*\Phi_{21} = M_{21} \, i_2(t) = \iint \vec{B_2} \cdot \overrightarrow{dS_1}$$

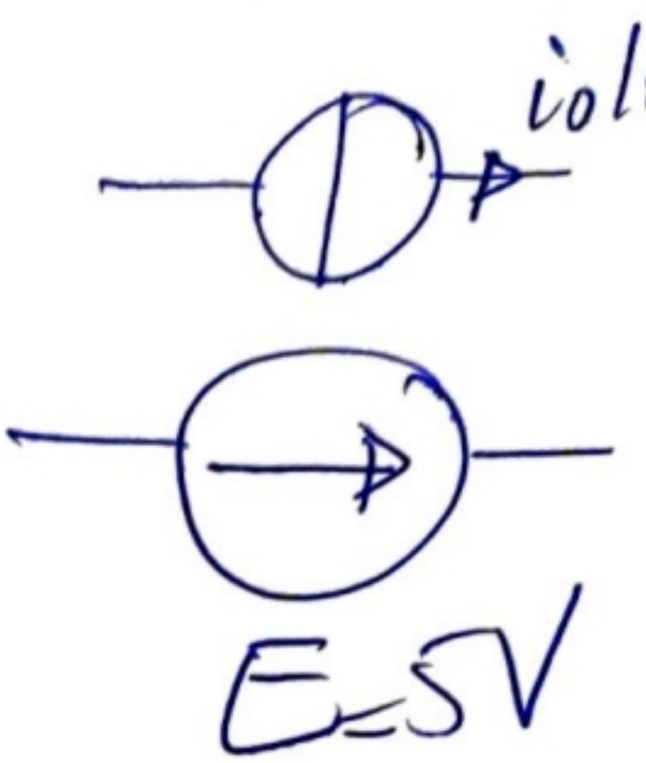
$$= \left[\mu_0 \frac{N_2}{l} i_2(t) \right] \pi r_2^2 N_1$$

$$\Rightarrow \boxed{M_{21} = M_{12} = M}$$

Theorème de Neuman

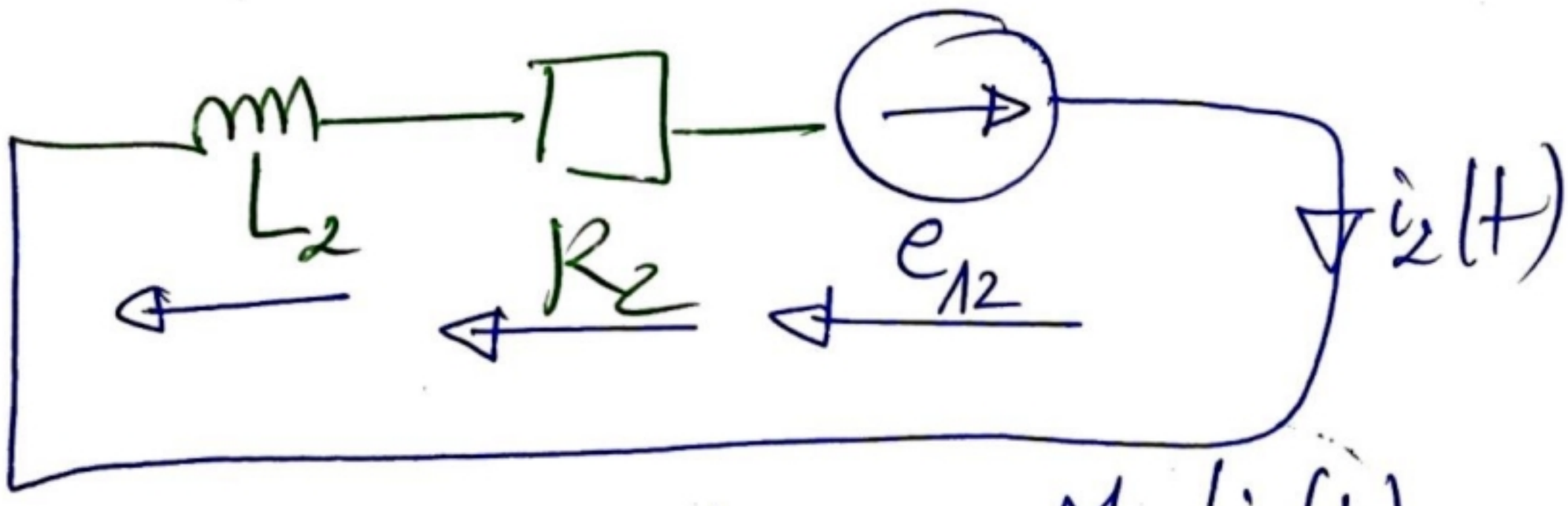


2) $i_0(t) = I_0 \cos(\omega t)$



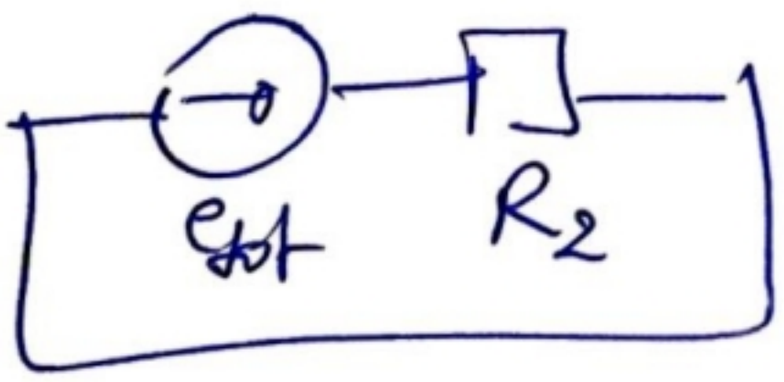
$\overline{B_1}(t) \longrightarrow \Phi_2 \Rightarrow$

$\Phi_{1\to 2}(t) = M\ i_1(t) = \Pi i_0(t)$



$e_{12} = f.é.m\ induite = -M \dfrac{di_0(t)}{dt}$

$e_{tot} = e_{propre} + e_{12}$;



$L_2 \dfrac{di_2}{dt} + R_2 i_2 - e_{12} = 0$

$$L_2 \frac{di_2}{dt} + R_2 i_2 = -M \frac{di_0(t)}{dt}$$

$(R_2 + jL_2\omega)\,\underline{i_2} = -jM\omega\,\underline{i_0}$

$$\frac{\underline{i_2}}{\underline{i_0}} = - \frac{jM\omega/R_2}{1 + j\frac{L_2}{R_2}\omega} = \frac{K\, j\omega/\omega_0}{1 + j\omega/\omega_0}$$

$$G_i(\omega) = \frac{|K|\,\omega/\omega_0}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

gain de courant G_i

$G_i\, G_r = G_{Puissance}$

$\omega_0 = \omega_c = \frac{R_2}{L_2}$; $\tau_L = \frac{L}{R}$, $\tau_c = RC$

$\frac{K}{\omega_0} = \frac{-M}{R_2} \Rightarrow K = \frac{-M}{R_2}\omega_0$

$K = -\frac{M}{R_2}\,\frac{R_2}{L_2} = -\frac{M}{L_2} = \frac{-N_1 N_2}{N_2^2}$

$K = -\frac{N_1}{N_2}$

b) $\underline{B_{tot}} = B_1(r<r_2) + B_2(r<r_2)$

$= \mu_0 \frac{N_1}{l}\underline{i_0} + \mu_0 \frac{N_2}{l}\underline{i_2}$

$$\underline{B_{tot}} = \mu_0 \frac{N_1}{\ell} \underline{i_0} \left(1 + \frac{\underline{i_2}}{\underline{i_0}} \frac{N_2}{N_1} \right)$$

$$\underline{B_{tot}} = \underline{B_1} \left[1 + \frac{N_2}{N_1} \cdot \frac{\left(-\frac{N_1}{N_2}\right) j\omega/\omega_c}{1 + j\omega/\omega_c} \right]$$

$$= \underline{B_1} \left[1 - \frac{j\omega/\omega_c}{1 + j\omega/\omega_c} \right]$$

$$\underline{B_{tot}} = \frac{\underline{B_1}}{1 + j\omega/\omega_c}$$

$\lim_{\omega \gg \omega_c} B_{tot} \simeq 0 \quad \forall \; r < r_2$

$\Rightarrow$ écrantage magnétique

$$\vec{J} = \sum_{i=1}^{N} n_i q_i \vec{v}_i = n_1 q_1 \vec{v}_1 + n_2 q_2 \vec{v}_2$$

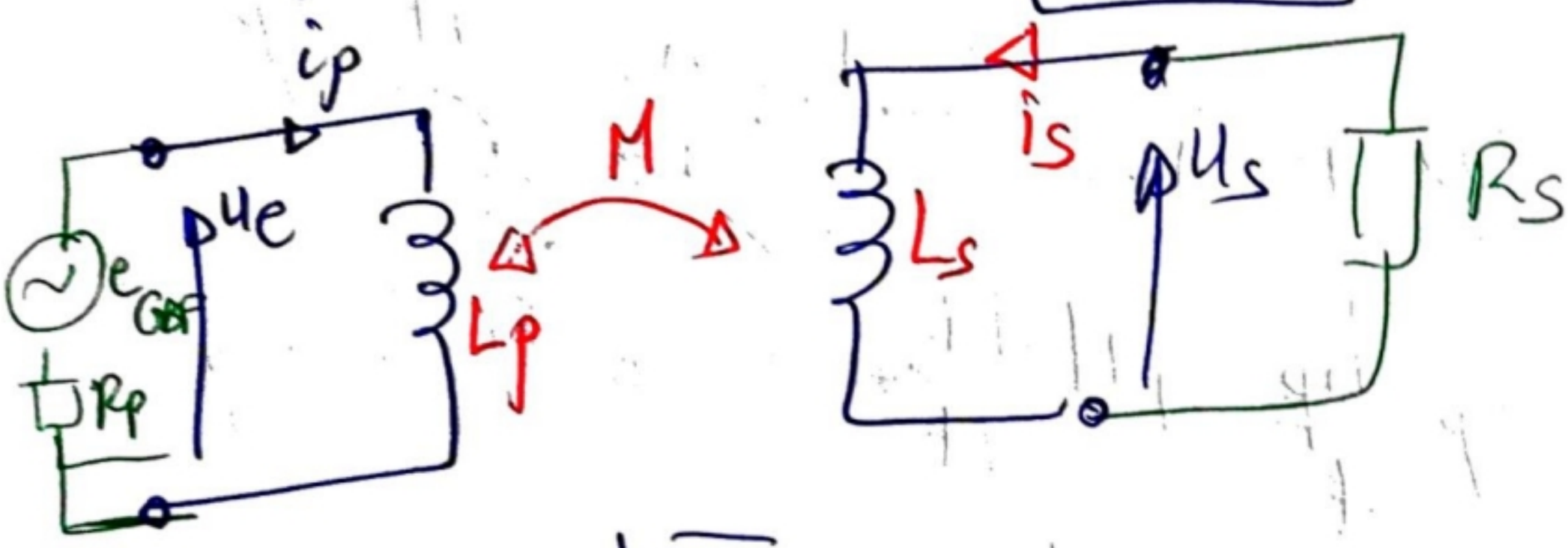
Transformateur

canaliser les $\vec{B}$, ferromagnétique.

$0 \leq k = \dfrac{|M|}{\sqrt{L_1 L_2}} \leq 1$

couplage parfait

$k = 1$



1) $e = -\dfrac{d\Phi}{dt}$

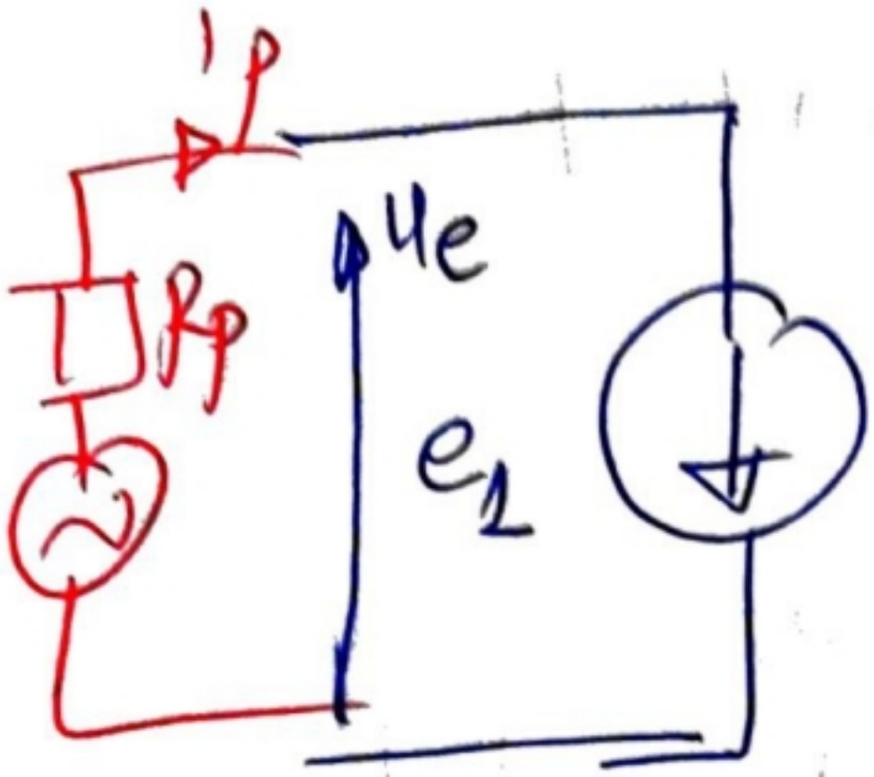
Lenz

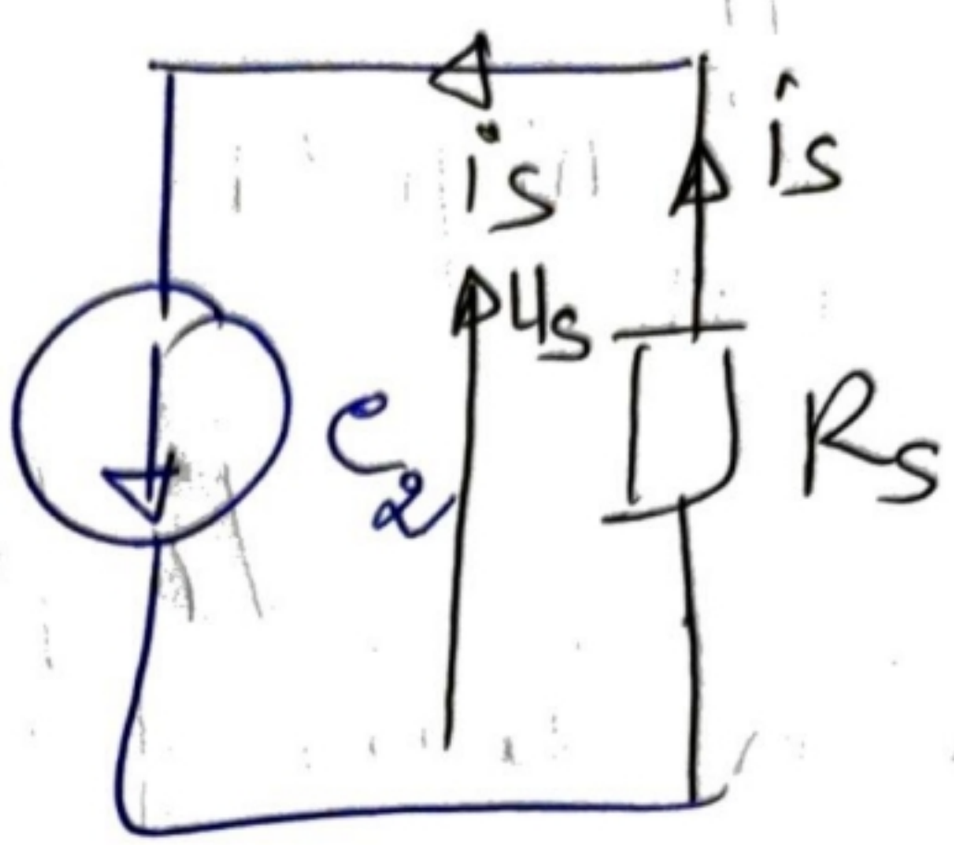
2) $i_p(t) \longrightarrow \vec{B}_p(t) \longrightarrow \Sigma_2 \longrightarrow i_s(t)$

$\Phi_{1\to 2} = \iint \vec{B}_p \, d\vec{S}_{\Sigma_2}$ dépend du temps

$\Phi_1 = \Phi_{11} + \Phi_{21} = L_p i_p + M i_s$

$\Phi_2 = \Phi_{22} + \Phi_{12} = L_s i_s + M i_p$





$$e_1 = e_{11} + e_{21} = -L_p \frac{di_p}{dt} - M \frac{di_s}{dt}$$

$$e_2 = e_{22} + e_{12} = -L_s \frac{di_s}{dt} - M \frac{di_p}{dt}$$

4) $$\begin{cases} u_e = L_p \frac{di_p}{dt} + M \frac{di_s}{dt} = e_{GBF}(t) - R_p i_p \\ u_s = L_s \frac{di_s}{dt} + M \frac{di_p}{dt} = -R_s i_s \end{cases}$$

5) $$(jL_p\omega + R_p)\underline{i_p} + jM\omega \underline{i_s} = E e^{j\omega t}$$

$$(jL_s\omega + R_s)\underline{i_s} + jM\omega \underline{i_p} = 0$$

$$\frac{\underline{i_s}}{\underline{i_p}} = \frac{-jM\omega}{R_s + jL_s\omega} \quad (2)$$

$$\left[\left(jL_p\omega + R_p\right) + \left(j\eta\omega\right)\left(\frac{-j\eta\omega}{R_s + jL_s\omega}\right)\right] i_p = E\, e^{j\omega t}$$

$$i_p = \frac{\left(R_s + jL_s\omega\right) E\, e^{j\omega t}}{R_s R_p + j\omega\left(L_s R_p + L_p R_s\right) - L_p L_s\omega^2 + \eta^2\omega^2}$$

si $k = 1$, $\eta = \sqrt{L_p L_s}$

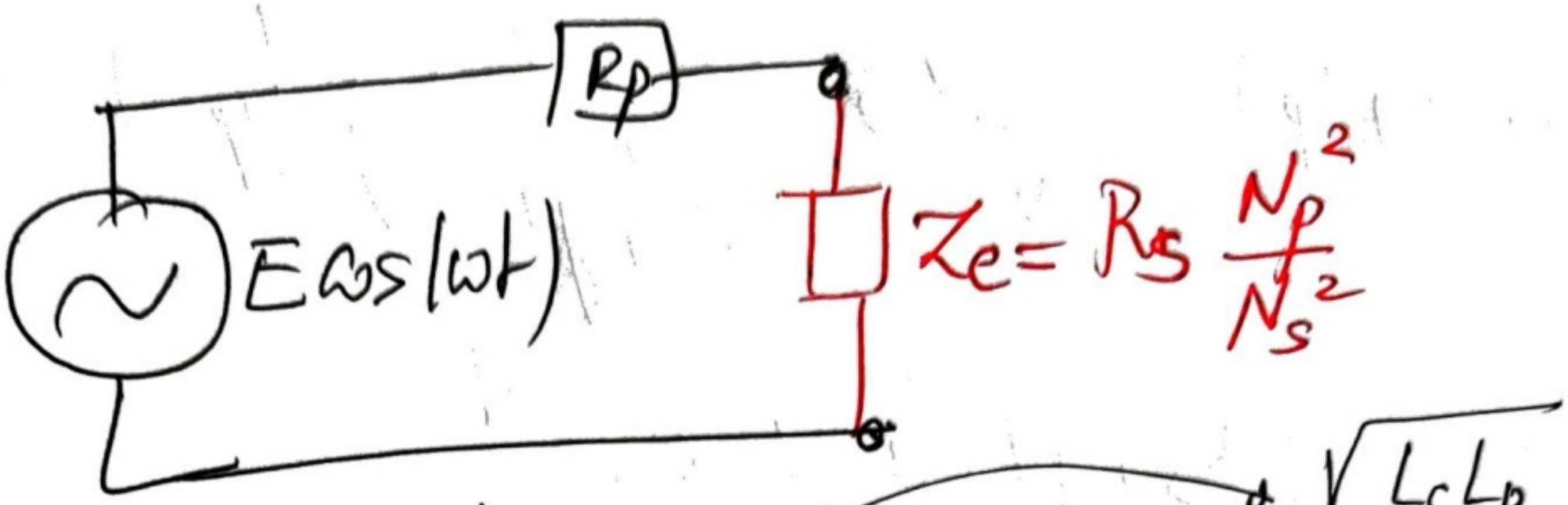
ou $R_s \ll \left(L_s\omega\right)$ et $R_p \ll \left(L_p\omega\right)$

$$i_p = \frac{jL_s\omega}{\left(L_s R_p + L_p R_s\right) j\omega} E\, e^{j\omega t}$$

$$i_p = \frac{E\, e^{j\omega t}}{\left(R_p + R_s \frac{L_p}{L_s}\right)}$$

$$E\, e^{j\omega t} = \left(R_p + \left(R_s \frac{N_p^2}{N_s^2}\right)\right) i_p$$

Z_c



$$\rightarrow \sqrt{L_s L_p}$$

$$\text{7)}\quad \frac{u_s}{u_e} = \frac{L_s \underline{i_s} + M \underline{i_p}}{L_p \underline{i_p} + M \underline{i_s}}$$

$$= \frac{\sqrt{L_s}\left(\sqrt{L_s}\,\underline{i_s} + \sqrt{L_p}\,i_p\right)}{\sqrt{L_p}\left(\sqrt{L_p}\,\underline{i_p} + \sqrt{L_s}\,\underline{i_s}\right)}$$

$$= \sqrt{\frac{L_s}{L_p}} = \boxed{\frac{N_s}{N_p} = m = \frac{u_s}{u_e}}$$

$m > 1$: élévateur de tension

$m < 1$: abaisseur de tension.

$m = 1$: masse flo Hente



$$\frac{i_s}{i_p} = \frac{-j\omega}{(R_s + jL_s\omega)} \approx -\frac{1}{L_s} \Rightarrow$$

$$\frac{i_s}{i_p} = \frac{-\sqrt{L_s L_p}}{L_s} = -\sqrt{\frac{L_p}{L_s}} = -\frac{N_p}{N_s}$$

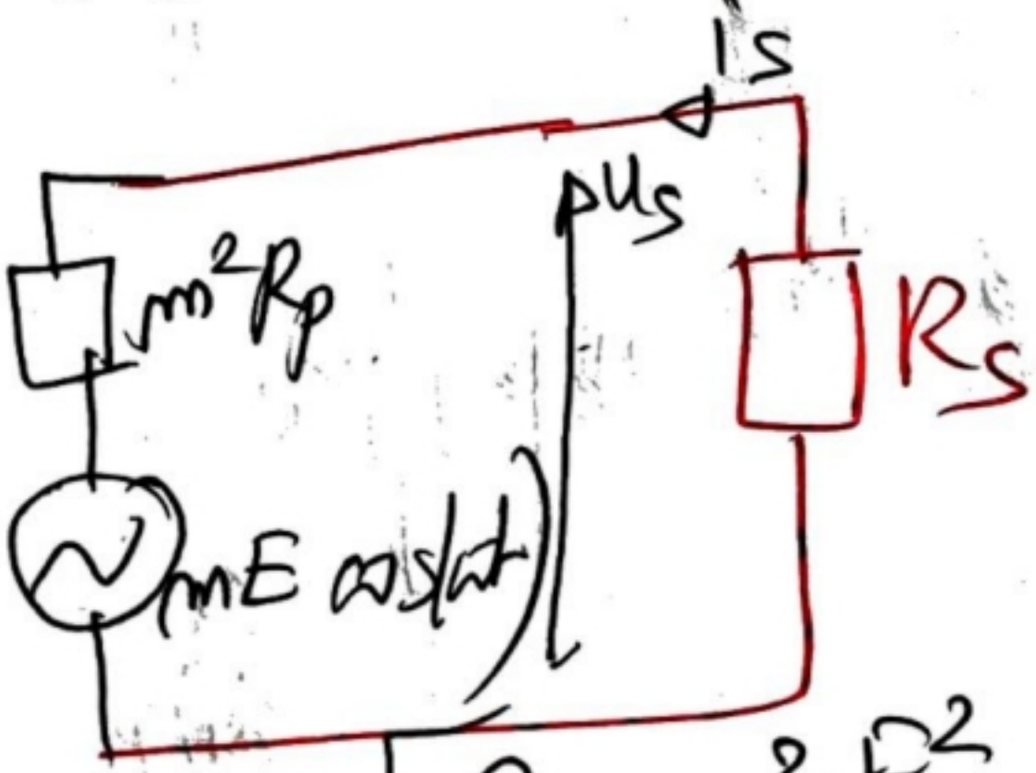
$$\frac{i_s}{i_p} = -\frac{1}{m}$$

$u_s \nearrow \quad i_s \searrow$

8) $u_s = m\,u_p = m\left(E\cos(\omega t) - R_p i_p\right)$

$u_s = m E\cos(\omega t) - \left(R_p m\right)\left(-m i_s\right)$

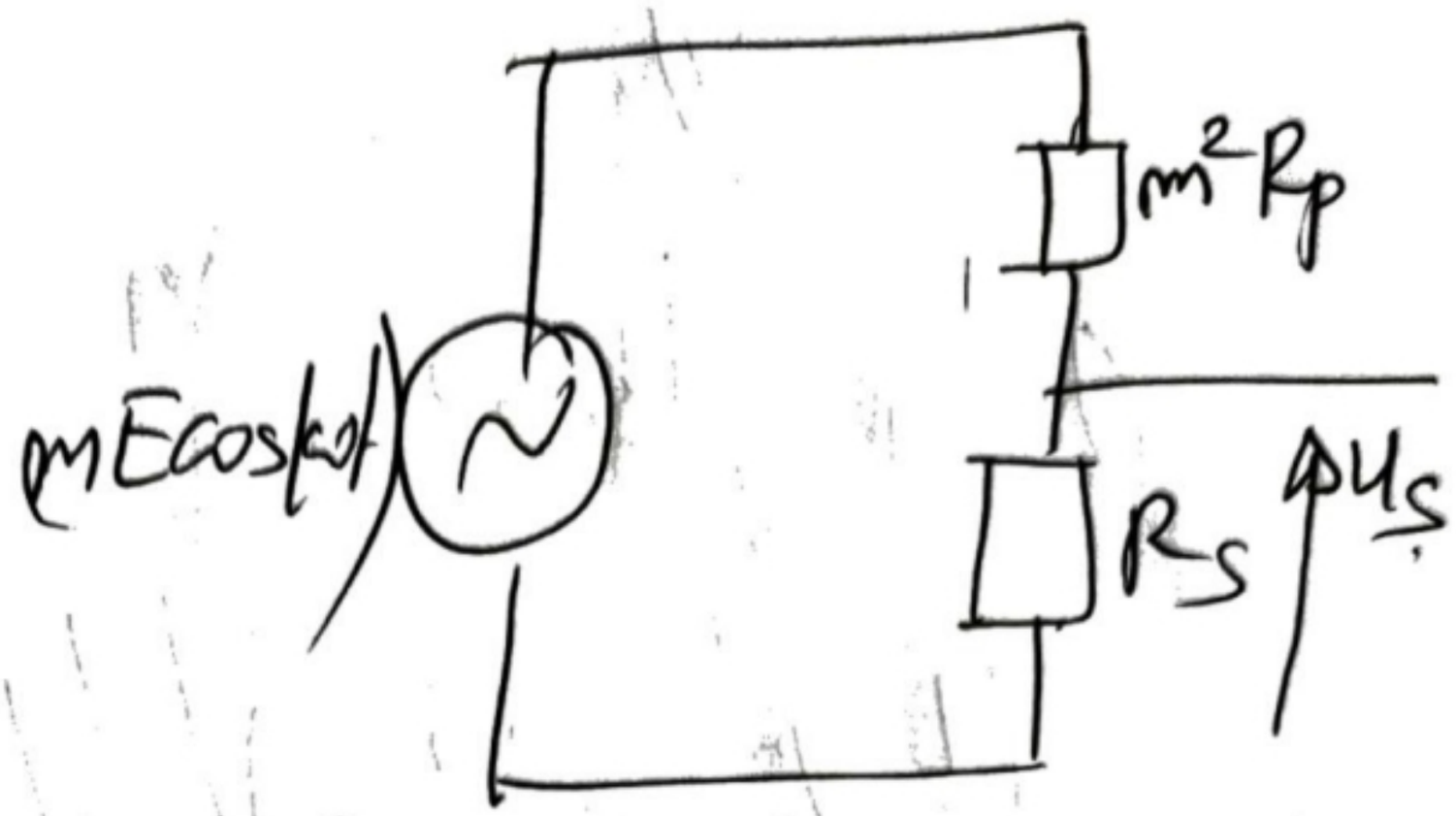
$u_s = m E\cos(\omega t) + \left(m^2 R_p\right) i_s$



$$P_s = \frac{u_s^2}{R_s}$$

$$u_s = \frac{R_s\, m E\cos(\omega t)}{(R_s + m^2 R_p)}$$

$$P_s = \left[\frac{R_s\, m^2 E^2}{(R_s + m^2 R_p)^2}\right]\cos^2(\omega t)$$

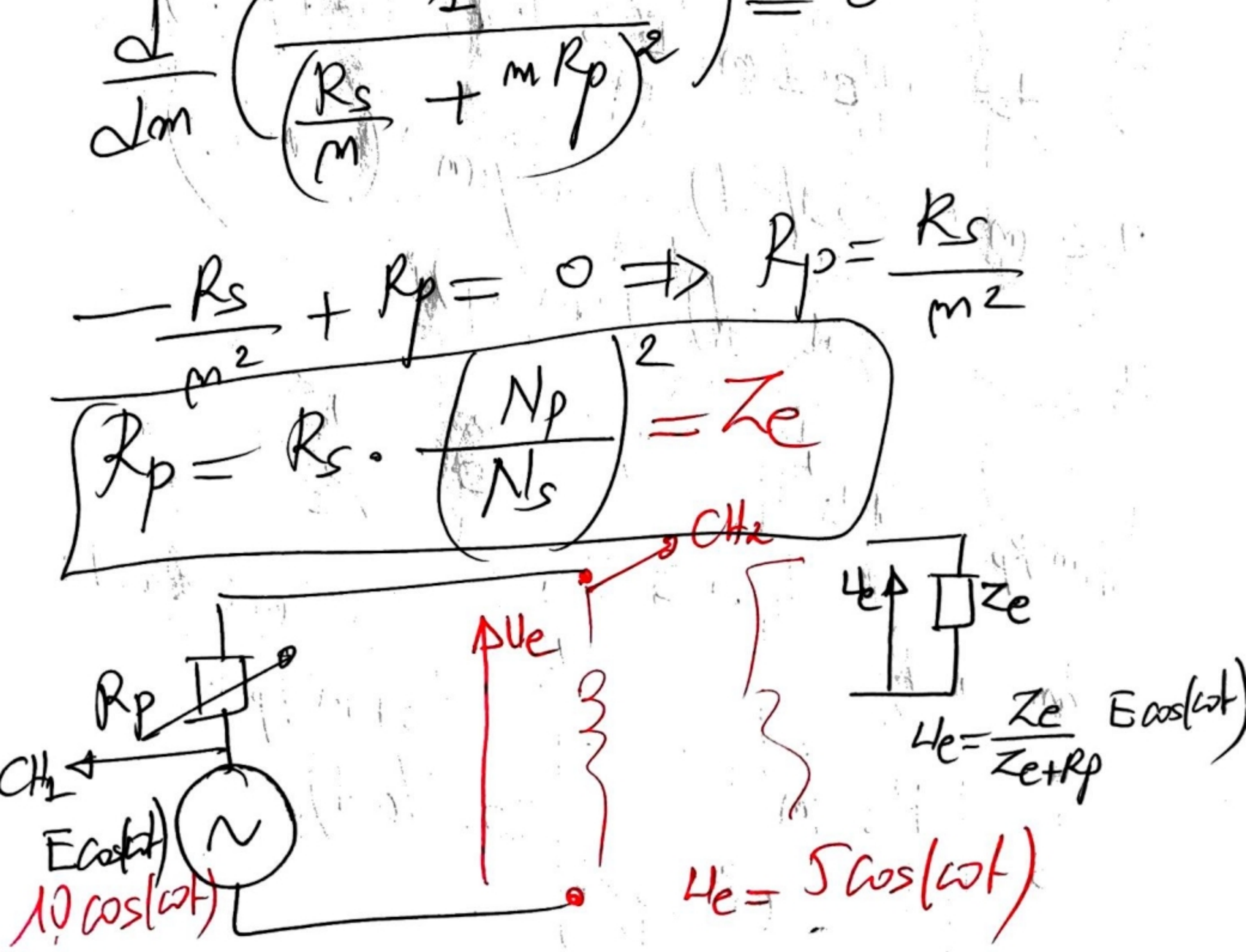


$$\langle P_s \rangle = \frac{R_s E^2}{2} \frac{m^2}{(m^2 R_p + R_s)^2}$$

$$\frac{d}{dm}\left(\frac{1}{\left(\frac{R_s}{m} + mR_p\right)^2}\right) = 0$$

$$-\frac{R_s}{m^2} + R_p = 0 \Rightarrow R_p = \frac{R_s}{m^2}$$

$$R_p = R_s \cdot \left(\frac{N_p}{N_s}\right)^2 = Ze$$



Ex4:

$$E_1 = R\, i_1 + L\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$E_2 = R\, i_2 + L\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$I = i_1 + i_2, \qquad J = i_1 - i_2$$

$$\frac{dI}{dt} + \frac{R}{(L+M)} I(t) = \frac{E_1 + E_2}{(L+M)}$$

$$E_1 - E_2 = R(J) + L\frac{dJ}{dt} - M\frac{dJ}{dt}$$

$$(L-M)\frac{dJ}{dt} + RJ = E_1 - E_2$$

$$\frac{dJ}{dt} + \frac{R}{L-M} J = \frac{E_1 - E_2}{L-M}$$

$$\frac{dJ}{dt} + \frac{J}{\tau_2} = \frac{E_1 - E_2}{L+M}$$

$$J(t) = A_2 e^{-t/\tau_2} + \frac{(E_1 - E_2)}{R} = i_1(t) - i_2(t)$$

$$I(t) = A_1 e^{-t/\tau_1} + \frac{E_1 + E_2}{R} = i_1 + i_2$$

$$i_2(t) = \frac{I(t) + J(t)}{2}$$

(EE) i_1 , (EE) i_2

$$E_1 i_1 + E_2 i_2 = R_1 i_1^2 + R_2 i_2^2 + \frac{d}{dt}\left(\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2\right)$$

2) a) $e_2(t) = \frac{q_1}{C} + L\frac{d^2 q_1}{dt^2} + M\frac{d^2 q_2}{dt^2}$

$$0 = \frac{q_2}{C} + L\frac{d^2 q_2}{dt^2} + M\frac{d^2 q_1}{dt^2}$$

b) $\ddot{q}_1 + \frac{M}{L}\ddot{q}_2 + \frac{1}{LC} q_1 = 0$

$$\ddot{q}_2 + \frac{M}{L}\ddot{q}_1 + \frac{1}{LC} q_2 = 0.$$

$$-\omega^2 q_1 + \frac{M}{L} q_2(-\omega^2) + \frac{1}{LC} q_1 = 0$$

$$\left(\frac{1}{LC} - \omega^2\right) q_1 = \frac{M}{L}\omega^2 q_2$$

Si $\omega = \omega_1$

$$\left[\frac{1}{LC} - \frac{1}{(L+M)C}\right] q_1 = \frac{M}{L}\frac{1}{(L+M)C} q_2$$

$$\frac{L+M-L}{(L+M)L} q_1 = \frac{M}{L(L+M)} q_2$$

$$\boxed{q_1 = q_2}$$

d) $M \ll L$ $\qquad \omega_1^2 = \frac{1}{(L+M)C} = \frac{1}{LC}\frac{1}{\left(1+\frac{M}{L}\right)}$

$$\omega_1^2 = \omega_0^2\left(1+\frac{M}{L}\right)^{-1}$$

$$\omega_1 = \omega_0\left[1+\frac{M}{L}\right]^{-1/2}$$

$$\boxed{\ddot{p} + \omega_1^2 p = 0} \Rightarrow p(t) = 2 p_0 \cos(\omega_1 t + \varphi_1)$$

$$\omega_1^2 = \frac{1}{(L+M)C}$$

$$\ddot{r}\left(1 - \frac{M}{L}\right) + \frac{1}{LC} r = 0$$

$$\ddot{r} + \frac{1}{C(L-M)} r = 0$$

$$\boxed{\ddot{r} + \omega_2^2 r = 0}, \quad \omega_2^2 = \frac{1}{C(L-M)}$$

$$r(t) = 2 r_0 \cos(\omega_2 t + \varphi_2)$$

$$q_1(t) = \frac{p(t) + r(t)}{2}$$

$$q_2 = p_0 \cos(\omega_1 t + \varphi_1) + r_0 \cos(\omega_2 t + \varphi_2)$$

$$q_2 = p_0 \cos(\omega_1 t + \varphi_1) - r_0 \cos(\omega_2 t + \varphi_2)$$

$\frac{q_1}{q_2} = +1$ synéchiga, $\omega = \omega_1$

$\frac{q_1}{q_2} = -1$ Antisy $\omega = \omega_2$

$$\omega_1 \simeq \omega_0 \left(1 - \frac{\Gamma}{2L}\right)$$

$$\omega_2 \simeq \omega_0 \left(1 + \frac{\Gamma}{2L}\right)$$

* $i_1(0) = 0 = -p_0 \omega_1 \sin\varphi_1 - r_0 \omega_2 \sin\varphi_2$

$i_2(0) = 0 = -p_0 \omega_1 \sin\varphi_1 + r_0 \omega_2 \sin\varphi_2$

$\sin\varphi_2 = 0$;

$\sin\varphi_1 = 0$

$\varphi_2 = 0$

$\varphi_1 = 0$

$q_1(t) = p_0' \cos(\omega_2 t) + r_0 \cos(\omega_2 t)$

$q_2(t) = p_0 \cos(\omega_1 t) - r_0 \cos(\omega_2 t)$

$q_2(0) = 0 \Rightarrow p_0 = r_0$

$q_1(t) = p_0 \left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$

$\frac{q_1(0)}{C} = u_0$;

$u_{c_1}(t) = \frac{u_0}{2}\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$

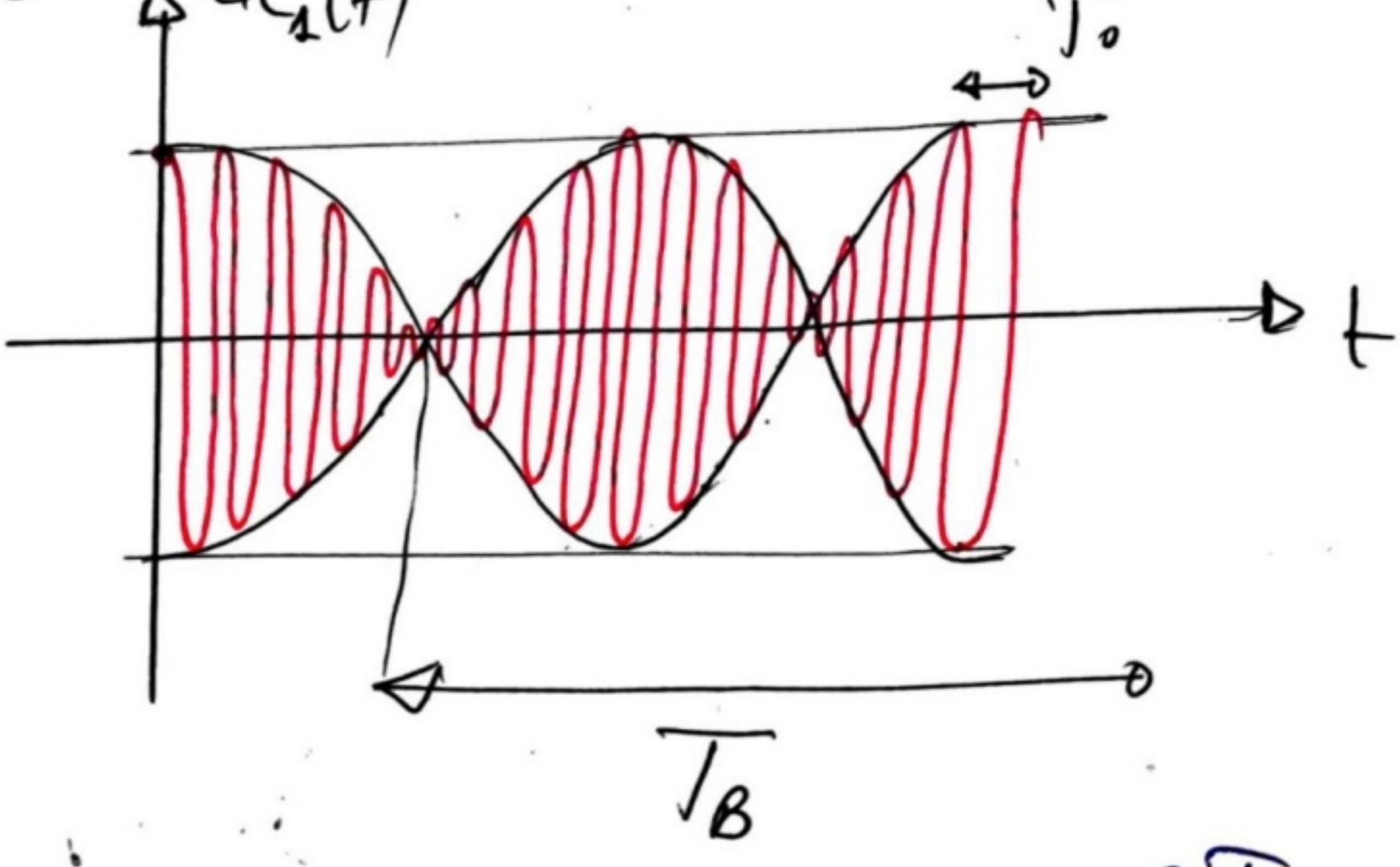
$u_{c_2}(t) = \frac{u_0}{2}\left[\cos(\omega_1 t) - \cos(\omega_2 t)\right]$

$$u_{c_1}(t) = u_0 \cos\left[\left(\frac{\omega_2-\omega_1}{2}\right)t\right] \cos\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right]$$

$$u_{c_1}(t) = u_0 \cos\left(\frac{M\,\omega_0}{2L}\,t\right) \cos(\omega_0 t)$$

$\frac{M}{2L}\omega_0 \ll \omega_0$

$\frac{M}{2L}$ fréquence faible $\Rightarrow$ T : période $\gg T_0$



$$\frac{M}{2L} \cdot \frac{2\pi}{T_0} = \frac{2\pi}{T_B} \Rightarrow \frac{2T_0}{T_B} = \frac{M}{L} = k$$

Battement

3) $\begin{cases} \ddot{q}_1 + C_c \ddot{q}_2 + \omega_0^2 q_1 = \dfrac{E}{L} e^{j\omega t} \\ \ddot{q}_2 + C_c \ddot{q}_1 + \omega_0^2 q_2 = 0 \end{cases}$

$\Rightarrow$ $\begin{cases} (\omega_0^2 - \omega^2) q_1 + (-\omega^2 C_c) q_2 = \dfrac{E}{L} e^{j\omega t} \\ -C_c \omega^2 q_1 + (\omega_0^2 - \omega^2) q_2 = 0 \end{cases}$

$$\Delta = \begin{vmatrix} (\omega_0^2 - \omega^2) & -\omega^2 C_c \\ -C_c \omega^2 & (\omega_0^2 - \omega^2) \end{vmatrix} \neq 0$$

$$\Delta = (\omega_0^2 - \omega^2)^2 - (\omega^2 C_c)^2$$

$$= \left[\omega_0^2 - \omega^2 - C_c\omega^2\right]\left[\omega_0^2 - \omega^2 + C_c\omega^2\right]$$

$$= \omega_0^4 \left[1 - \frac{(1+C_c)\omega^2}{\omega_0^2}\right]\left[1 - \frac{(1-C_c)\omega^2}{\omega_0^2}\right]$$

$$= \omega_0^4 \left[1 - \frac{\omega^2}{\omega_1^2}\right]\left[1 - \frac{\omega^2}{\omega_2^2}\right]$$

$$\frac{q_1}{C} = \frac{\begin{vmatrix} \frac{E}{LC} e^{j\omega t} & -\omega^2 C_c \\ 0 & (\omega_0^2 - \omega^2) \end{vmatrix}}{\Delta}$$

$$\frac{q_2}{C} = \frac{\begin{vmatrix} \omega_0^2 - \omega^2 & \frac{E}{LC} e^{j\omega t} \\ -C_c \omega^2 & 0 \end{vmatrix}}{\Delta}$$

$$\begin{cases} \alpha x + \beta y = \gamma \\ \alpha' x + \beta' y = \gamma' \end{cases}$$

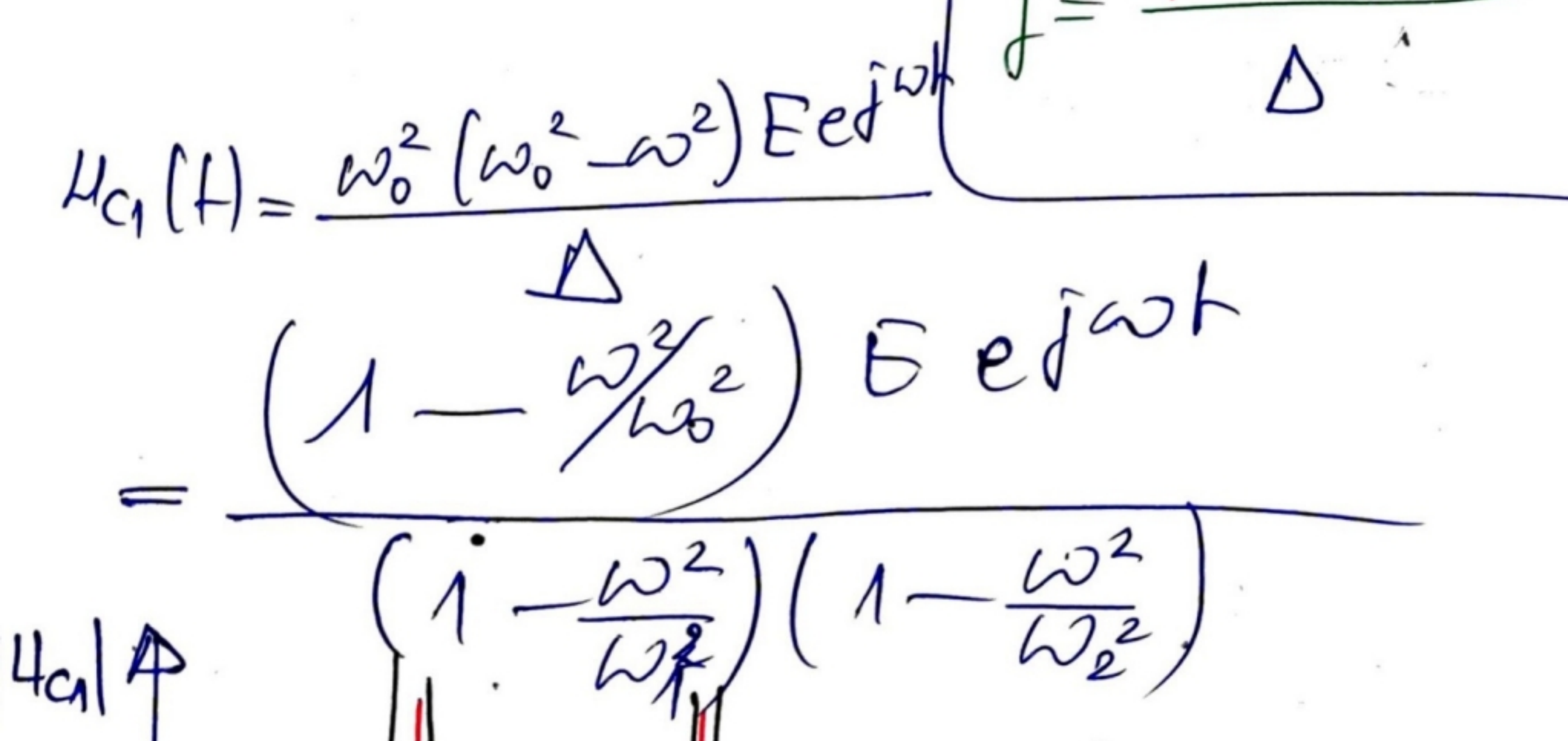
$$\Delta = \begin{vmatrix} \alpha & \beta \\ \alpha' & \beta' \end{vmatrix} \neq 0$$

$$x = \frac{\begin{vmatrix} \gamma & \beta \\ \gamma' & \beta' \end{vmatrix}}{\Delta}$$

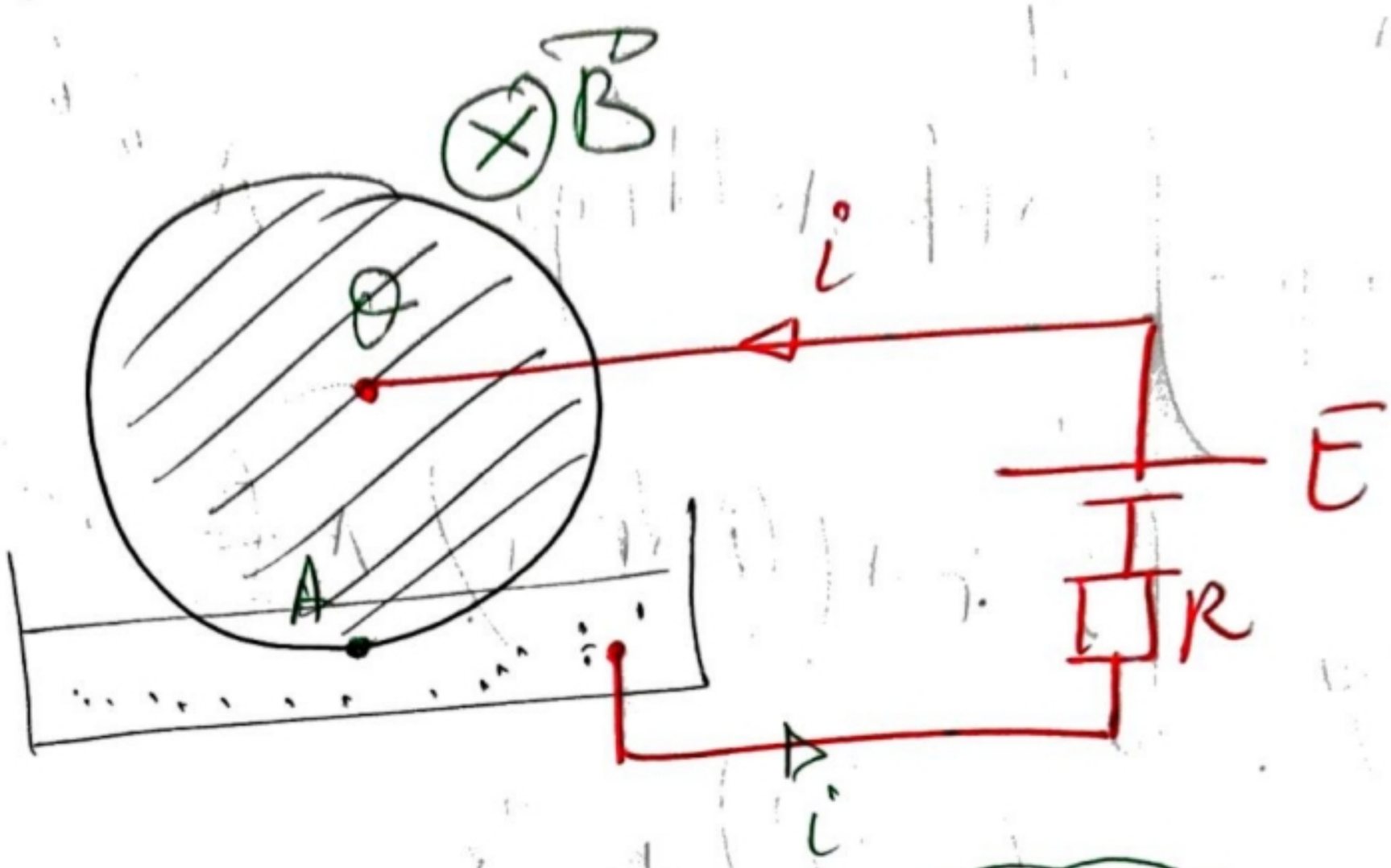
$$y = \frac{\begin{vmatrix} \alpha & \gamma \\ \alpha' & \gamma' \end{vmatrix}}{\Delta}$$

$$U_{C_1}(t) = \frac{\omega_0^2 (\omega_0^2 - \omega^2) E e^{j\omega t}}{\Delta}$$

$$= \frac{\left(1 - \omega^2/\omega_0^2\right) E e^{j\omega t}}{\left(1 - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right)}$$

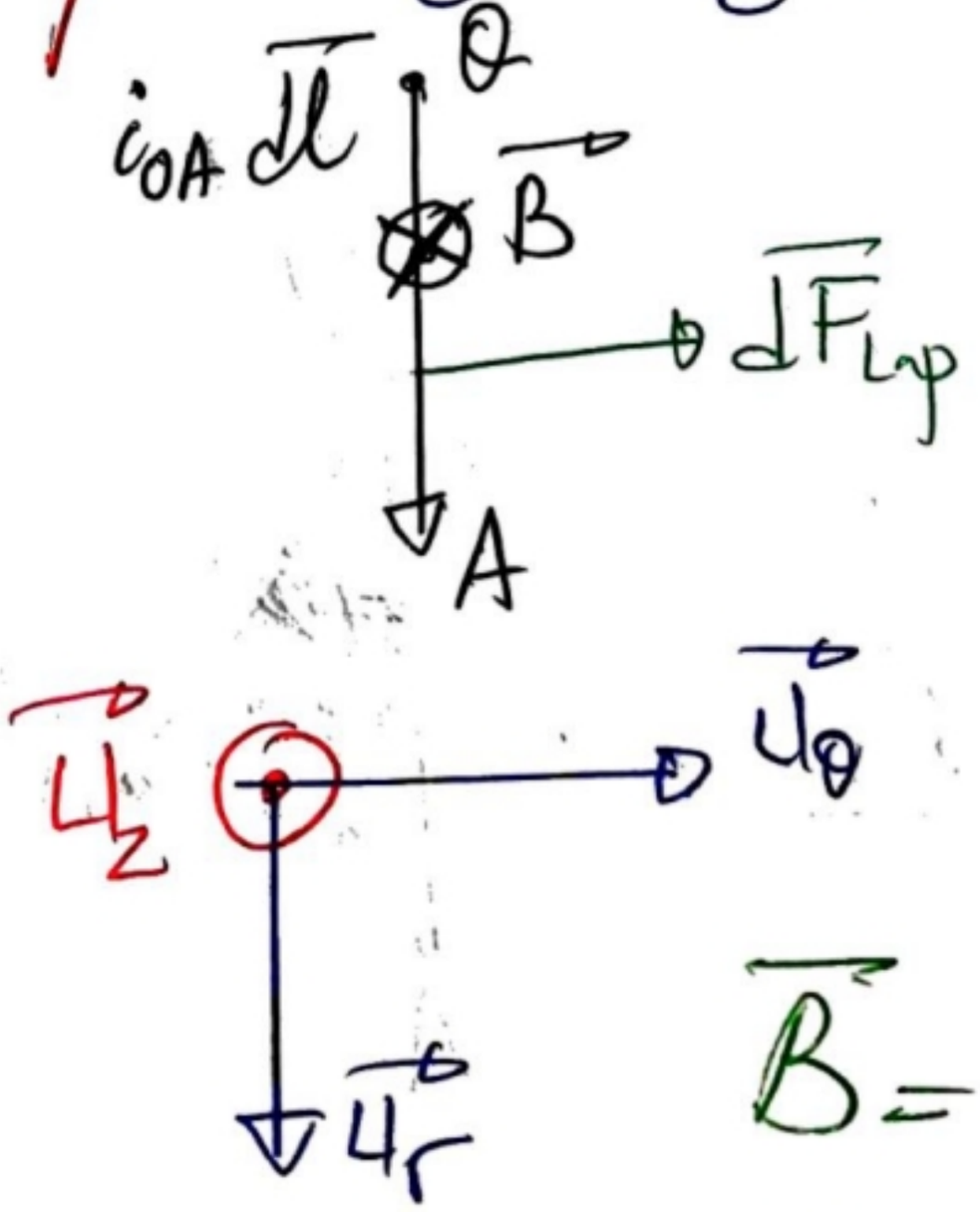


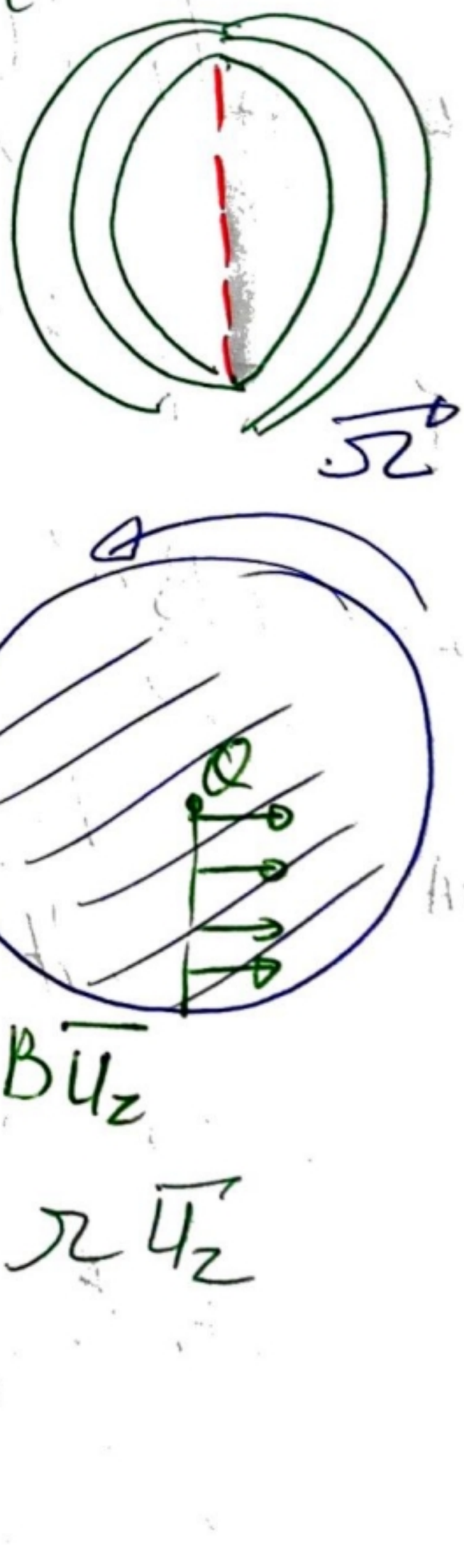
Roue de Barlow



$$d\vec{F}_{Lap} = i\, d\vec{\ell} \wedge \vec{B}$$

$d\vec{\ell}$: من O إلى A

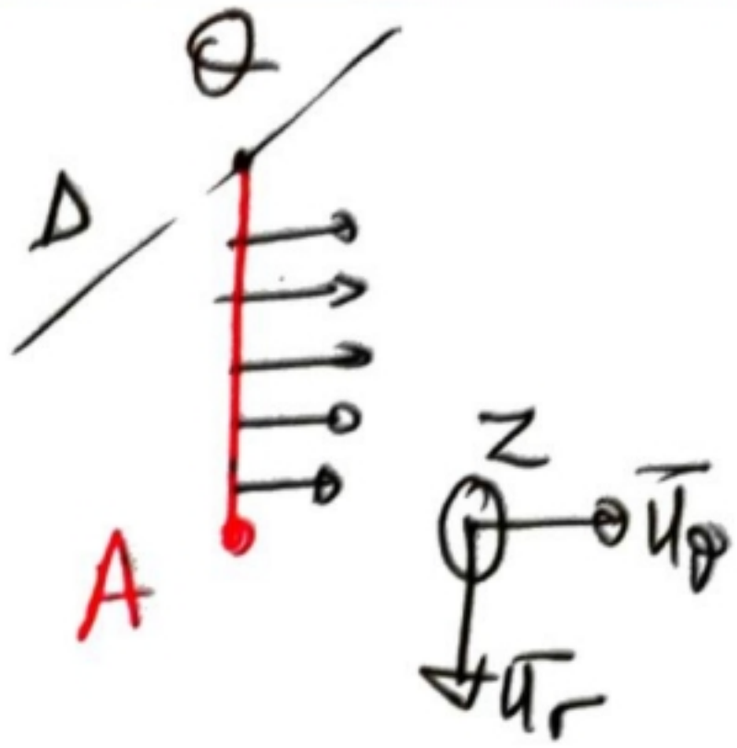




$$\vec{B} = -B\vec{u}_z$$

$$\vec{\Omega} = +\Omega\vec{u}_z$$

2) $e_{OA} = -\frac{d\Phi}{dt}$



$$\vec{dM}_{Lap} = \int \vec{OM} \wedge d\vec{F}_{Lap}$$

$$\vec{dM}_{Lap} = \int_0^a r\vec{e}_r \wedge \left((i\, dr\, \vec{e}_\theta) \wedge (-B\vec{e}_z) \right)$$

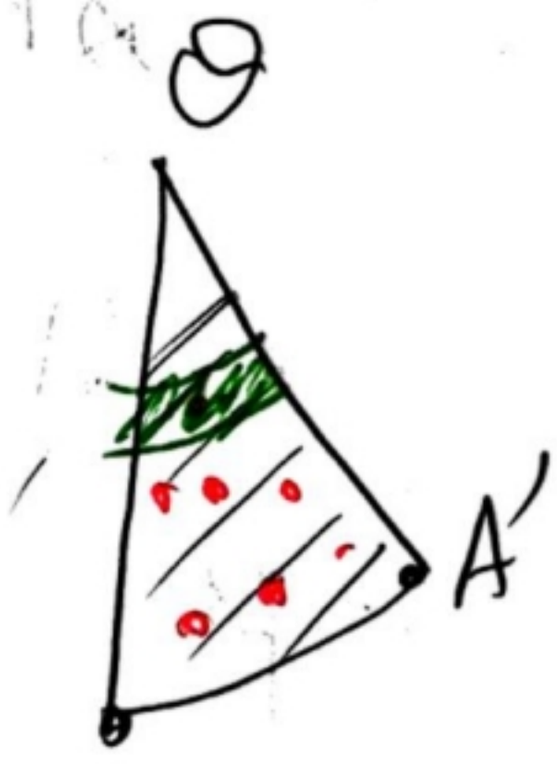
$$\vec{dF}_{Lap} = (i\, \vec{dl}) \wedge \vec{B}$$

$$\vec{M}_{Lap} = +i B \int_0^a r\, dr\, \vec{e}_z$$

$$\vec{M}_{Lap} = \frac{iBa^2}{2}\vec{e}_z$$

- $e_{OA} = -\frac{d\,\Phi_{Lapl\acute{e}e}}{dt}$

$$d\Phi_c = \vec{B}\cdot d\vec{S}_c$$



$$= -B\vec{e}_z \cdot \left[dr\,\vec{e}_r \wedge r\,d\theta\,\vec{e}_\theta \right]$$

$$= -B\vec{e}_z \cdot \left[r\,d\theta\,\vec{u}_\theta \wedge dr\,\vec{u}_r \right]$$

$$d^2\Phi_c = +B\, r\, d\theta\, dr$$

$$d\Phi_c = \int d^2\Phi_c = \int_0^a B\, 2r\, dr\, d\theta$$

$$d\Phi_c = \frac{Ba^2}{2}\, d\theta \;,\quad \boxed{e_{OA} = -\frac{Ba^2}{2}\omega}$$

• 2ème methode $\boxed{P_{Lap} + e_{OA}\, i = 0}$

Lois ← Lorentz

$$e_{OA} = -\frac{P_{Lap}}{i} = -\frac{d\vec{M}\cdot\vec{\omega}}{i}$$

$$\boxed{e_{OA} = -\frac{Ba^2}{2}\omega}$$

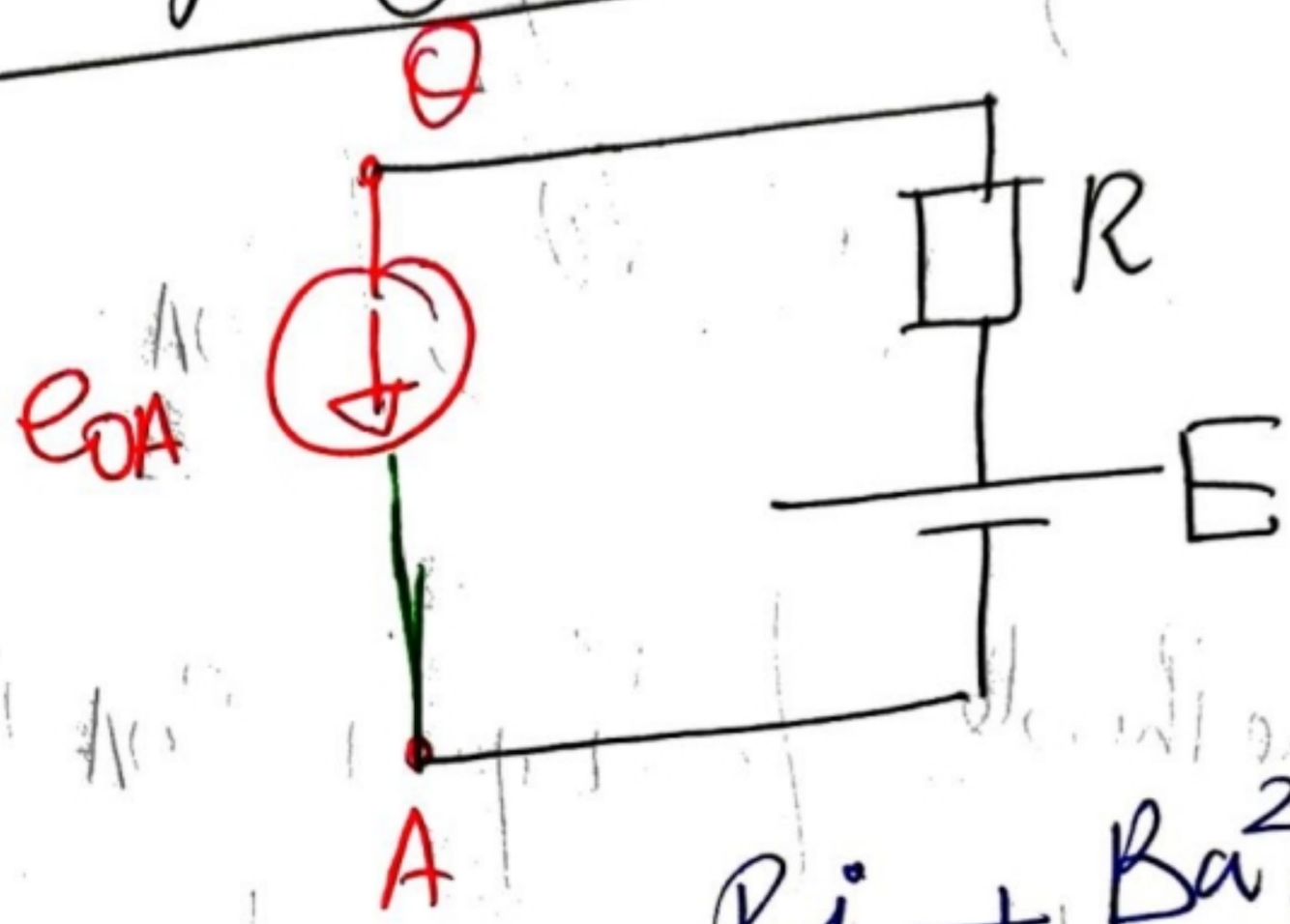
$$\delta W = \vec{F}\cdot d\vec{l} + \frac{\partial M}{\partial \theta}\cdot d\theta$$

$$P = \vec{F}\cdot\vec{v} + \frac{\partial M}{\partial t}\cdot\vec{\omega}$$

$$e_{OA} = \int \vec{E}_m \cdot \vec{dl} = \int (\vec{v} \wedge \vec{B}) \cdot \vec{dl}$$

Hors programme

c)



- $E = Ri - e_{OA} = Ri + \frac{Ba^2\omega}{2} = E$

- Eq mécanique : T.M.C

$$\sum M_O(\vec{F}_{ext}) = \frac{d\vec{L_O}}{dt}$$

$$\vec{L_O} = \vec{\sigma_O} = \begin{cases} \vec{OM} \wedge m\vec{v} \\ J\vec{\omega} \\ \hookrightarrow \text{moment d'inertie} \end{cases}$$

R.F.D : $\sum \vec{F}_{ext} = \frac{d\vec{p}}{dt}$

$$\vec{M}_O(\vec{F}_{Lap}) + \vec{M}_O(\vec{R}) + \vec{M}_O(\vec{P}) = J\frac{d\vec{\omega}}{dt}$$

$\vec{M}_O(\vec{R}) = \vec{0}$, $\vec{M}_O(\vec{P}) = \vec{0}$

спирає $|\Delta|$

$$\frac{Ba^2 i_\kappa}{2} = J\frac{d\omega}{dt} \quad (E.M)$$

$$E = Ri + \frac{Ba^2\omega}{2} \quad (EE)$$

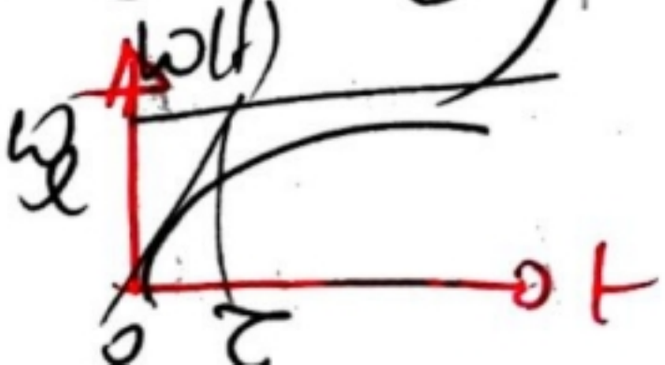
$$J\frac{d\omega}{dt} = \frac{Ba^2}{2}\left(\frac{E}{R} - \frac{B^2a^2\omega}{2R}\right)$$

$$\frac{J\omega}{dt} + \frac{B^2a^4}{4RJ}\omega = \frac{Ba^2E}{2RJ}$$

$$\frac{d\omega}{dt} + \frac{\omega}{\tau} = \frac{\omega_0}{\tau}$$

$$\omega(t) = \omega_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$\omega(0) = 0$;



$$\frac{\omega_l}{\tau} = \frac{Ba^2E}{2RJ} \quad , \quad \omega_l = \frac{Ba^2E}{2RJ} \times \frac{4RJ}{B^2a^4}$$

$$\tau = \frac{4RJ}{B^2a^4} \quad ; \quad \omega_l = \frac{2E}{Ba^2}$$

$$\frac{dJ}{Ei} = Ri^2 + \frac{Ba^2\omega}{2} i$$

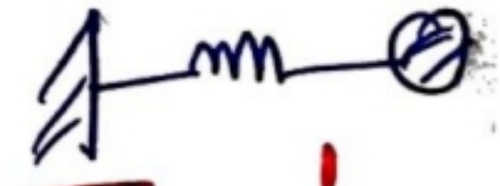
$$J\omega\frac{d\omega}{dt} = \frac{Ba^2 i^\circ \omega}{2}$$

$$Ei = Ri^2 + \frac{d}{dt}\left(\frac{1}{2}J\omega^2\right)$$

Ei → Générateur ; Ri^2 → puissance calorifique ; $\frac{d}{dt}\left(\frac{1}{2}J\omega^2\right)$ → puissance cinétique de rotation

$$E_C = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2$$

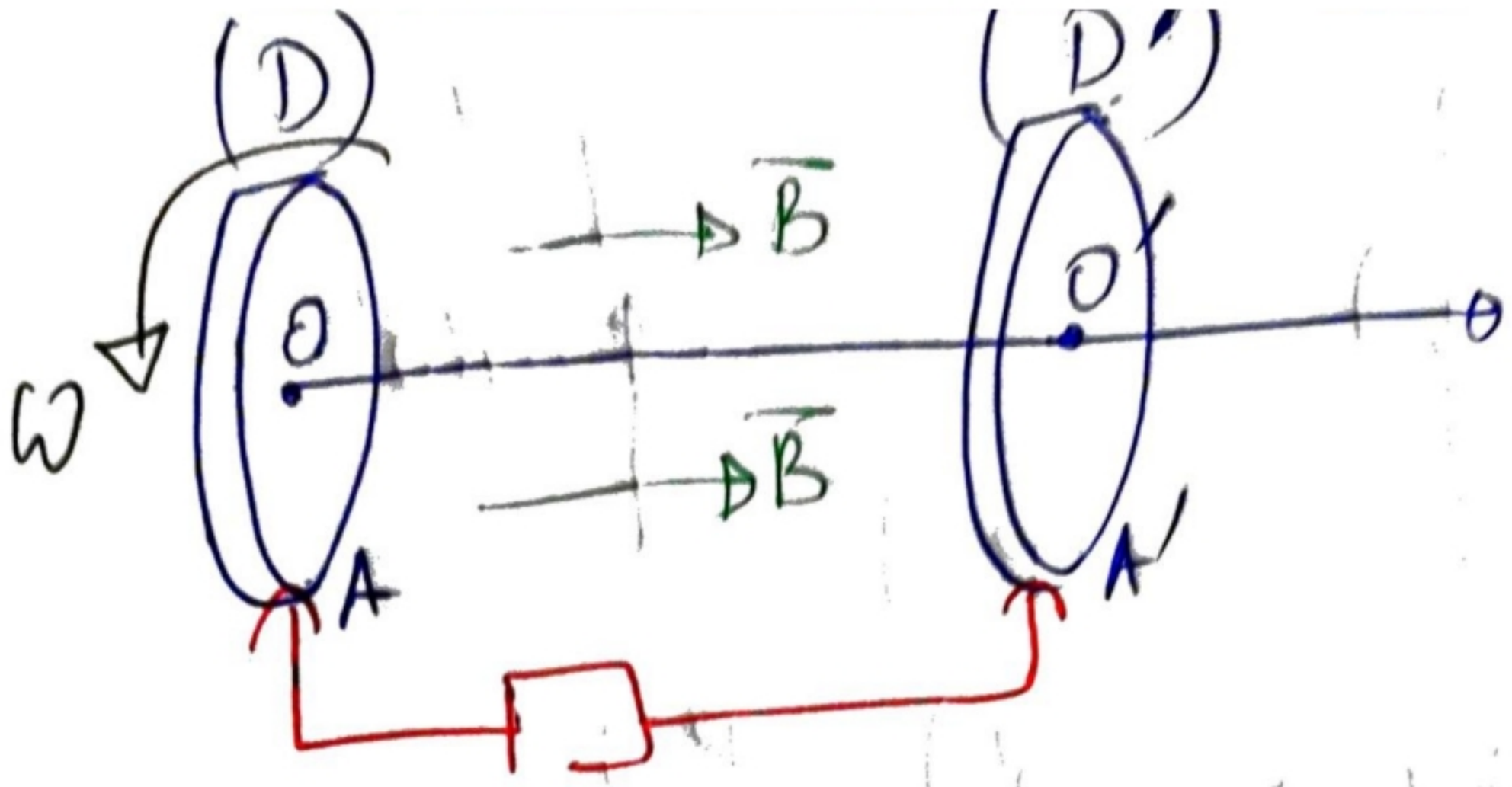
$$E_{Pelastique} = \frac{1}{2}kx^2 + \frac{1}{2}C\theta^2$$



$$T = -kx$$



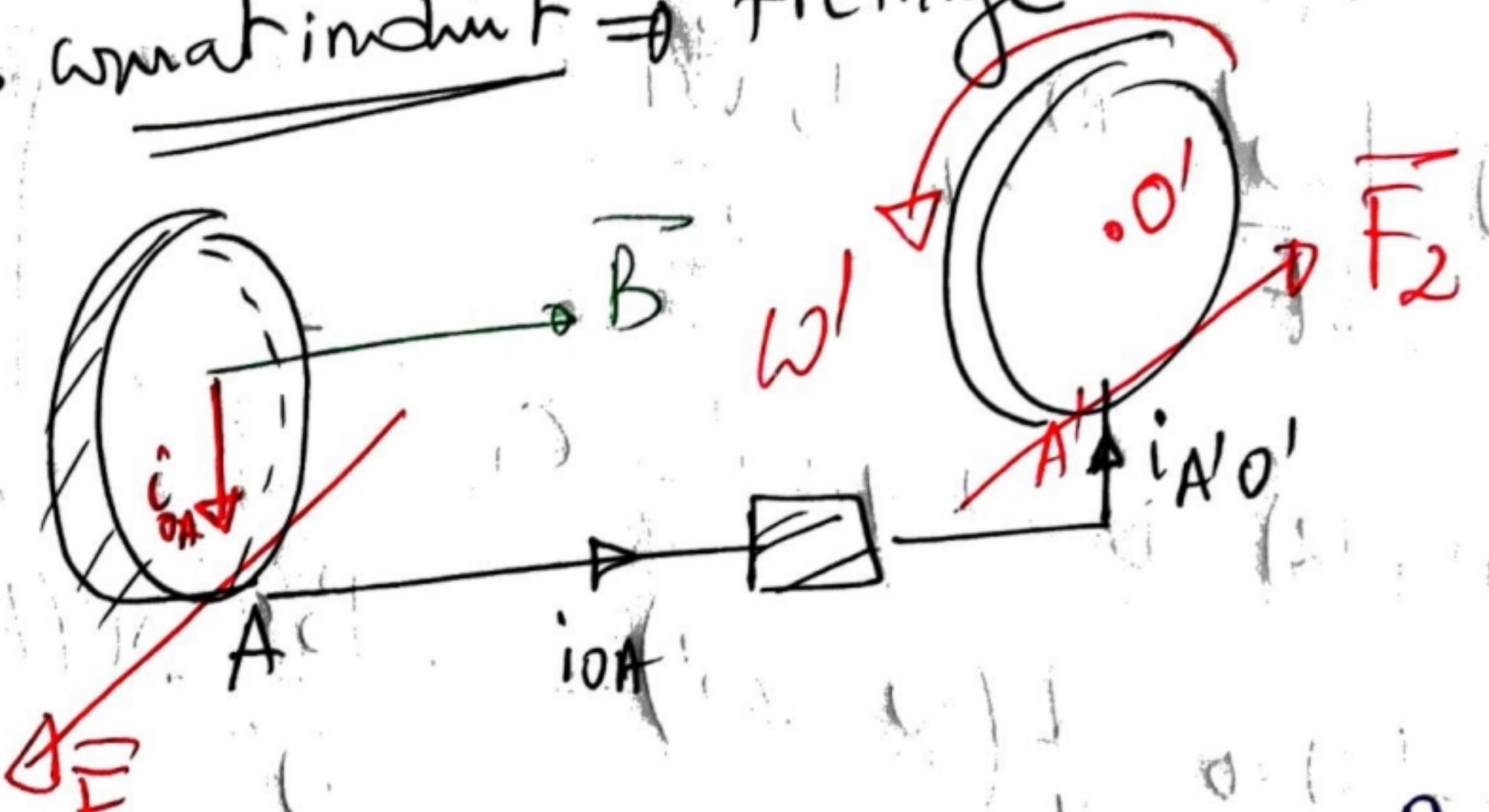
$$\vec{\Gamma} = -C\theta\vec{k}$$



$\omega(0) = \omega_0$ et $\omega'(0) = 0$

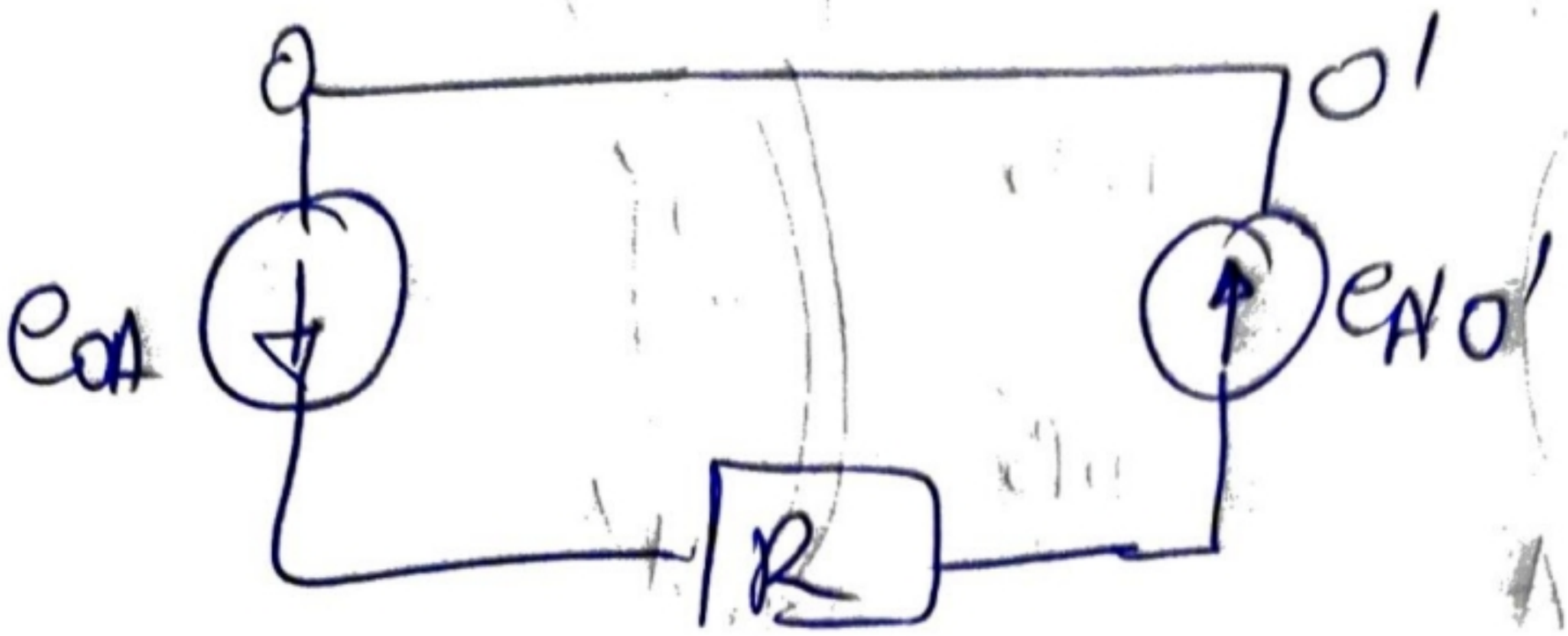
(D): circuit mobile dans un champ uniforme

→ aimant induit ⇒ freinage



$$\vec{\mathcal{M}}_1 = -\frac{iBa^2}{2}\vec{e}_z \qquad e_{OA} = +\frac{Ba^2\omega}{2}$$

$$\vec{\mathcal{M}}_2 = +\frac{iBa^2}{2}\vec{e}_z \qquad e_{A'O'} = -\frac{Ba^2\omega'}{2}$$



b) $\frac{Ba^2}{2}(\omega - \omega') = Ri$

c) $J\frac{d\omega}{dt} \overset{(1)}{=} -\frac{Ba^2}{2}i$

$J\frac{d\omega'}{dt} \overset{(2)}{=} +\frac{Ba^2 i}{2}$

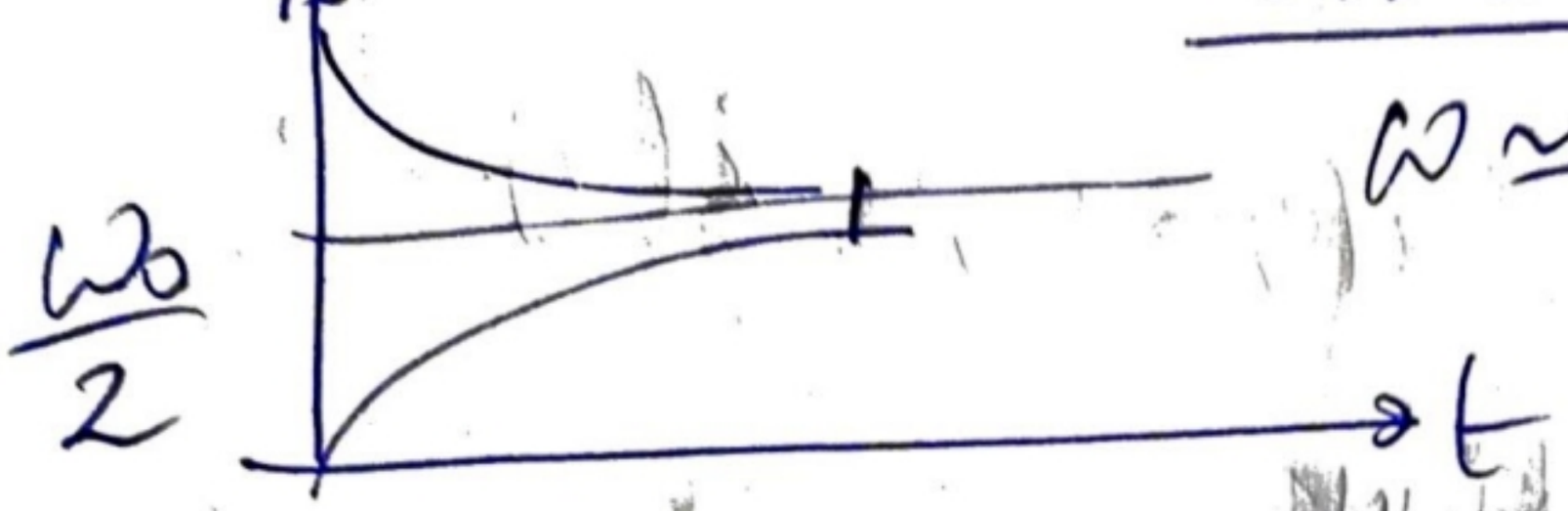
d) $(1)+(2) \Rightarrow \boxed{\omega + \omega' = C_1}$

$(1)-(2)=0 \quad \frac{d}{dt}(\omega - \omega') = -\frac{Ba^2}{J}i(t)$

$\frac{d}{dt}(\omega - \omega') = -\frac{B^2a^4}{2JR}(\omega - \omega')$

$\boxed{\omega - \omega' = C_2 \exp\left(-\frac{t}{\tau}\right)}$

$$\begin{cases} \omega(t) = \frac{\omega_0}{2}\left(1 + e^{-t/\tau}\right) \\ \omega'(t) = \frac{\omega_0}{2}\left(1 - e^{-t/\tau}\right) \end{cases}$$



$t \gg \tau$

$\omega \simeq \omega' \simeq \frac{\omega_0}{2}$

$Ri(\infty) \to 0$,

e) $\frac{d}{dt}\left(\frac{1}{2}J\omega^2\right) = \frac{d}{dt}\left(\frac{1}{2}J\omega'^2\right) + Ri^2$

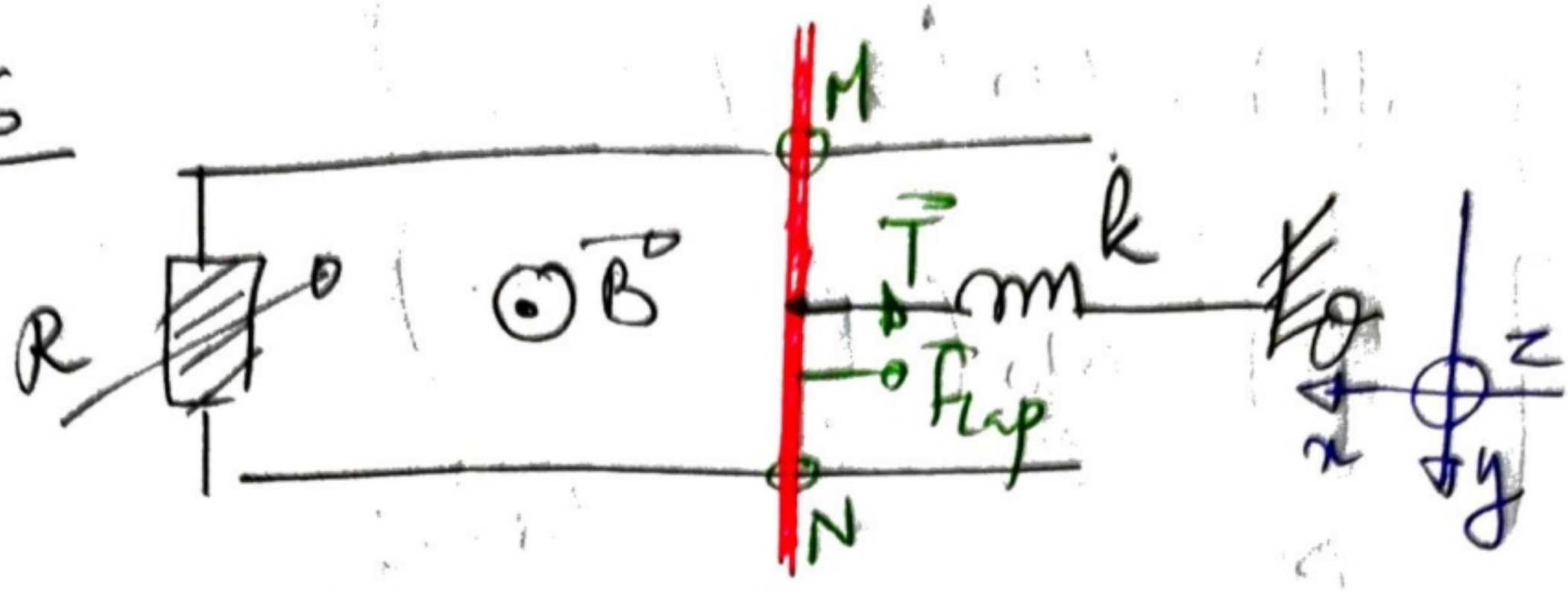
Effet thermique

$E_{CR}(D)$

$E_{CR}(D')$

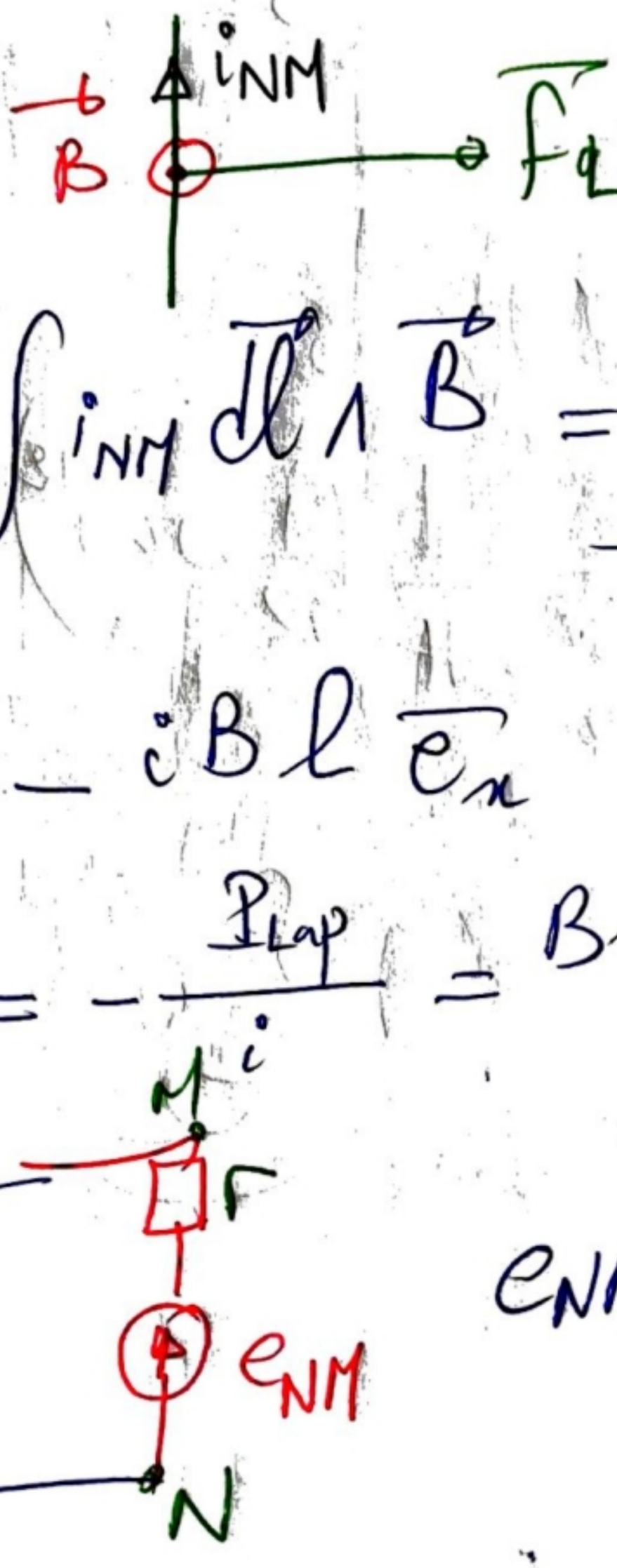
Diminuo

Ex6



$\underline{NM = l}$, $x(0) = a$; $\dot{x}(0) = 0$

a)



$$\vec{f}_{Lap} = \int i_{NM} \, d\vec{l} \wedge \vec{B} = \int_{y_N}^{y_M} i \, dy \, \vec{e}_y \wedge B \vec{e}_z$$

$$\vec{f}_{Lap} = -iBl\vec{e}_x$$

$$e_{NM} = -\frac{f_{Lap}}{i} = Blv$$

$$e_{NM} = (R+r)i = Blv$$

$$m\frac{d\vec{v}}{dt} = \vec{F_{Lap}} + m\vec{g} + \vec{T} + \vec{R}$$

(ox): $m\frac{dv}{dt} = -iBl - kx$

$$m\ddot{x} + Bl\dot{i} + kx = 0$$

$$m\ddot{x} + \frac{B^2l^2}{(R+r)m}\dot{x} + \frac{k}{m}x = 0 .$$

b) $$\alpha^2 + \frac{B^2l^2}{m(R+r)}\alpha + \frac{k}{m} = 0$$

$$\Delta = \left(\frac{B^2l^2}{m(R+r)}\right)^2 - \frac{4k}{m} < 0$$

والنظام يتذبذب فيسمى Critique :

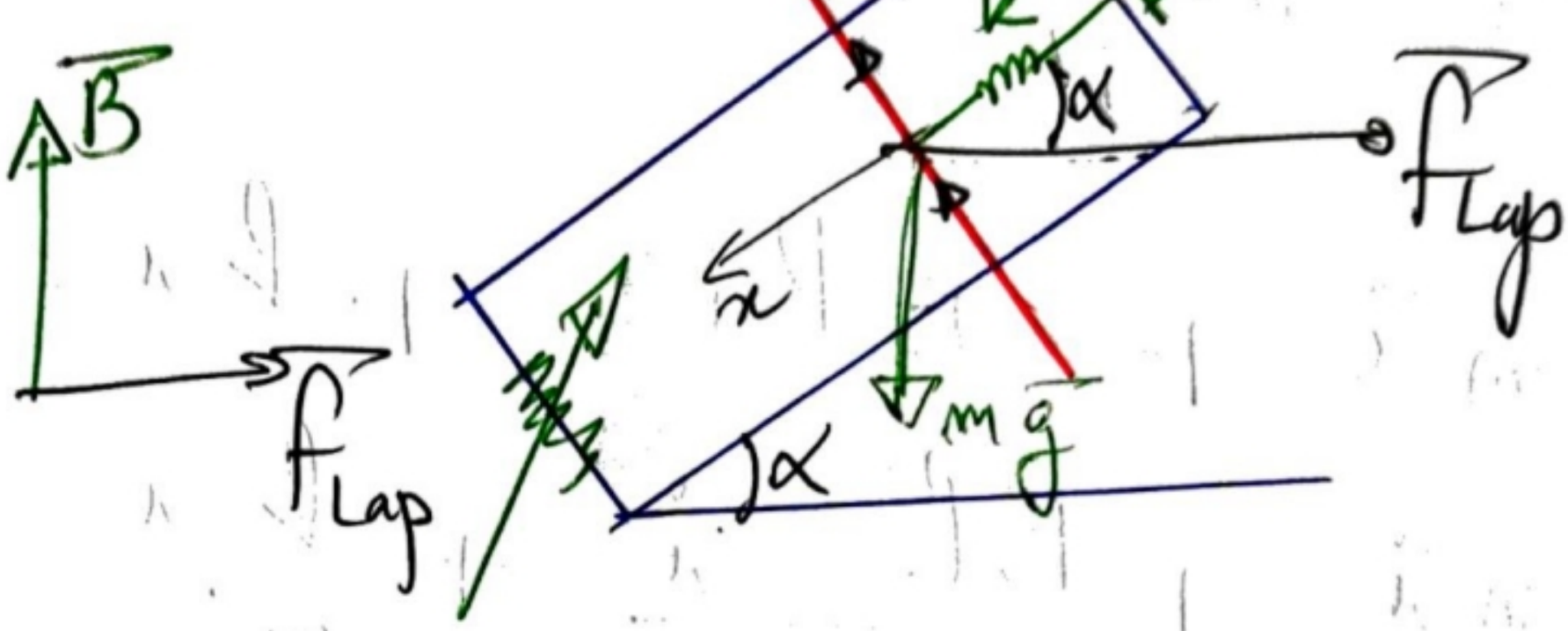
$$\alpha_1 = \alpha_2 = -\frac{B^2l^2}{2m(R+r)} = -\sqrt{\frac{k}{m}} = -\lambda$$

$$x(t) = (A_1t + A_2)e^{-\lambda t}$$

$x(0) = a$, $\dot{x}(0) = 0$

$$x(t) = a\left(\sqrt{\frac{ak}{m}}\,t + 1\right) \exp\left(-\frac{t\sqrt{k}}{\sqrt{m}}\right)$$

2)



$$\left(\vec{f}_{Lap}\right)_x = -iBl\cos\alpha$$

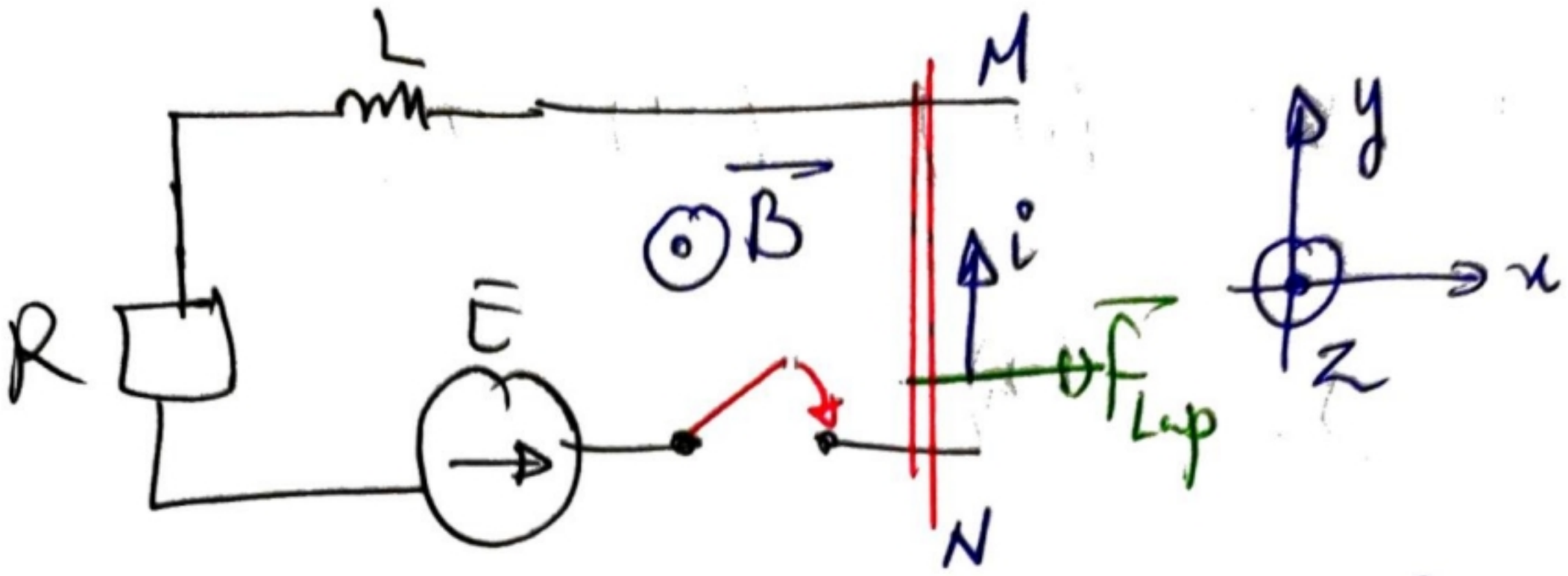
$$e_{NM} = -\frac{f_{Lap}}{i} = -\frac{\vec{F}_{Lap}\cdot\vec{v}}{i}$$

$$e_{NN} = +Blv\cos\alpha$$

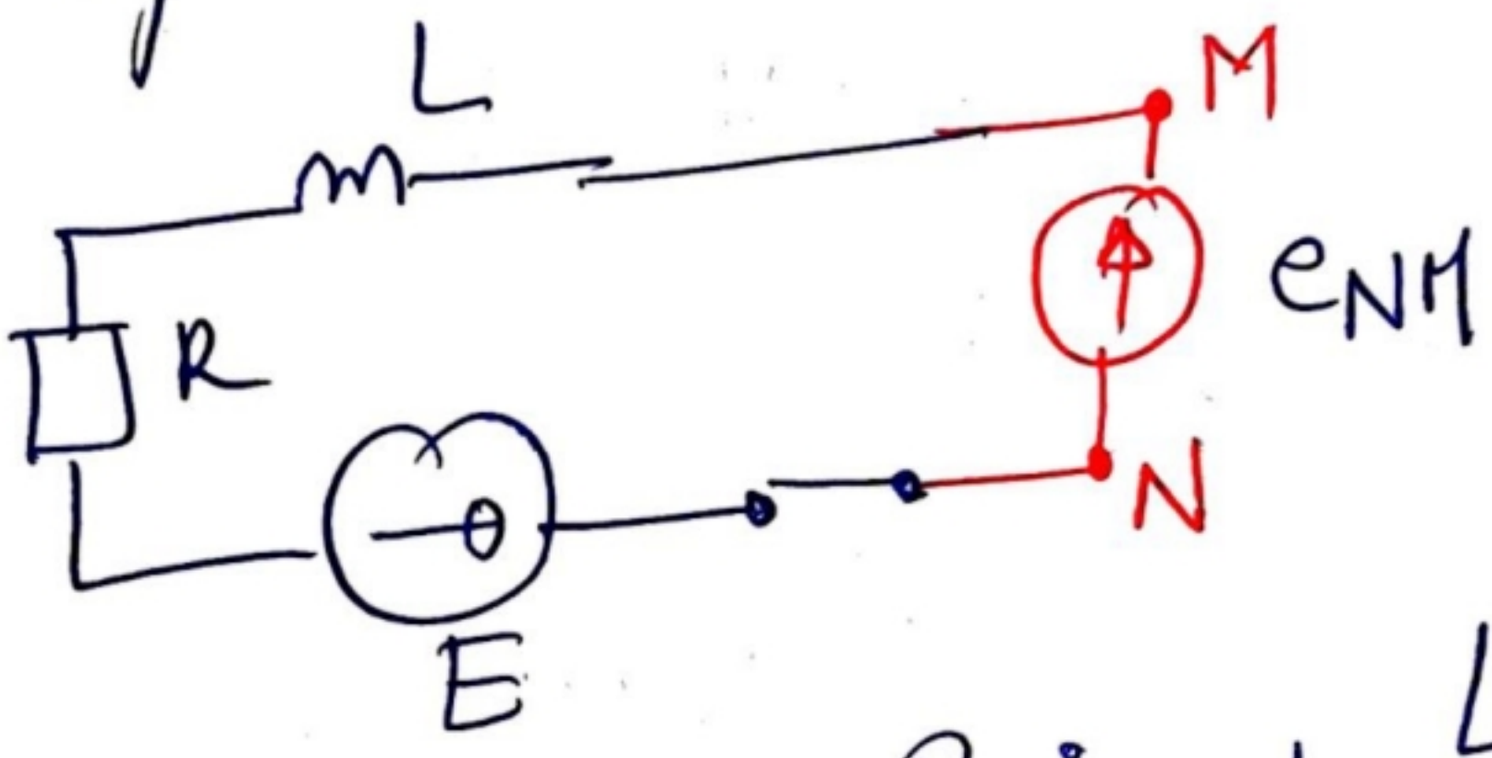
$$i = \frac{Blv}{(R+r)}\cos\alpha$$

$$m\frac{dv}{dt} = -k(x + x_0) + mg\sin\alpha - iBl\cos\alpha$$

$$m\ddot{x} = -kx - iBl\cos\alpha$$



$\vec{f}_{Lap} = iBl\vec{e_x}$; $e_{NM} = -Blv$



$$* \quad \bar{E} + e_{NM} = Ri + L\frac{di}{dt}$$

$$E = Ri + L\frac{di}{dt} + Blv$$

$$* \quad m\frac{dv}{dt} = Bli$$

$$\textcircled{*} \quad L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{B^2l^2}{m}i = 0$$

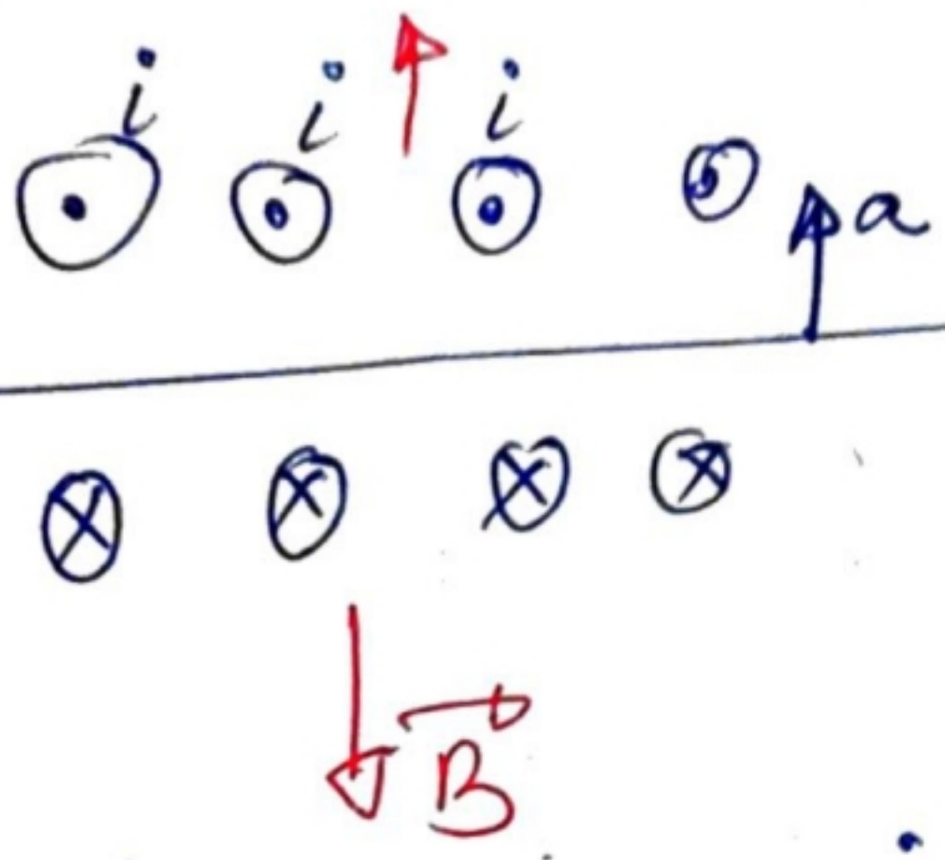
$\Delta > 0$ apériodique ; $\Delta < 0$ pseudo-p.

$\Delta = 0$ critique

à $t=0^-$: $i(0^-) = i(0^+) = 0$

à $t=0^+$, $v(0) = 0$

$$\left(\frac{di(t)}{dt}\right)_{0^+} = \frac{E}{L} \quad \leftarrow (E.E)$$



MP2
2010

circuit parcouru par i et placé $\vec{B}$

$$\vec{F}_{Lap} = \int (i\, d\vec{l}) \wedge \vec{B}$$

$$= \int_0^{2\pi N} i\, a\, d\theta\, \vec{e}_\theta \wedge B\, \vec{e}_r$$

$$= (N 2\pi a)\, i\, B\, (-\vec{e}_z)$$

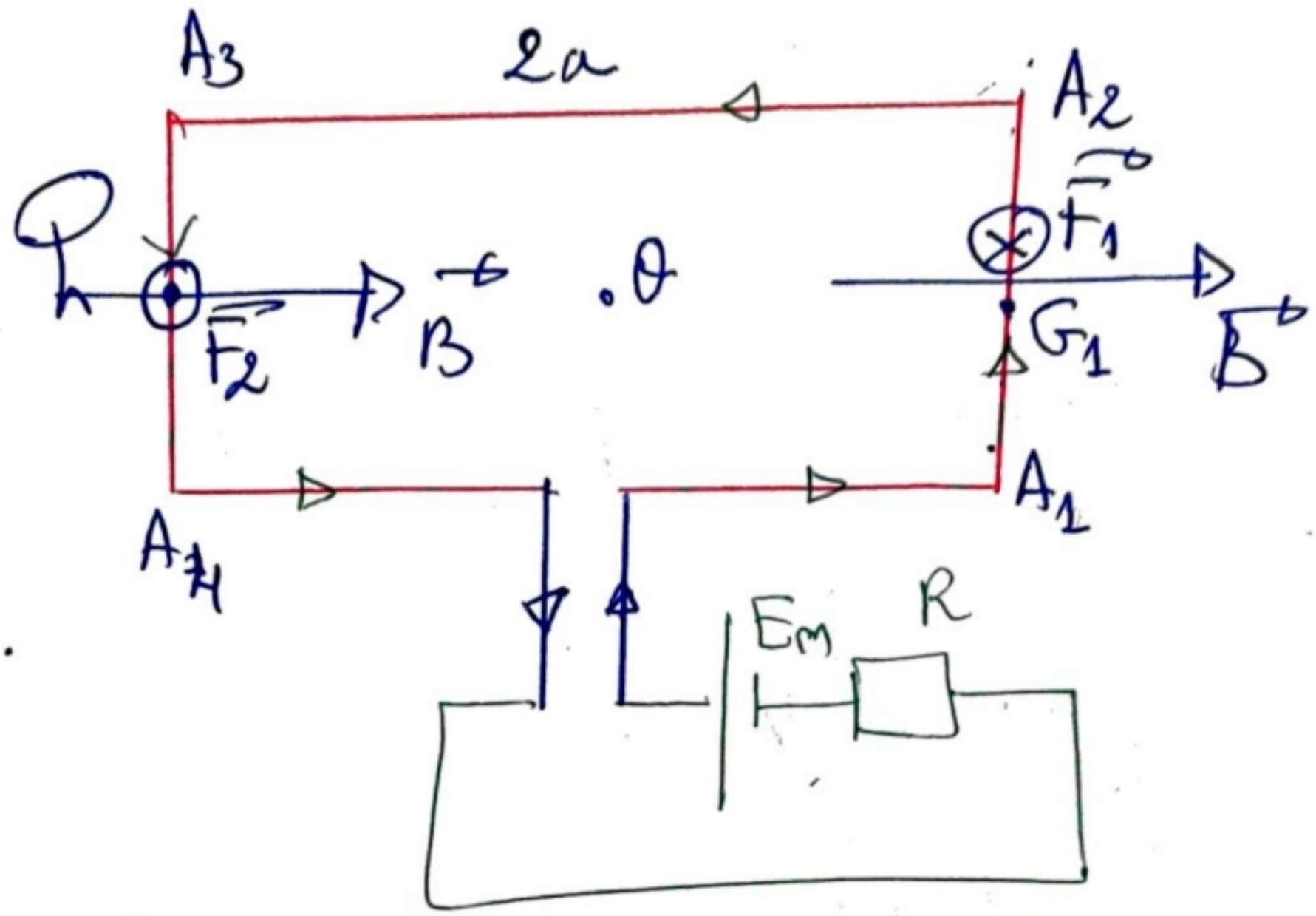
$$\vec{F}_{Lap} = -\,B\, i\, l_b\, \vec{e}_z$$

↳ bobinage

$$e = -\frac{P_{Lap}}{i} = B l v$$

$$e = \oint \vec{E}_m\, d\vec{l} = \int_{Conspire} (\vec{v} \wedge \vec{B}) \cdot d\vec{l}$$

Galvanomètre



$$\vec{F_2} = \int i\, dz\, \vec{e_z} \wedge B\vec{e_r} = i\ell h B \vec{e_\theta}$$

$$\vec{\mathcal{M}}(\vec{F_1}) = \overrightarrow{OG_1} \wedge \vec{F_1} = \frac{a}{2}\vec{e_r} \wedge i\ell h B \vec{e_\theta}$$

$$\vec{\mathcal{M}}(\vec{F_1}) = \frac{a\ell h B i}{2}\vec{e_z}$$

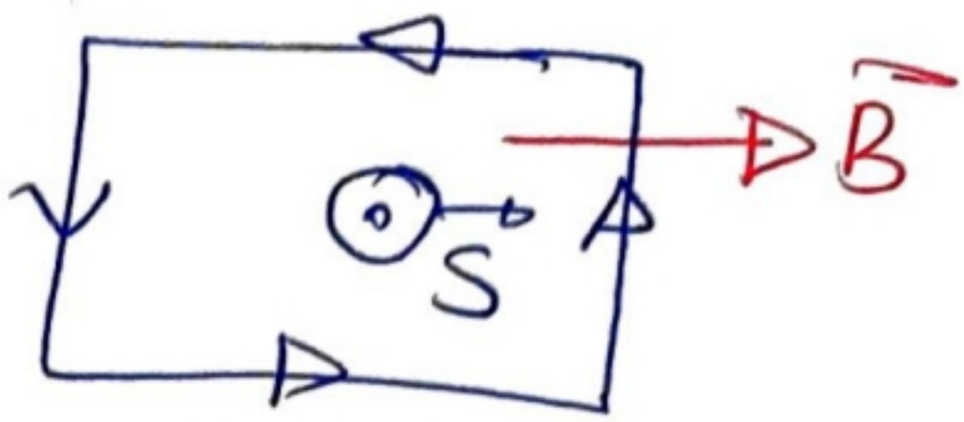
$$\vec{\mathcal{M}}(\vec{F_2}) = a\ell h B i \vec{e_z}$$

$$\vec{\Gamma} = (2a\ell h) B i \vec{e_z}$$

circuit indéformable, rigide ; et plongé dans un champ $\vec{B}$ uniforme : $\vec{\Gamma} = \vec{\mathcal{M}} \wedge \vec{B}$

$\vec{M}$ = moment magnétique = $i\vec{S}$

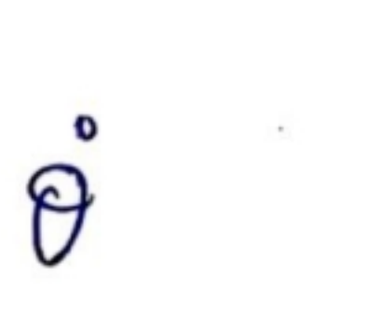
$= iS\vec{n}$



$\vec{\Gamma} = N\, iSB\, \vec{e_z} = \Phi_0\, i\, \vec{e_z}$

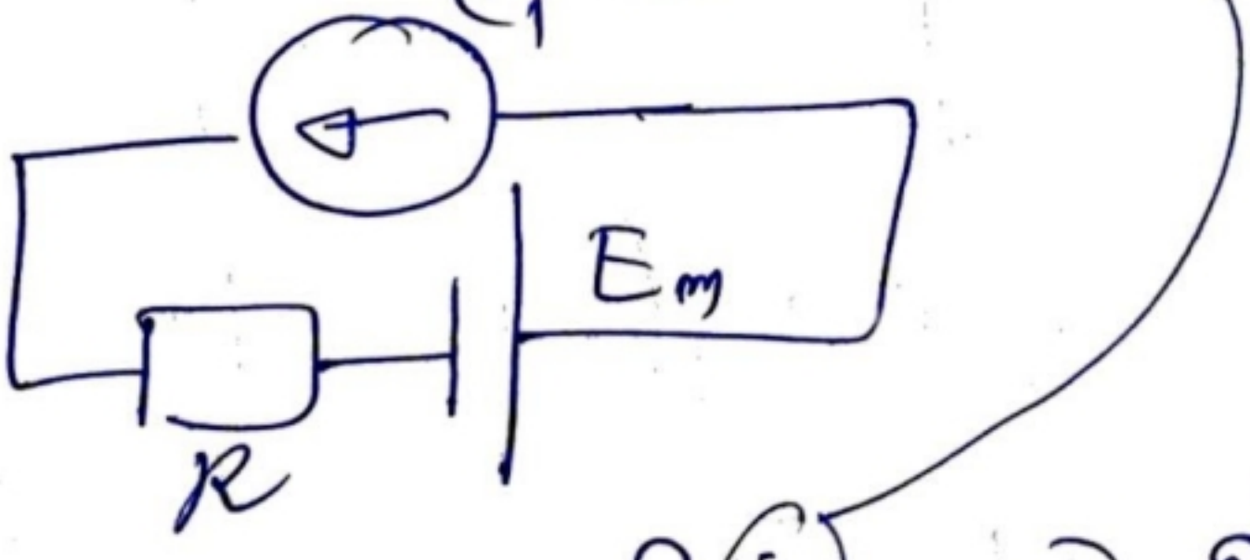
$e_{A_1A_2A_3A_4} = e_1 = -\dfrac{P_{Lap}}{i^6} = -\dfrac{\vec{\Gamma}\cdot\vec{\omega}}{i}$

$$\boxed{e_1 = -\Phi_0\,\omega}$$



1) $J\dfrac{d^2\theta}{dt^2} = \Phi_0\, i - C\theta$

2)



$E_m = Ri - e_1 = Ri + \Phi_0\,\dot{\theta}$

$J\ddot{\theta} = \Phi_0\left(\dfrac{E_m - \Phi_0\dot{\theta}}{R}\right) - C\theta$

$\ddot{\theta} + \dfrac{\Phi_0^2}{RJ}\dot{\theta} + \dfrac{C}{J}\theta = \dfrac{E_m\Phi_0}{RJ}$ ($\dfrac{C}{J} \to \omega_0^2$)

$$\overline{dM_\Delta} = \overline{dM_\theta} \cdot \overline{u_\Delta}$$

Champ tournant

$$\vec{B} = B_0 \left[\cos(\omega t)\, \vec{e_x} + \sin(\omega t)\, \vec{e_y} \right]$$

$$\vec{B}_{1\,bobine} = \left[\frac{\mu_0 n}{2} (\cos\theta_1 - \cos\theta_2) \right] i(t)\, \vec{e_x}$$

$$\vec{B}_{solénoïde} = (\mu_0 n)\, i_1(t)\, \vec{e_x} = k\, i_1(t)\, \vec{e_x}$$

$$u(t) = U_{eff}\sqrt{2}\cos(\omega t + \varphi_0)$$

1) $$\underline{u} = \left[R + j\left(L\omega - \frac{1}{C\omega} \right) \right] \underline{i_1}$$

$$\underline{i_1} = I_{01} \exp j(\omega t + \varphi_0 - \varphi_1)$$

$$= \frac{U_{eff}\sqrt{2}\; e^{j(\omega t + \varphi_0)}}{R + j\left(L\omega - \frac{1}{C\omega} \right)}$$

$$\underline{I_{01}} = \frac{U_{eff}\sqrt{2}}{R + j\left(L\omega - \frac{1}{C\omega} \right)} = I_{01} e^{-j\varphi_1}$$

$$|\underline{I}_{01}| = I_{02} = \frac{U_{eff}\sqrt{2}}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}}$$

$$\arg(\underline{I}_{01}) = -\varphi_1 = 0 - \operatorname{arctg}\left(\frac{L\omega-1/C\omega}{R}\right)$$

$$\varphi_2 = \operatorname{arctg}\left(\frac{L\omega-1/C\omega}{R}\right)$$

دەم

$$\left.\begin{array}{l} |\underline{I}_{02}| = I_{02} = \dfrac{U_{eff}\sqrt{2}}{\sqrt{R^2+(L\omega)^2}} \\ \varphi_2 = \operatorname{arctg}\left(\dfrac{L\omega}{R}\right) \end{array}\right\}$$

2) $I_{02} = I_{01} \Rightarrow (L\omega)^2 = \left(L\omega - \frac{1}{C\omega}\right)^2$

$$-L\omega = L\omega - \frac{1}{C\omega} \Rightarrow$$

$$\boxed{2L\omega = \frac{1}{C\omega}}$$

$\varphi_2 > 0$ نسکرفمّتک

$$\varphi_1 = +\operatorname{arctg}\frac{(L\omega - 2L\omega)}{R}$$

$$\boxed{\varphi_1 = -\varphi_2 = -\operatorname{arctg}(L\omega/R)}$$

3) $\varphi_2 - \varphi_1 = \frac{\pi}{2}$ en outre

$2\varphi_2 = \frac{\pi}{2} \Rightarrow \boxed{\varphi_2 = \frac{\pi}{4}}$

$tg\,\varphi_2 = 1$, $\boxed{L\omega = R}$

4) $\overrightarrow{B_1} = (k I_{01}) \cos(\omega t + \varphi_0 - \varphi_1)\vec{e}_x$

$\overrightarrow{B_1} = B_0 \cos(\omega t)\vec{e}_x$, $\varphi_0 = \varphi_1$

$\overrightarrow{B_2} = B_0 \cos(\omega t + \varphi_1 - \varphi_2)\vec{e}_y$

$= B_0 \cos(\omega t - \frac{\pi}{2})\vec{e}_y$

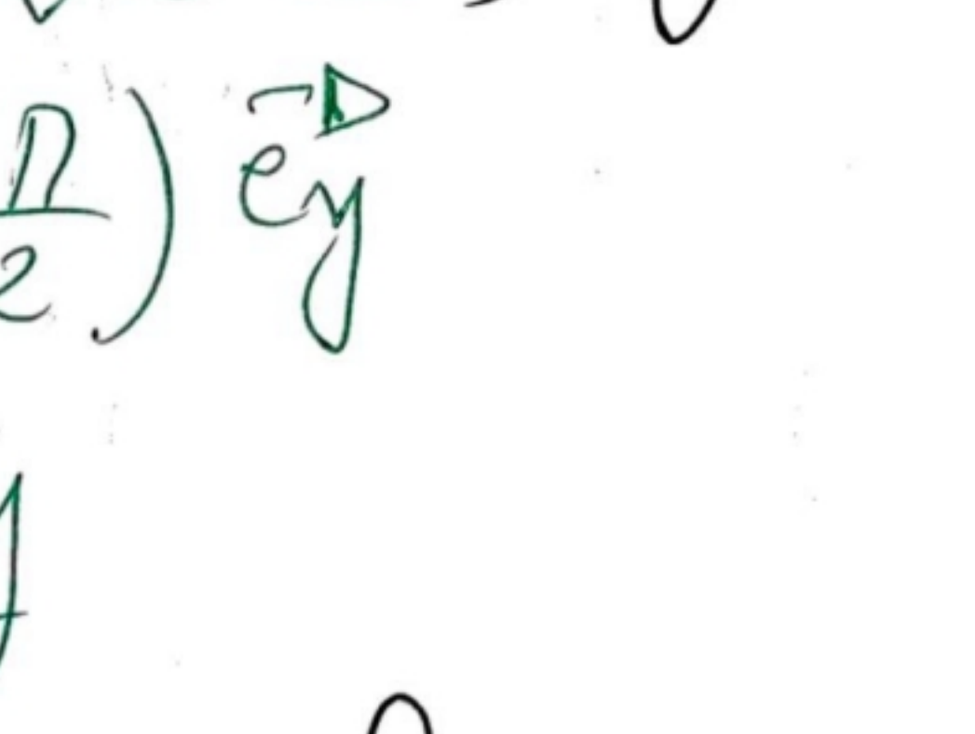
$\overrightarrow{B_2} = B_0 \sin(\omega t)\vec{e}_y$

5) $\left(\frac{B_x}{B_0}\right)^2 + \left(\frac{B_y}{B_0}\right)^2 = 1$ cercle

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

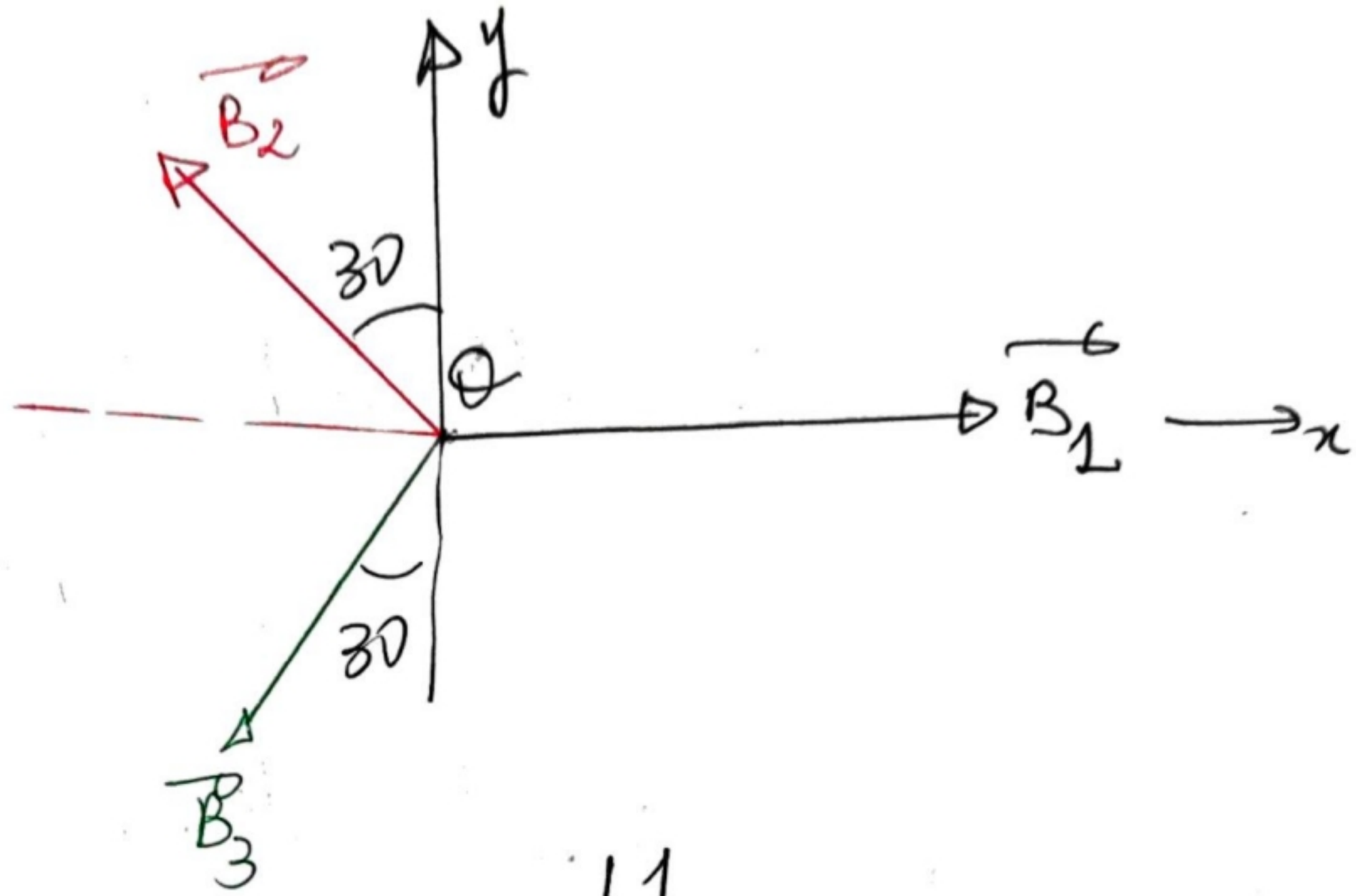
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\varphi = \sin^2\varphi$

Lissajous



$\sin\varphi = \frac{AB}{ACD}$

B)



1)

$$\vec{B_1} = kI\sqrt{2}\cos(\omega t) \;\Big|\; \begin{matrix}1\\0\\0\end{matrix}$$

$$\vec{B_2} = kI\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3}\right) \;\Big|\; \begin{matrix}-\frac{1}{2}\\+\frac{\sqrt{3}}{2}\\0\end{matrix}$$

$$\vec{B_3} = kI\sqrt{2}\cos(\omega t) \;\Big|\; \begin{matrix}-\frac{1}{2}\\-\frac{\sqrt{3}}{2}\\0\end{matrix}$$

2)

$$\vec{B_2} = kI\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3}\right) \;\Big|\; \begin{matrix}-\frac{1}{2} \rightarrow \cos\left(\frac{2\pi}{3}\right)\\+\frac{\sqrt{3}}{2} \rightarrow \sin\left(\frac{2\pi}{3}\right)\\0\end{matrix}$$

$$\vec{B_3} = kI\sqrt{2}\cos\left(\omega t + \frac{2\pi}{3}\right) \;\Big|\; \begin{matrix}-\frac{1}{2} \leftarrow \cos\left(\frac{4\pi}{3}\right)\\-\frac{\sqrt{3}}{2} \leftarrow \sin\left(\frac{4\pi}{3}\right)\\0\end{matrix}$$

méthode d'affixe $\overrightarrow{OM} \longrightarrow z$

$$\overrightarrow{OM} = \begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases} \longrightarrow z = \rho e^{i\theta}$$

$$\overrightarrow{B_2} \longrightarrow b_2 = kI\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3}\right) e^{j\frac{2\pi}{3}}$$

$$\overrightarrow{B_3} \longrightarrow b_3 = kI\sqrt{2}\cos\left(\omega t - \frac{4\pi}{3}\right) e^{j\frac{4\pi}{3}}$$

$$\overrightarrow{B_1} \longrightarrow b_1 = kI\sqrt{2}\cos(\omega t)\, e^{j0}$$

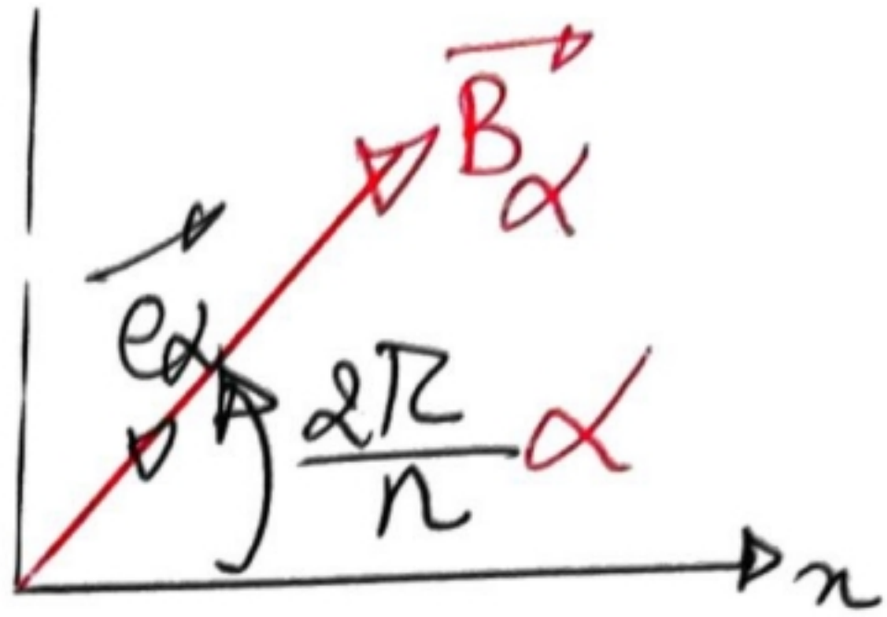
$$b = b_1 + b_2 + b_3$$

$$= \frac{kI\sqrt{2}}{2}\left[e^{j\omega t}(1+1+1) + e^{-j\omega t}\underbrace{\left(1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}}\right)}_{0} \right]$$

$$b = \frac{3}{2} I k\sqrt{2}\, e^{j\omega t}$$

$$\overrightarrow{B} = \frac{3}{2} I k\sqrt{2}\left(\cos(\omega t)\,\vec{e_x} + \sin(\omega t)\,\vec{e_y}\right)$$

3) $\alpha \in [0, n-1]$



$$\vec{B}_\alpha = k I_{max} \cos\left(\omega t - \frac{2\pi}{n}\alpha\right) \vec{e}_\alpha$$

$$(B_\alpha)_x = k I_{max} \cos\left(\omega t - \frac{2\pi\alpha}{n}\right) \cos\left(\frac{2\pi}{n}\alpha\right)$$

$$(B_\alpha)_y = k I_{max} \sin\left(\frac{2\pi\alpha}{n}\right)$$

$$B_x = \sum_{\alpha=0}^{n-1} \frac{(k I_{max})}{2} \left[\cos(\omega t) + \cos\left(\omega t - \frac{4\pi\alpha}{n}\right)\right]$$

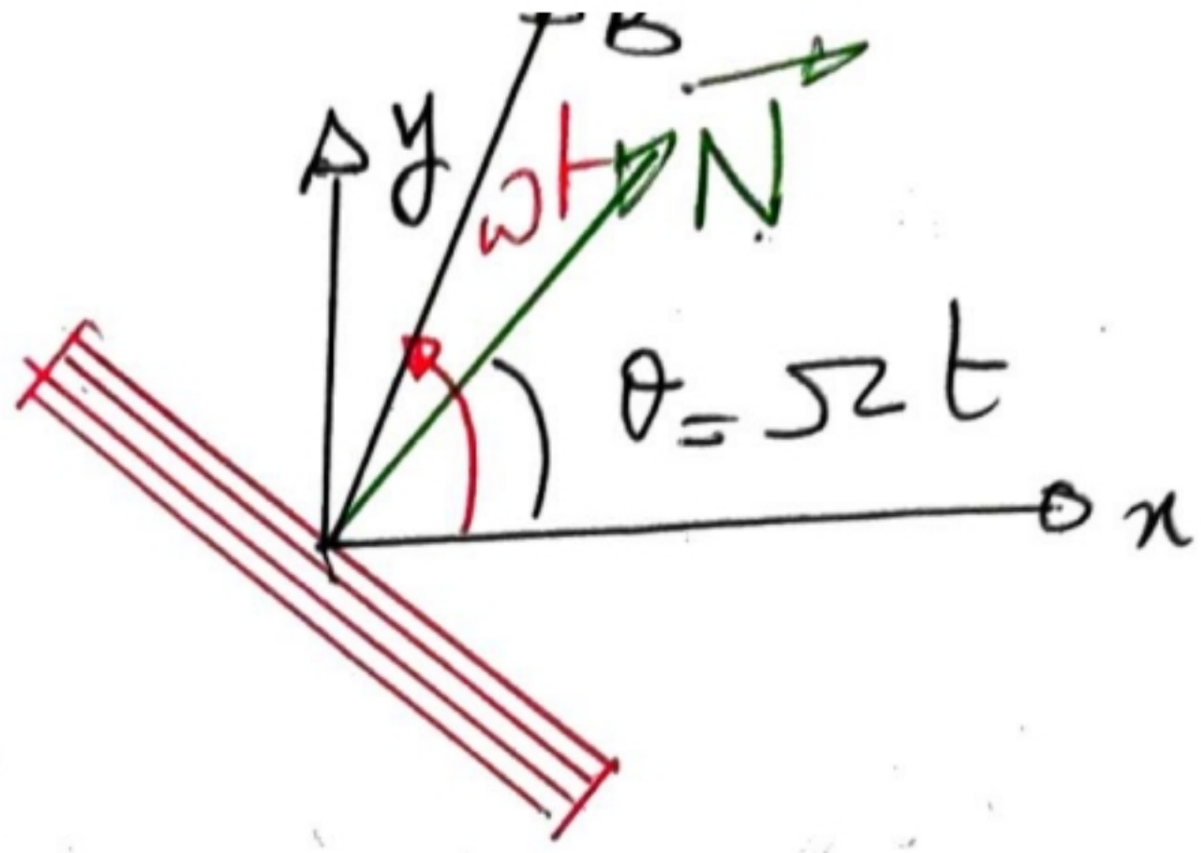
$$\boxed{B_x = n \frac{k I_{max}}{2} \cos(\omega t)}$$

$$\boxed{B_y = n \frac{k I_m}{2} \sin(\omega t)}$$

$$\sum_{0}^{n-1} Rel\left[\left(e^{j\omega t}\right)\left(e^{-j + \frac{4\pi\alpha}{n}}\right)\right]$$

$$Rel\left[\left(e^{j\omega t}\right) \sum e^{\left(-j\frac{4\pi\alpha}{n}\right)}\right] = 0.$$

Exy



1) $e = -\dfrac{d\Phi}{dt}$

$\Phi = n\iint \vec{B}(t)\cdot d\vec{S} = n\,\vec{B}\cdot S\vec{N}$

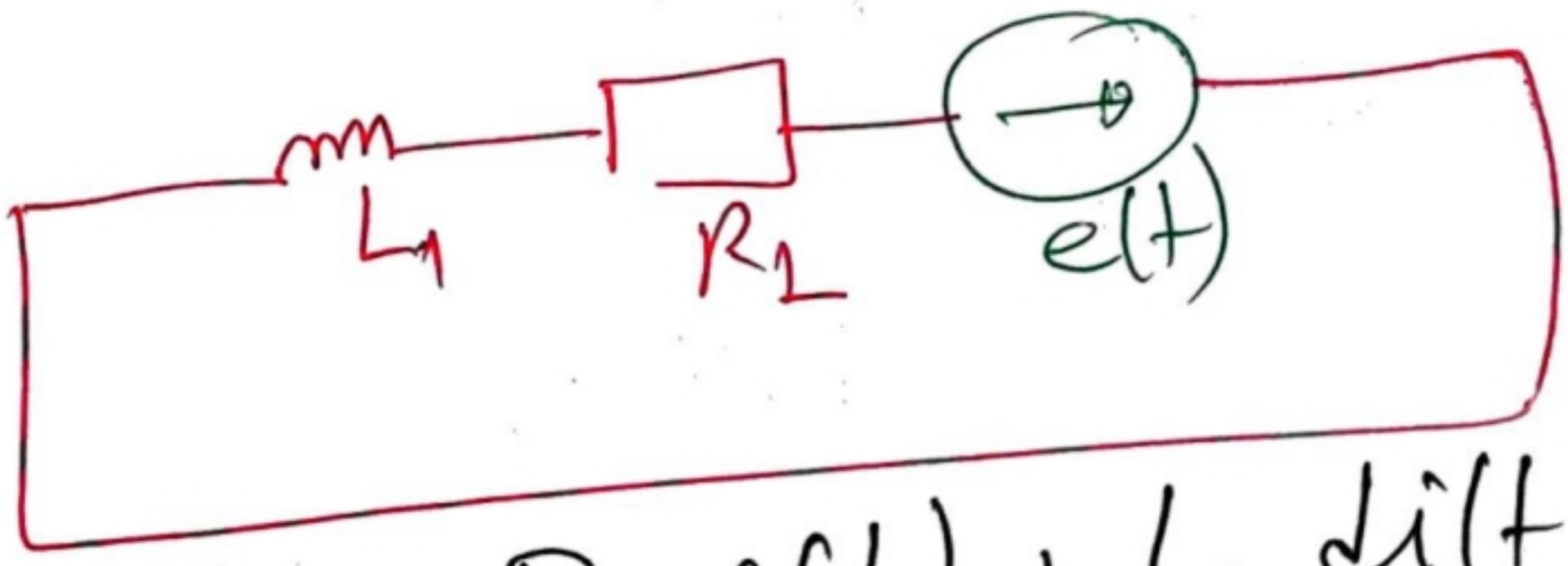
$\Phi = (n B_0 S)\cos[(\omega - \Omega)t]$

$\Phi = \Phi_0 \cos(\omega - \Omega)t$

$\iint_A \vec{J}\cdot d\vec{S} = I$

Weber

$e = +\Phi_0(\omega - \Omega)\sin[(\omega - \Omega)t]$



$\underline{e}(t) = R_1\,\underline{i}(t) + L_1\dfrac{d\underline{i}(t)}{dt}$

$\underline{e} = (R_1 + jL_1(\omega - \Omega))\,\underline{i}(t)$

$$\underline{i} = \frac{\Phi_0(\omega - \Omega)}{R_1 + j L_1(\omega - \Omega)} e^{j\left[(\omega-\Omega)t - \frac{\pi}{2}\right]}$$

$$= \underline{I_0} \exp j\left[(\omega - \Omega)t - \frac{\pi}{2}\right]$$

$$\underline{I_0} = \frac{\Phi_0(\omega - \Omega)}{R_1 + j L_1(\omega - \Omega)} = I_0 e^{-j\varphi}$$

$$|\underline{I_0}| = \frac{\Phi_0(\omega - \Omega)}{\sqrt{R_1^2 + L_1^2(\omega - \Omega)^2}}$$

$$\arg \underline{I_0} = -\varphi = 0 - \operatorname{arctg} \frac{L_1(\omega - \Omega)}{R_1}$$

$$\varphi = +\operatorname{arctg}\left(\frac{L_1(\omega - \Omega)}{R_2}\right)$$

2) $\vec{\Gamma} = \vec{m} \wedge \vec{B}(t) = n i S \vec{N} \wedge \vec{B}$

$$\vec{\Gamma} = \Phi_0 \, i(t) \sin\left[(\omega - \Omega)t\right] \vec{e}_z$$

$$\Gamma(t) = \Phi_0 I_0 \sin\left((\omega - \Omega)t - \varphi\right) \sin\left((\omega - \Omega)t\right)$$

$$\langle \Gamma(\Omega) \rangle = \frac{\mathcal{E}_0 I_0}{2} \cos\varphi$$

$$= \frac{\mathcal{E}_0^2}{2} \frac{(\omega - \Omega)}{\sqrt{R_1^2 + L_1^2(\omega-\Omega)^2}} \cos\varphi$$

$$\text{or} \quad \varphi = \operatorname{arctg}\left(\frac{L_1(\omega-\Omega)}{R_1}\right)$$

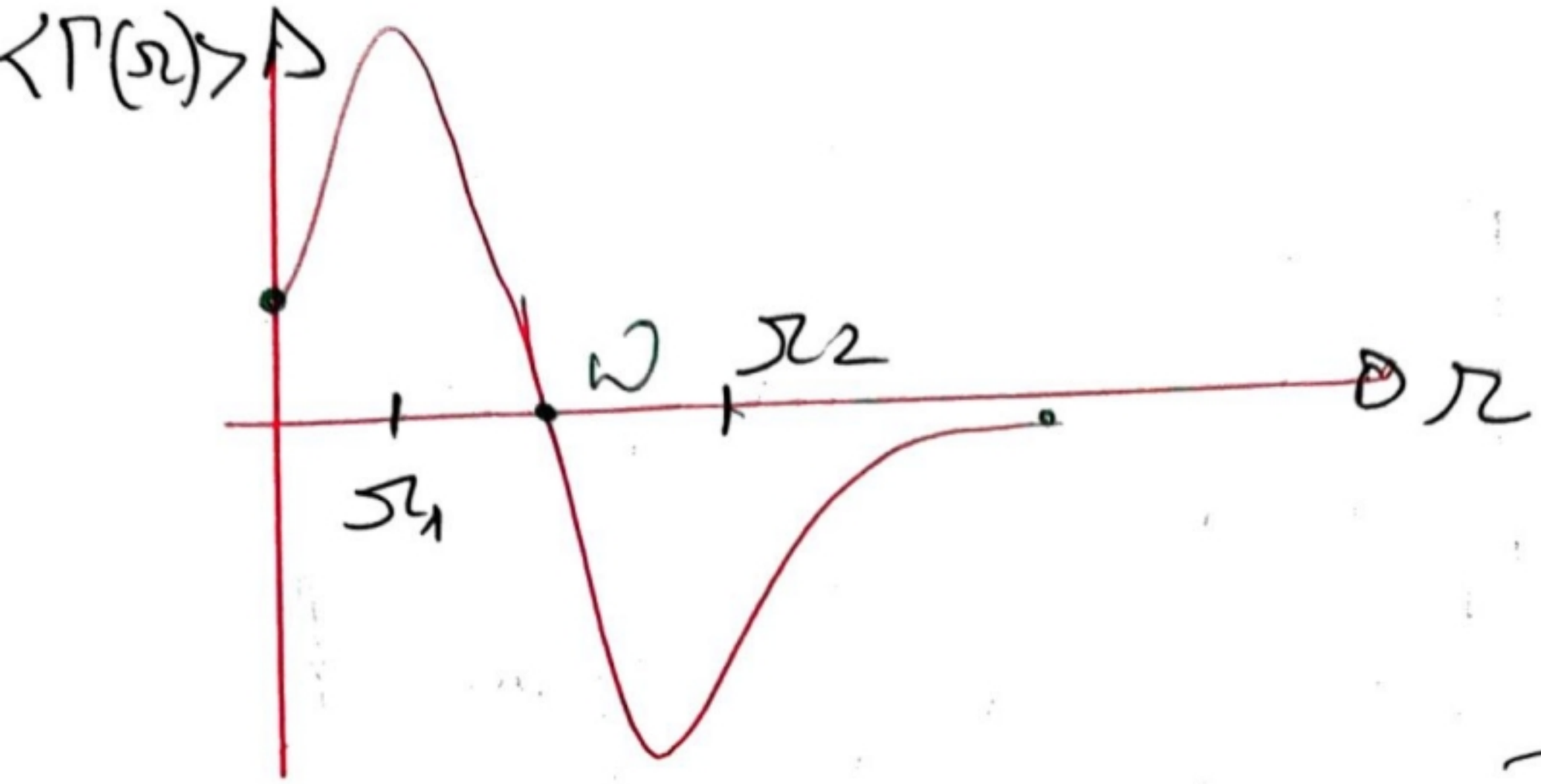
$$\langle \Gamma(\Omega) \rangle = \frac{\mathcal{E}_0^2}{2} \frac{R_1(\omega-\Omega)}{(R_1^2 + L_1^2(\omega-\Omega)^2)}$$

$$\cos^2\varphi = \frac{1}{1+\operatorname{tg}^2\varphi} = \frac{R_1^2}{R_1^2 + L_1^2(\omega-\Omega)^2}$$

$$\langle \Gamma(0) \rangle = \frac{\mathcal{E}_0^2 R_1 \omega}{2(R_1^2 + L_1^2\omega^2)}$$

$$\langle \Gamma(\Omega) \rangle = 0 \quad \text{si} \quad \Omega = \omega$$

$$\lim_{\Omega \to \infty} \langle \Gamma(\Omega) \rangle = 0.$$



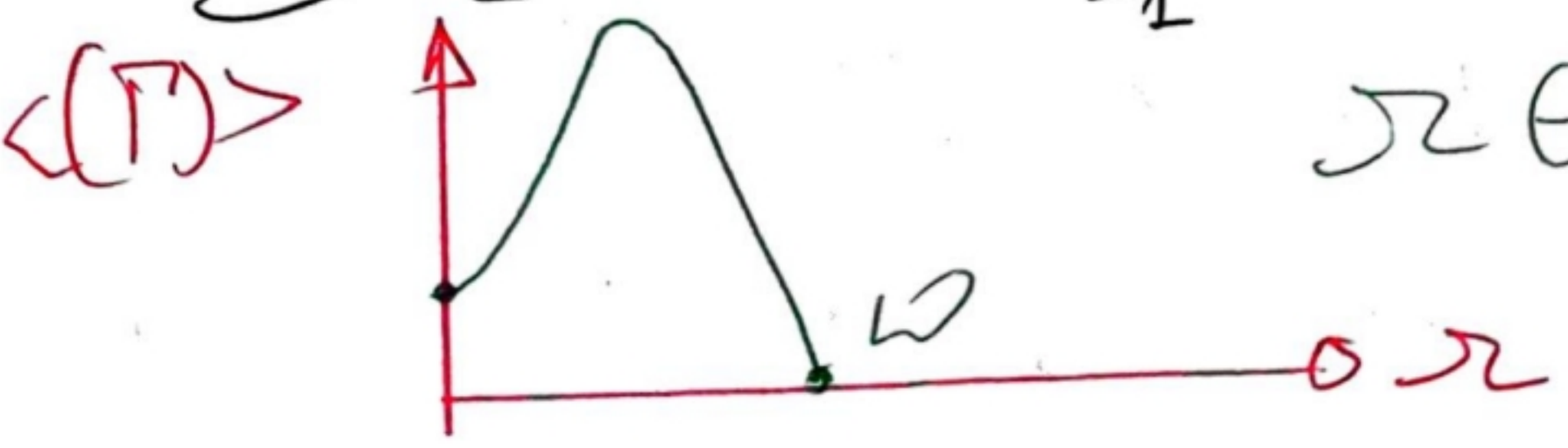
$$x = (\omega - x) \; ; \quad \frac{d}{dx}\left[\frac{x}{\left(R_1^2 + L_1^2 x^2\right)}\right] = 0$$

$$\frac{d}{dx}\left(\frac{1}{\frac{R_1^2}{x} + L_1^2 x}\right) = 0$$

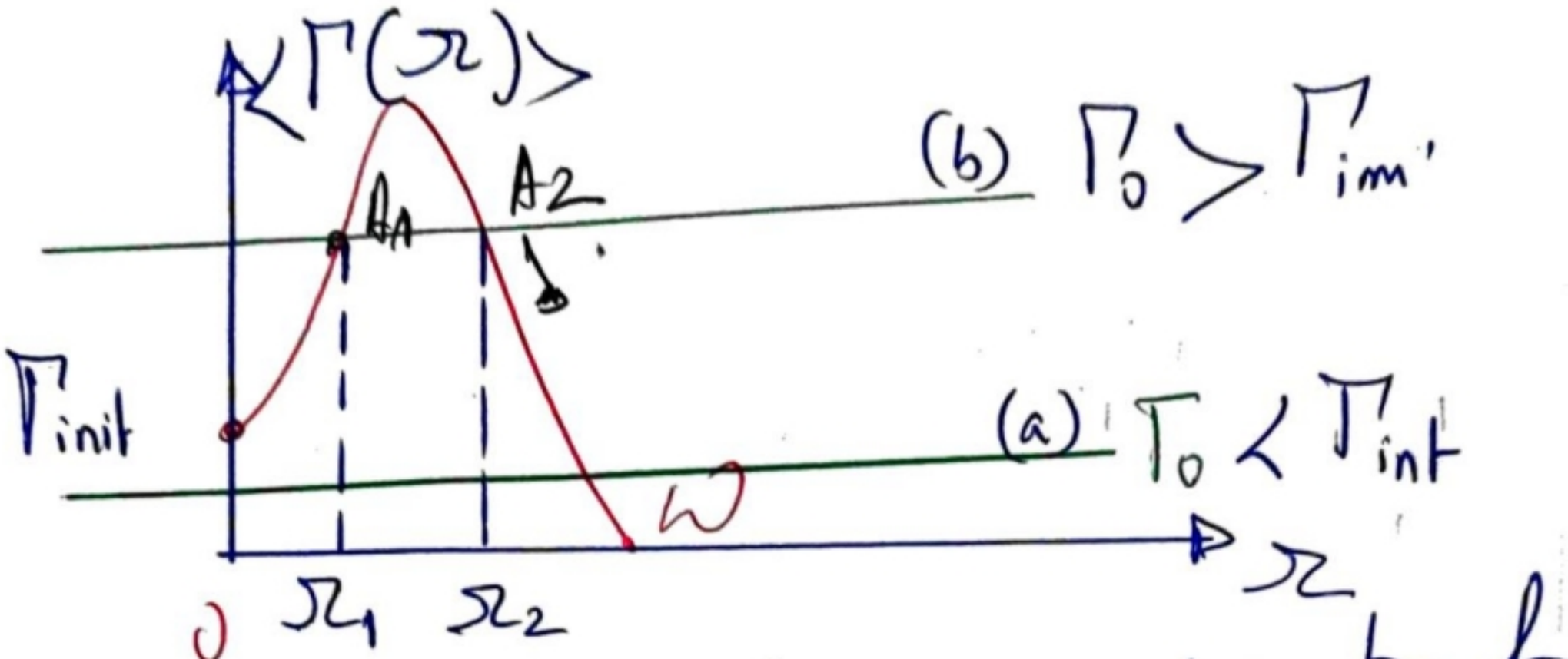
$$-\frac{R_1^2}{x^2} + L_1^2 = 0 \Rightarrow x = \pm \frac{R_1}{L_1}$$

$$\Omega_1 = \left(\omega - \frac{R_1}{L_1}\right) > 0$$

$$\Omega_2 = \omega + \frac{R_1}{L_1}$$



$$\Omega \in [0, \omega[$$



(a): le moteur démarrage spontanément.

(b): il faut lancer le moteur

en régime permanent $\Omega = cts$.

$$J\frac{d\Omega}{dt} = 0 = \langle \Gamma(\Omega) \rangle - \Gamma_0$$

$$\langle \Gamma(\Omega) \rangle = \Gamma_0$$

Etude de la stabilité

(A2) si le pt A_2 se déplace vers la droite:

$\Omega \nearrow$; $\langle \Gamma(\Omega) \rangle \nearrow \Rightarrow \left(\frac{d\Omega}{dt}\right) > 0$

$\Rightarrow \Omega \nearrow$ instable

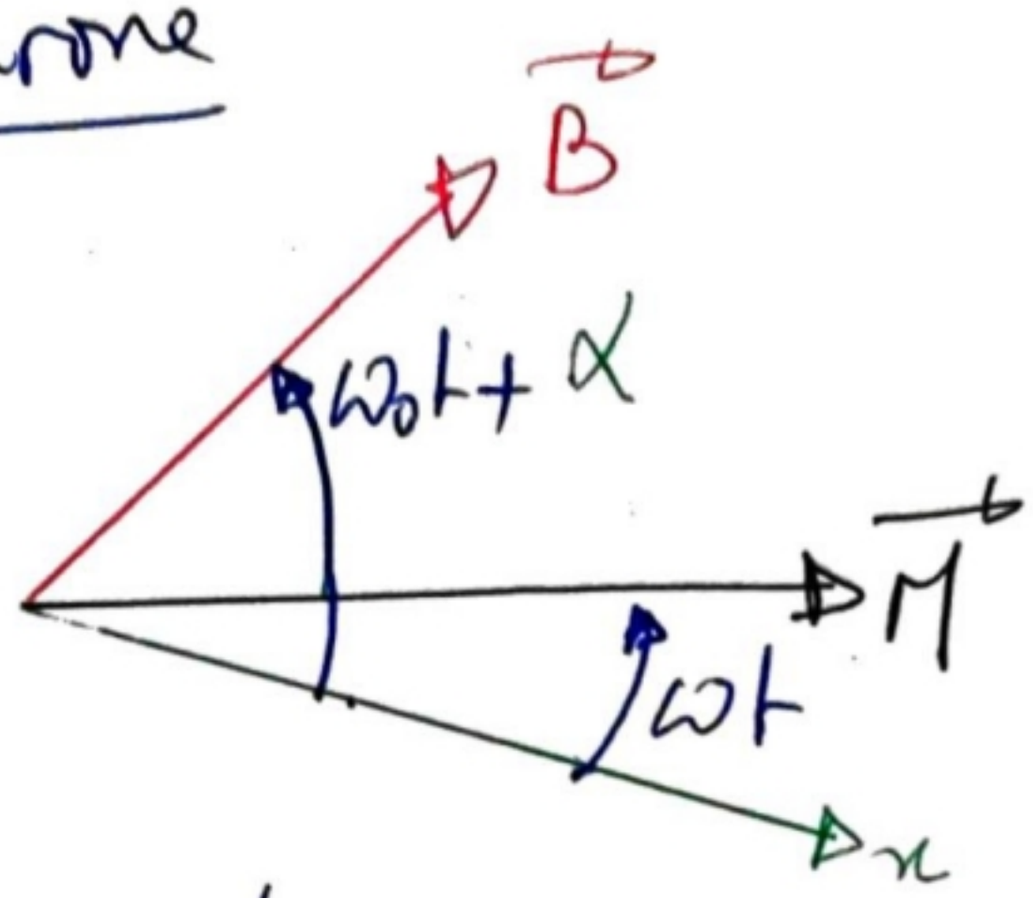
⊙ A_2 : le pt A_2 se déplace vers le bas

$\Omega \searrow$, $\langle \Gamma(\Omega) \rangle \searrow \Rightarrow \left(\frac{d\Omega}{dt}\right) < 0 \Rightarrow \Omega \searrow$ stable

Moteur synchrone

$(\vec{M}, \vec{B}) = \alpha$

$\vec{a}\ t = 0$



a) $\theta = (\omega_0 - \omega)t + \alpha$

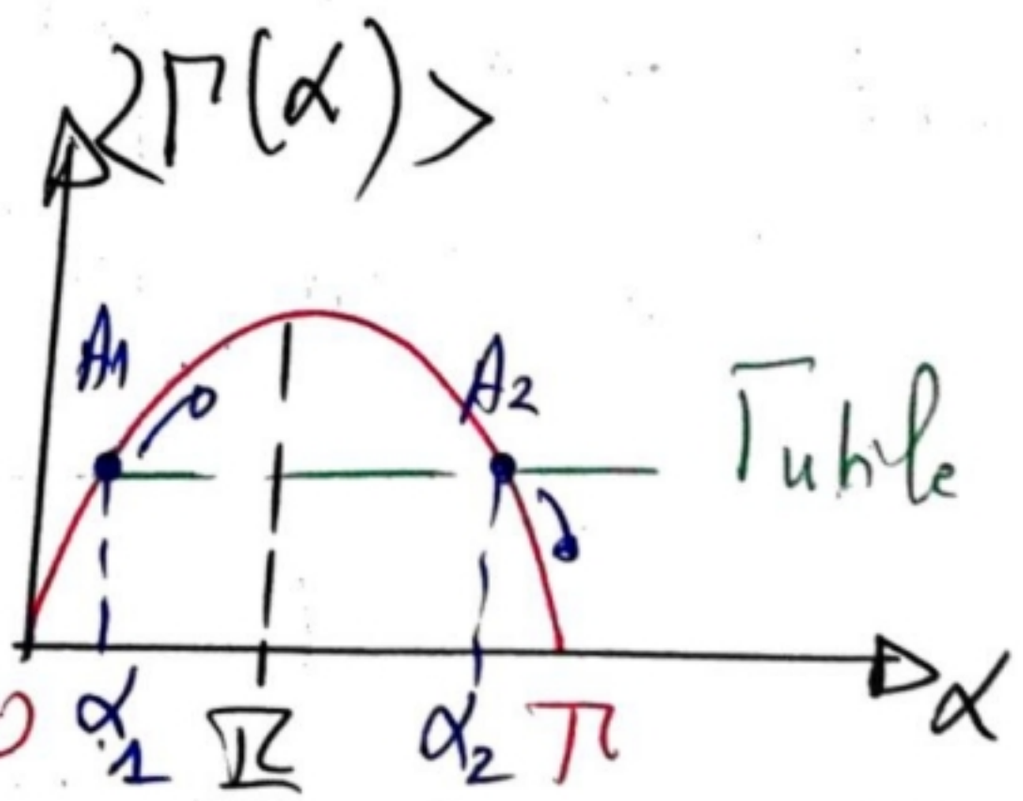
$\vec{\Gamma}(t) = \vec{M} \wedge \vec{B} = M B_0 \sin\left((\omega_0 - \omega)t + \alpha\right) \vec{e}_z$

$\langle \vec{\Gamma}(t) \rangle = \vec{0}$, $\omega \neq \omega_0$

$\langle \Gamma(\alpha) \rangle \neq 0$ $\omega = \omega_0$

b) $\langle \Gamma(\alpha) \rangle = \Gamma M B_0 \sin\alpha$

$\alpha \in]0, \pi[$



$P_{mis\,en} = \langle \Gamma(\alpha) \rangle \cdot \omega$

$= M B_0 \omega \sin\alpha$

$\underline{P_{max} = M B_0 \omega}$

$\alpha = \frac{\pi}{2}$

c) $J \frac{d\omega}{dt} = \langle \Gamma(\alpha) \rangle - \Gamma_{utile}$

permanent : $\omega = cte \Rightarrow$

$\langle \Gamma(\alpha) \rangle = \Gamma_{utile}$.

A_1 : se déplace vers le haut : $\alpha > 0$;

$\Sigma \Gamma(\alpha) > 0 \Rightarrow \left(\frac{d\omega}{dt}\right) > 0 \Rightarrow \omega > 0$

$\alpha \searrow$

équilibre stable

---

l_0 = à vide

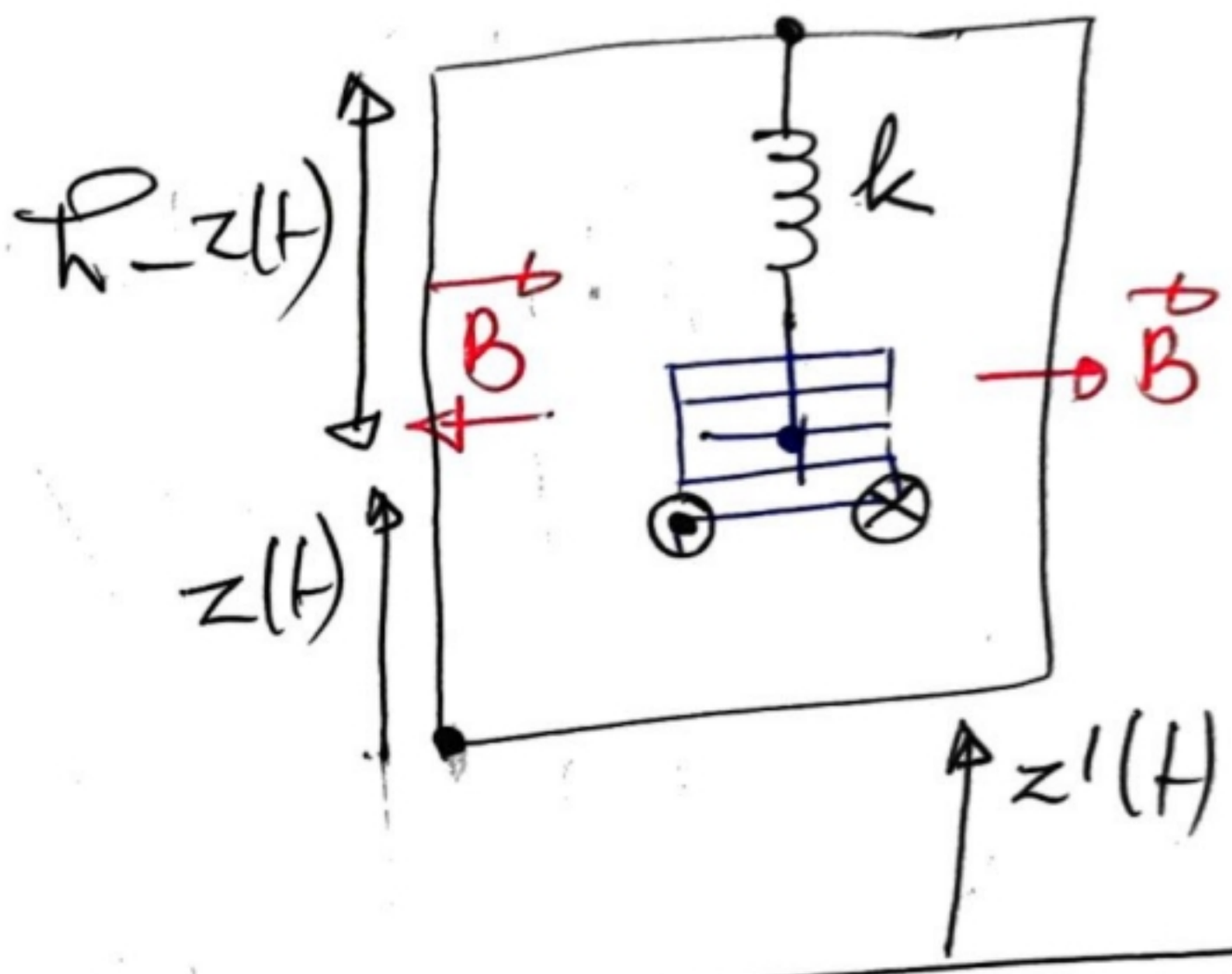


bobine : (L, R)

lors du séisme

bobine se met en

mvt dans un champ $\vec{B}$



création d'un courant induit $i(t)$

$$e = -\frac{d\Phi}{dt}$$

$$d\Phi_c = B\vec{e_r} \cdot \left[d\ell\vec{p} \wedge \vec{sens\ cnt} \right]$$

$$= B\vec{e_r} \left[dz\,\vec{e_z} \wedge a\,d\theta\,\vec{e_\theta} \right]$$

$$d^2\Phi_c = -aB\,dz\,d\theta$$

$$d^k\Phi_c = -\,aB\,dz\int_0^{2\pi N} d\vartheta$$

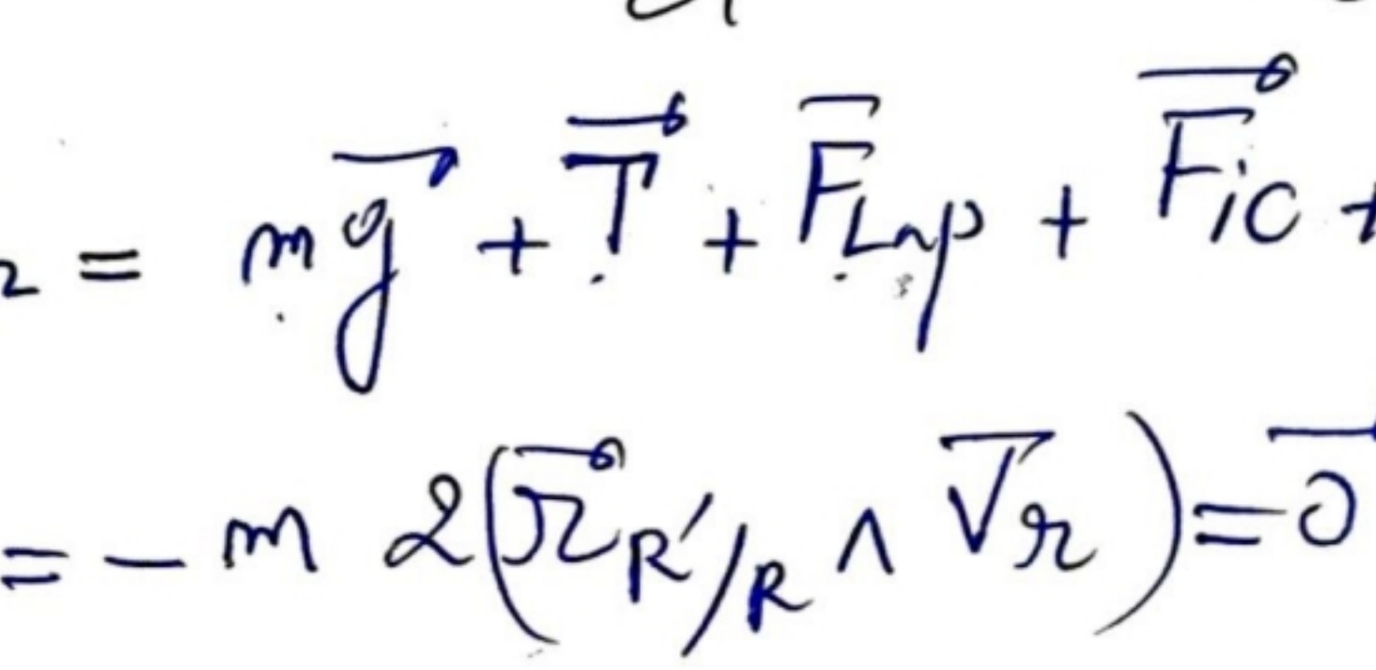
$$d^2\Phi_c = -(2\pi a N)\,B\,dz$$

$$e = +\,B\,l_b\frac{dz}{dt}$$

- $\vec{f}_{Lap} = \int i\,\vec{dl}\wedge\vec{B} = \int_0^{2\pi N} i\,a\,d\vartheta\,\vec{e_\varphi}\wedge B\,\vec{e_r}$

$$\boxed{\vec{f}_{Lap} = -\,i\,B\,l_b\,\vec{e_z}}$$

$\vec{f}_{Lap} + \vec{v} + e\,i = 0$.



3) $e = R\,i + L\frac{di}{dt} = B l_b \frac{dz}{dt}$

4) $m\vec{a}_R = m\vec{g} + \vec{T} + \vec{F}_{Lap} + \vec{F}_{ic} + \vec{F}_{ie}$

$\vec{F}_{ic} = -\,m\,2(\vec{\Omega}_{R'/R}\wedge\vec{v}_R) = \vec{0}$

$$\overrightarrow{F_{ie}} = -m\overrightarrow{a_e}$$

$$= -m\left[\frac{d^2\overrightarrow{OO'}}{dt^2} + \overrightarrow{\Omega}_{R'/R} \wedge \left(\overrightarrow{\Omega}_{R'/R} \wedge \overrightarrow{O'M}\right) + \cdots\right.$$

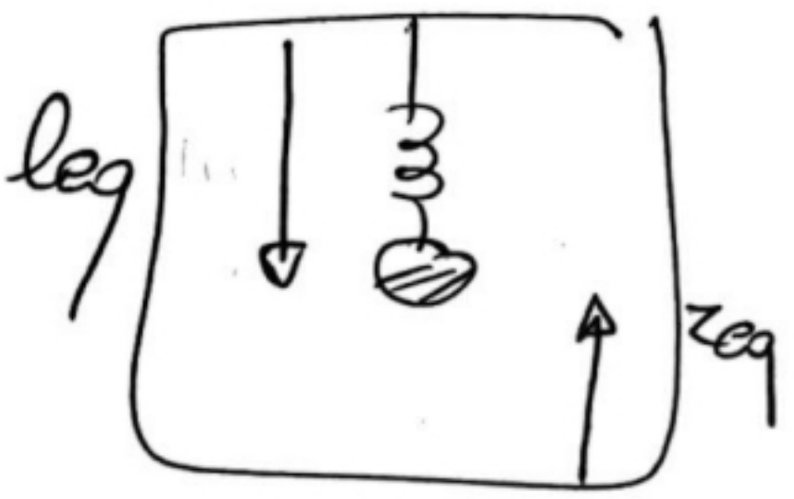
$$= -m\ddot{z}'(t)\,\vec{e}_z$$

$$m\ddot{z} = +k(l(t) - l_0) - mg - iBl_b - m\ddot{z}'$$

$$0 = k(l_{eq} - l_0) - mg \quad \text{---} \quad 0 \quad \text{---} \quad \circ$$

$$m\ddot{z} = +k(l(t) - l_0) - k(l_{eq} - l_0) - iBl_b - m\ddot{z}'$$

$$= -k(l_{eq} - l(t)) - iBl_b - m\ddot{z}'$$



or $h = l_{eq} + z_{eq}$

$$= l(t) + z(t)$$

$$l_{eq} + z_{eq} = l(t) + z(t)$$

$$l_{eq} - l(t) = z(t) - z_{eq} = x$$

Donc

$$m\ddot{z} = -kx - iBl_b - m\ddot{z}' = m\ddot{x}$$

$$\boxed{m\ddot{x} + kx + Bl_b i = -m\ddot{z}'(t)}$$

5) $(EE): \left(R + j\omega L\right)\underline{i} = j B l_b \omega \underline{x}$

$$(EM): \left(-\omega^2 + \omega_0^2\right)\underline{x} + \frac{Bl_b}{m}\underline{i} = +\omega^2 \underline{z}'$$

$$\underline{H}(j\omega) = \frac{\underline{i}(t)}{\underline{z}'(t)}$$

passe — haut