

Champ électrique

Déterminer $\vec{E}(r)$ et $V(r)$

$$(M \vec{e}_r \vec{e}_z) \quad \text{PSL} \quad \vec{E} \in$$

$$(M \vec{e}_r \vec{e}_\theta) \quad \text{PSL} \quad \vec{E} \in$$

$$\vec{E}(r) = E_r \vec{e}_r = E(r) \vec{e}_r$$

$$\oint \vec{E} d\vec{S} = 2\pi r h E(r)$$

$$= \frac{\int \lambda dz}{\epsilon_0}$$

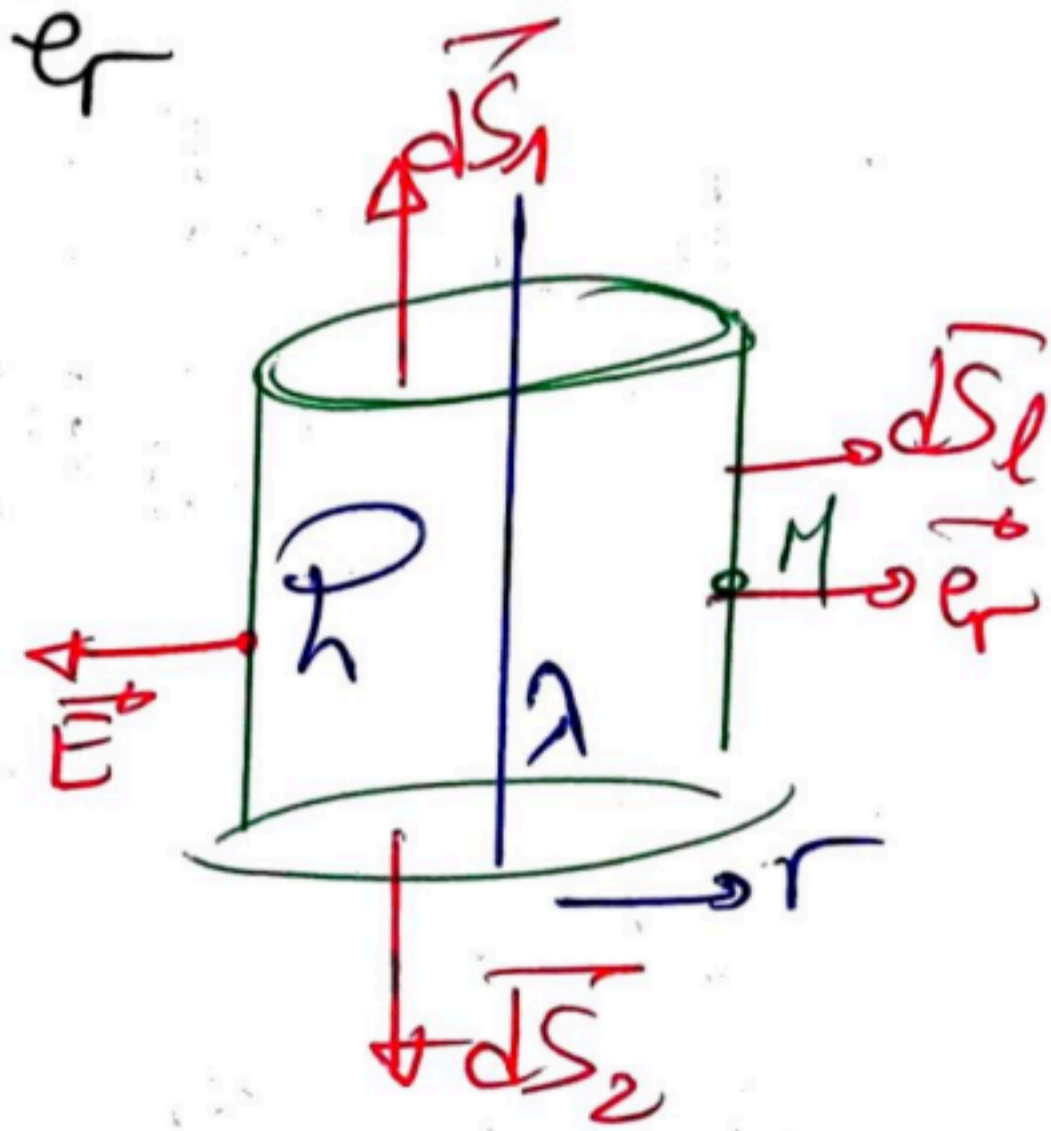
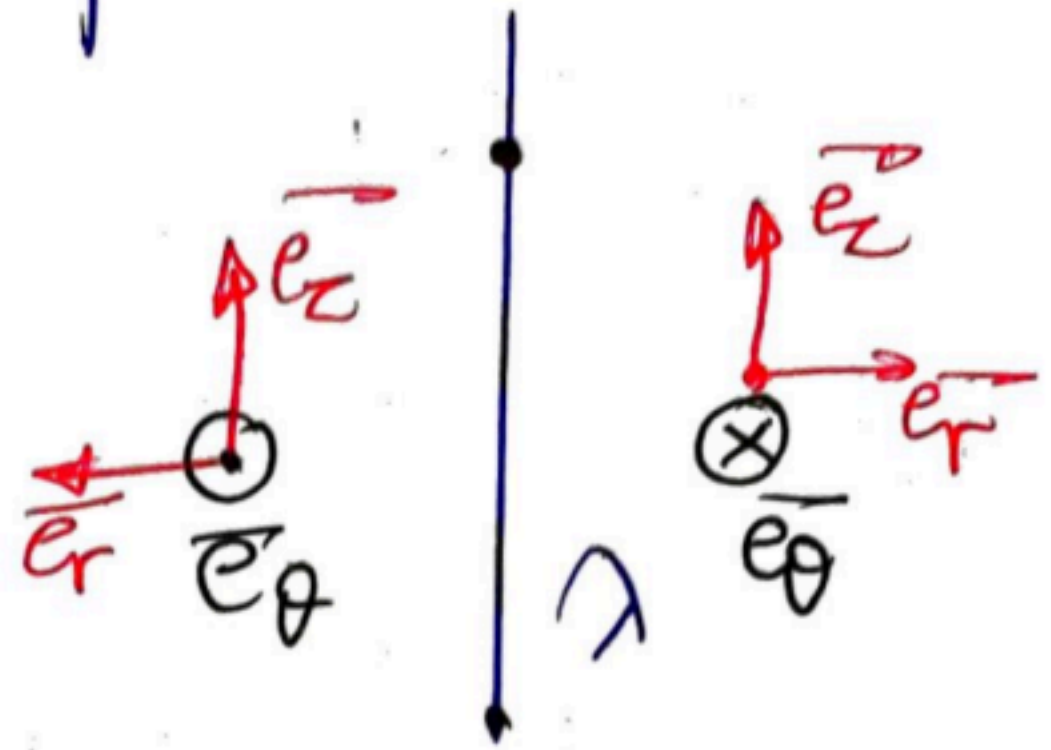
$$= \frac{\lambda h}{\epsilon_0}$$

$$\Rightarrow \vec{E}_{fil} = \frac{\lambda}{2\pi\epsilon_0 r} \quad \vec{e}_r = -\frac{dV}{dr} \vec{e}_r$$

$$V(r) = - \int \frac{\lambda}{2\pi\epsilon_0} \frac{dr}{r}$$

$$V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln r + \text{cte}$$

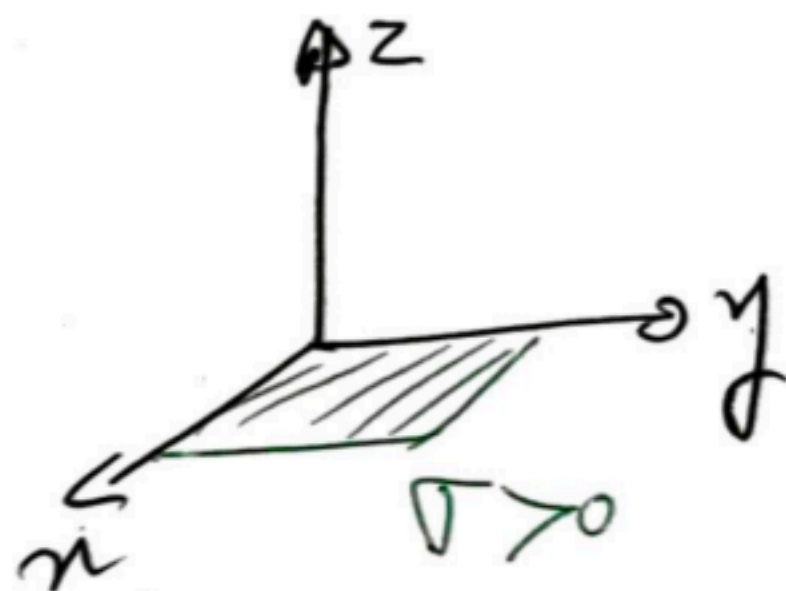
suchant $V(a) = 0$



Plan

$(M \vec{u}_x \vec{u}_z)$ PSL

$(M \vec{u}_y \vec{u}_z)$ PSL



$$\Rightarrow \vec{E} = E(x, y, z) \vec{u}_z$$

source transl^o (on) et (oy) —

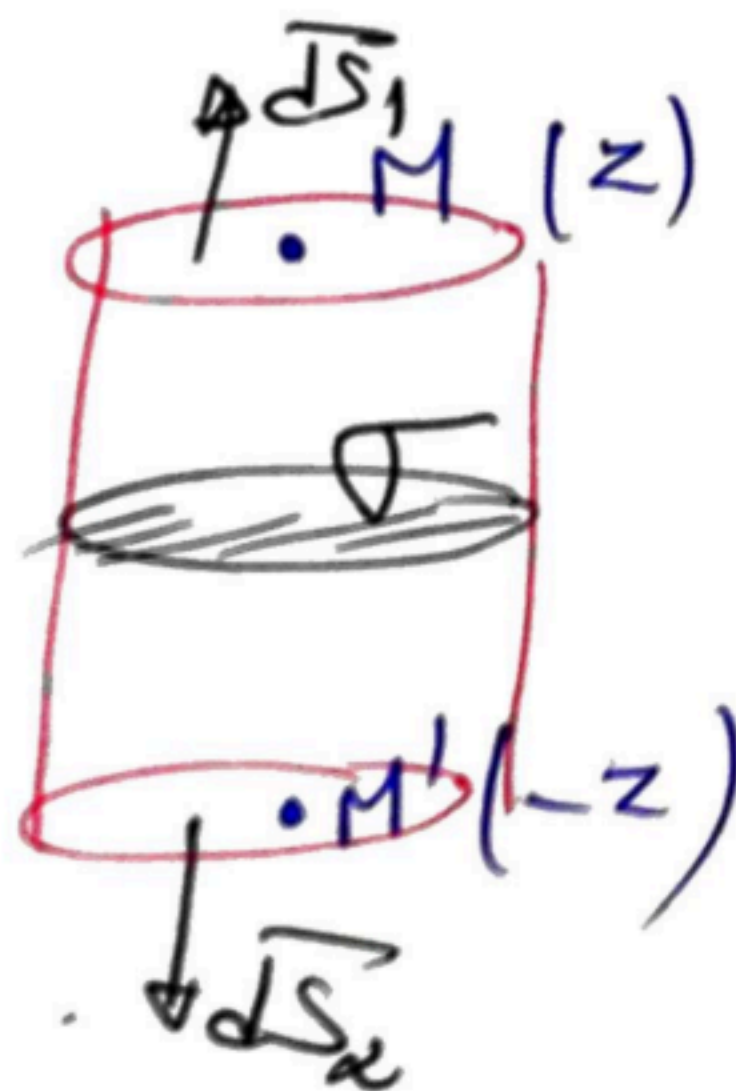
$$\vec{E} = E(z) \vec{u}_z$$

(noy) PSL pour $\sigma \Rightarrow E(-z) = -E(z)$

$$* Q_{int} = \sigma S_B$$

$$* \oint \vec{E} \cdot d\vec{S} = [E(z) - E(-z)] S_B$$

$$= 2E(z) S_B = \frac{\sigma S_B}{\epsilon_0}$$



$$\boxed{\vec{E}(z) = \frac{\sigma}{2\epsilon_0} \vec{u}_z} \quad z > 0$$

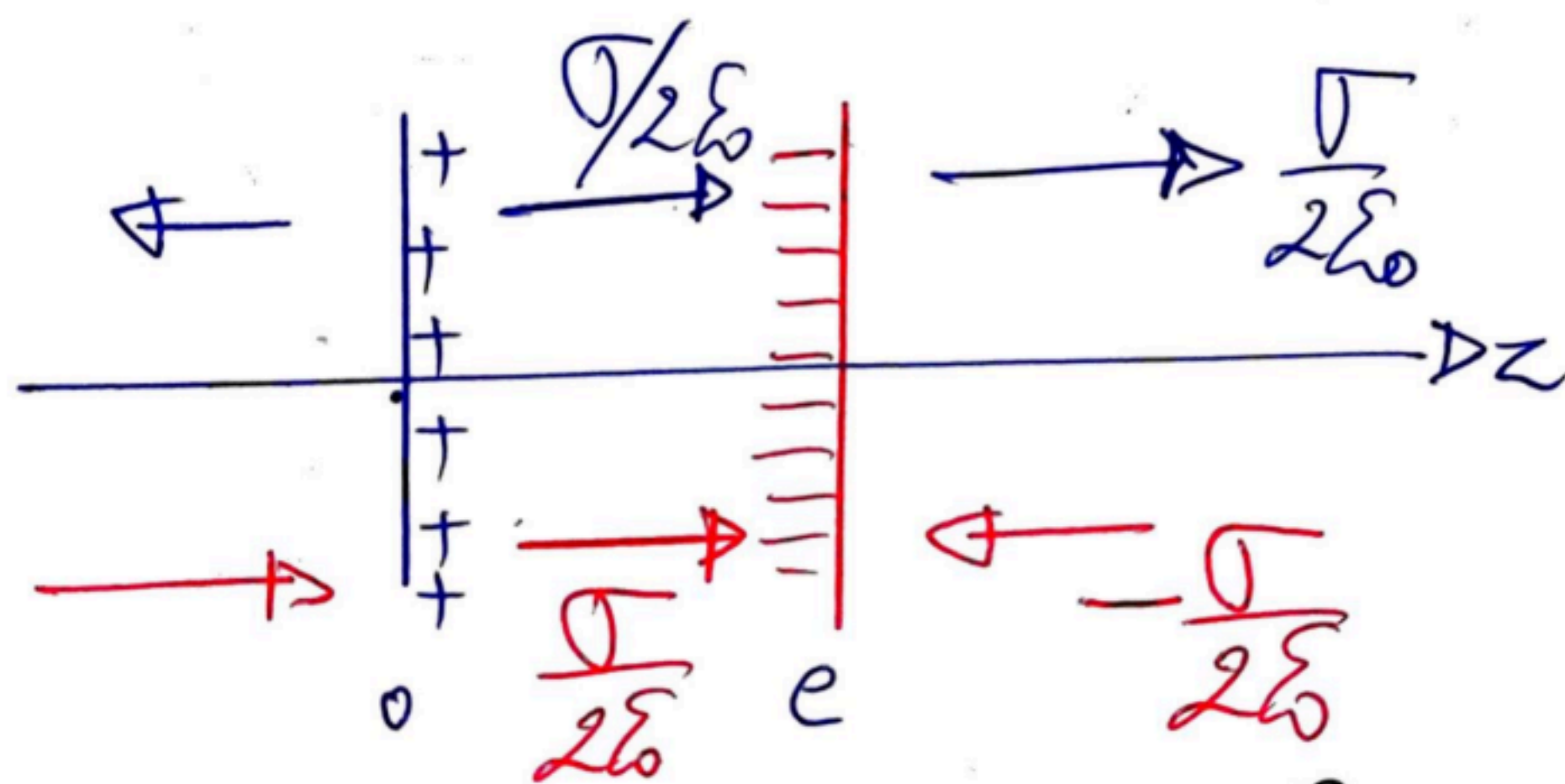
$$\vec{E}(z) = \frac{\sigma}{2\epsilon_0} \frac{z}{|z|} \vec{u}_z$$

$$\begin{cases} \vec{n}_{12} \wedge (\vec{E}_2 - \vec{E}_1) = \vec{0} & \text{tg} \\ \vec{n}_{12} \cdot [\vec{E}_2 - \vec{E}_1] = \frac{\sigma}{\epsilon_0} & \text{normale} \end{cases}$$

$$* \vec{E}_2(z > 0) - \vec{E}_1(z < 0) = \frac{\sigma}{\epsilon_0} \vec{n}_{12}$$

$$2\vec{E}(z > 0) = \frac{\sigma}{\epsilon_0} \vec{u}_z$$

$$\vec{E}(z > 0) = \frac{\sigma}{2\epsilon_0} \vec{u}_z$$



$$\vec{E}_{int} = \frac{\sigma}{\epsilon_0} \vec{u}_z = \frac{Q}{S \epsilon_0} \vec{u}_z = - \frac{dV}{dz} \vec{u}_z$$

$$\int_{V^+}^{V^-} dV(z) = - \frac{Q}{\epsilon_0 S} \int_0^e dz$$

$$V^+ - V^- = U = - \frac{Q}{\epsilon_0 S} e = \frac{Q}{C}$$

$$C = \frac{\epsilon_0 S}{e}$$

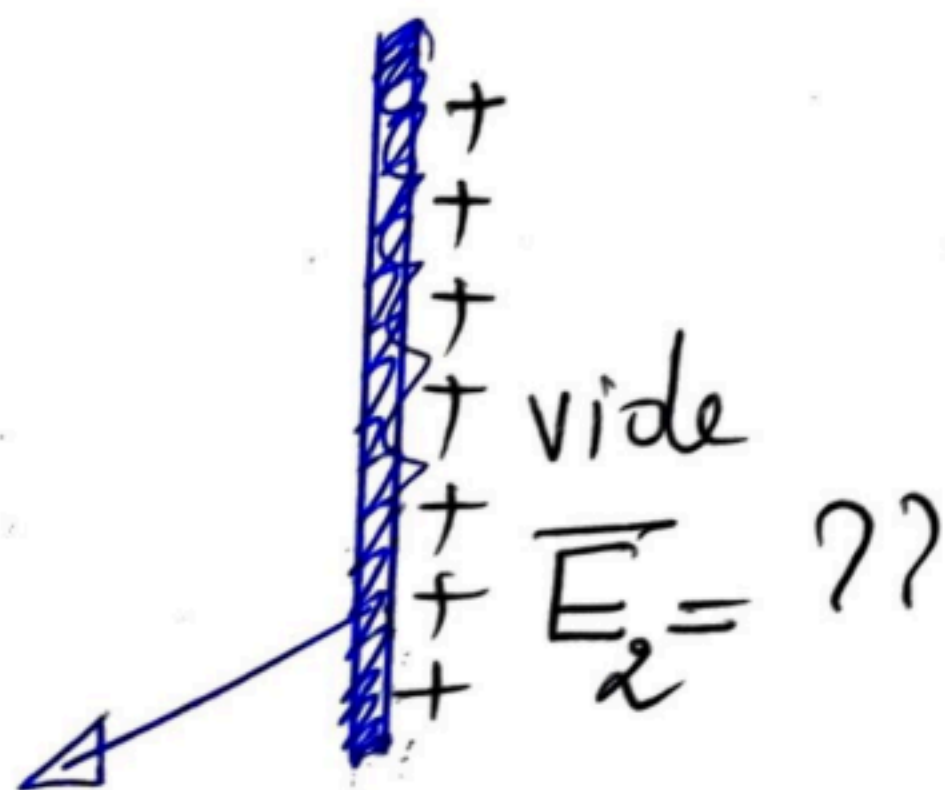
2^{em} méthode $W_{ele} = \frac{1}{2} \frac{Q^2}{C} = \iiint \frac{\epsilon_0 E^2}{2} dz$

$\frac{\epsilon_0 E^2}{2}$ = densité d'énergie volume.

$$\frac{\epsilon_0}{2} \iiint \left(\frac{Q}{S \epsilon} \right)^2 dx dy dz$$

$$= \frac{1}{2} \frac{Q^2}{\epsilon_0 S^2} S e = \frac{1}{2} \frac{Q^2}{\left(\frac{\epsilon_0 S}{e} \right)}$$

$$\vec{E}_{int} = \vec{\sigma}$$



métal $\vec{E}_1 = \vec{0}$

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{\epsilon_0} \quad \vec{n} = \vec{E}_2$$

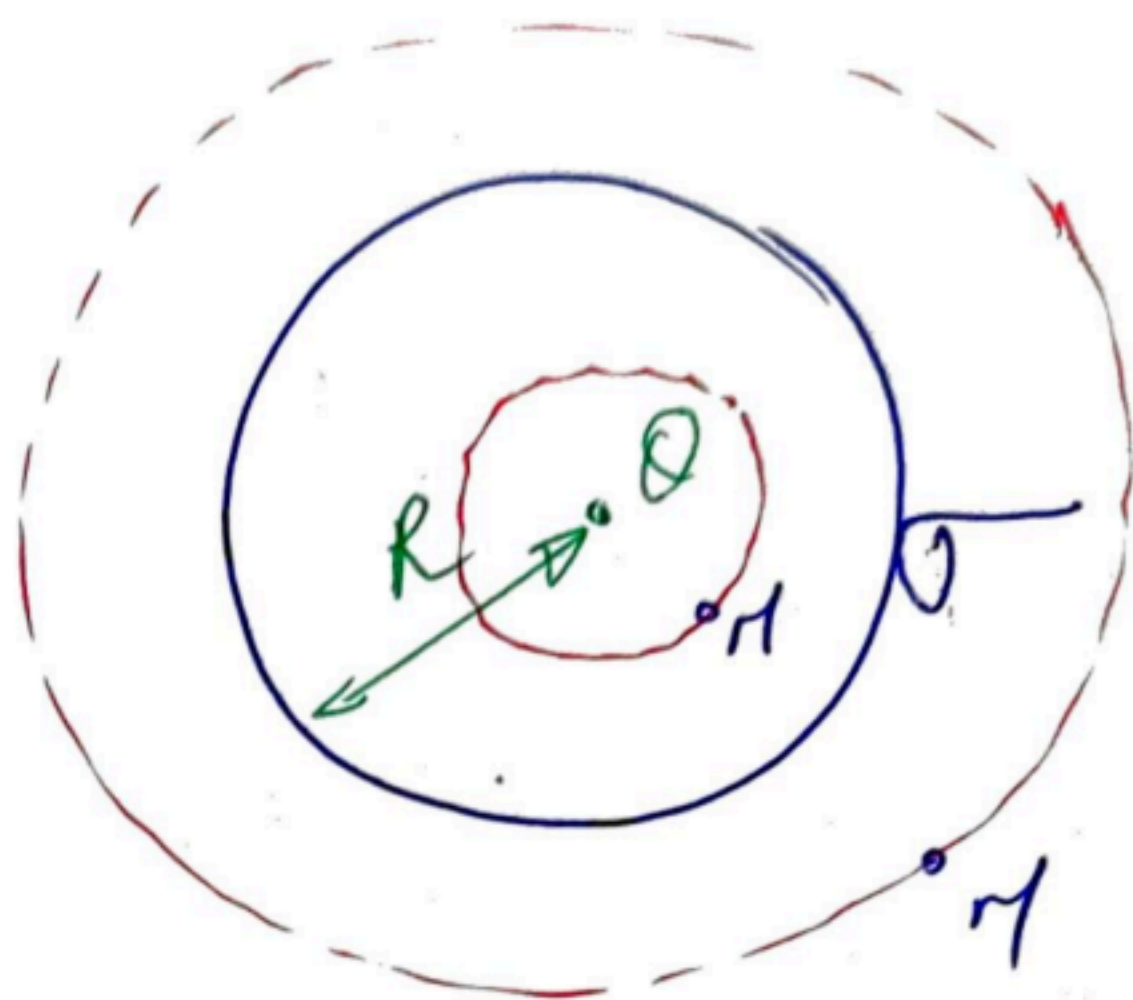
Coulomb

sphere creuse

$$(M, \vec{e}_r, \vec{e}_\theta) \text{ PSP}$$

$$(M, \vec{e}_r, \vec{e}_\varphi) \text{ PSP}$$

$$\vec{E} = E(r) \vec{e}_r$$



$$\oint \vec{E} \cdot d\vec{S} = 4\pi r^2 E(r)$$

$\int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\varphi$

$$r < R \quad Q_{\text{int}} = 0$$

$$r > R \quad Q_{\text{int}} = Q = \sigma 4\pi R^2$$

$$\Rightarrow \vec{E}(r < R) = 0$$

$$\vec{E}(r > R) = \frac{Q}{4\pi \epsilon_0 r^2} \vec{e}_r$$

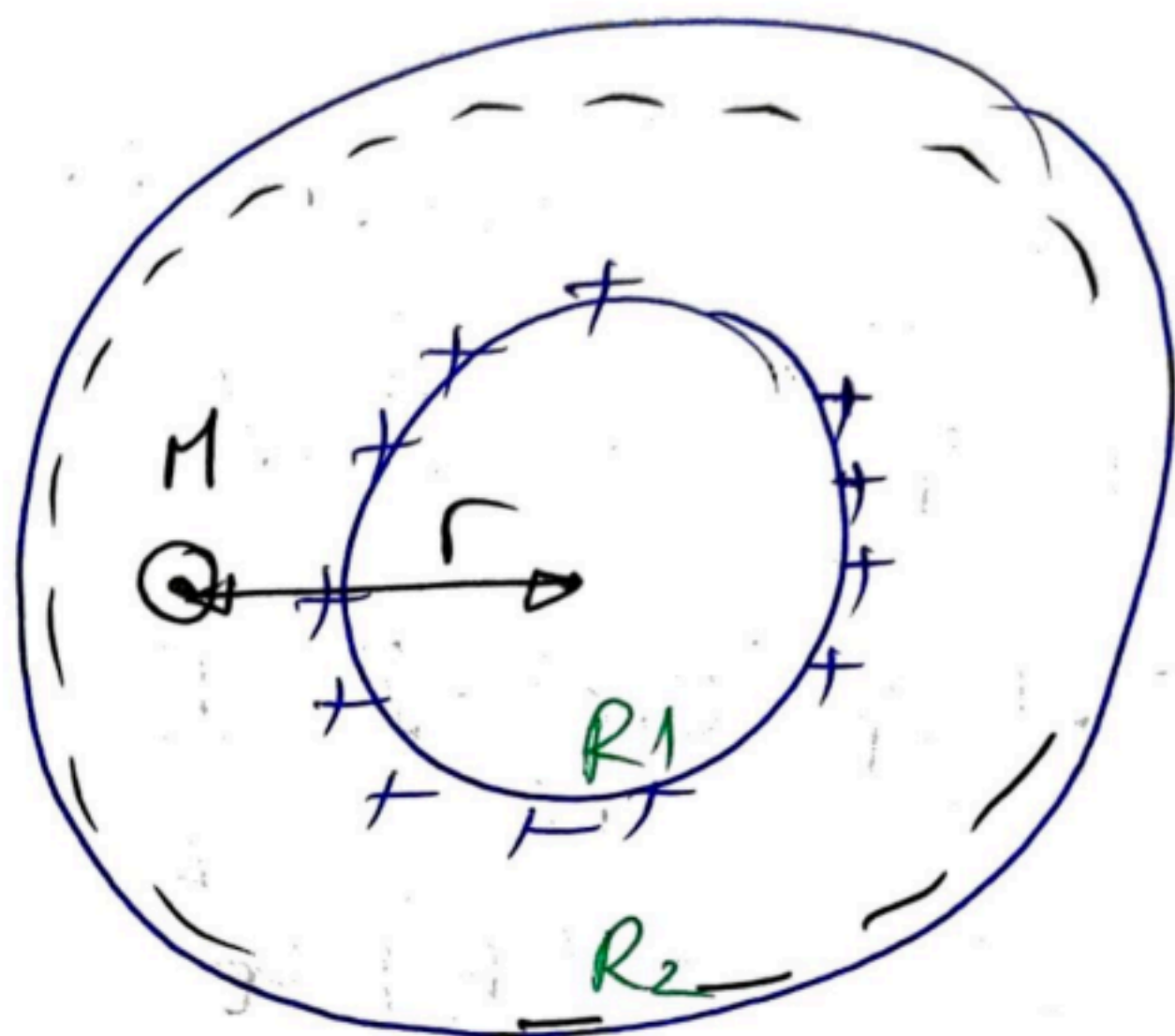
$$= \frac{\sigma R^2}{\epsilon_0 r^2} \vec{e}_r$$

$$* E(R^+) - E(R^-) = \frac{\sigma}{\epsilon_0} - 0 = \frac{\sigma}{\epsilon_0}$$

$$Q_1 = \sigma_1 \cdot 4\pi R_1^2 = Q$$

$$Q_2 = \sigma_2 \cdot 4\pi R_2^2 = -Q$$

$$\sigma_1 R_1^2 = |\sigma_2| R_2^2$$



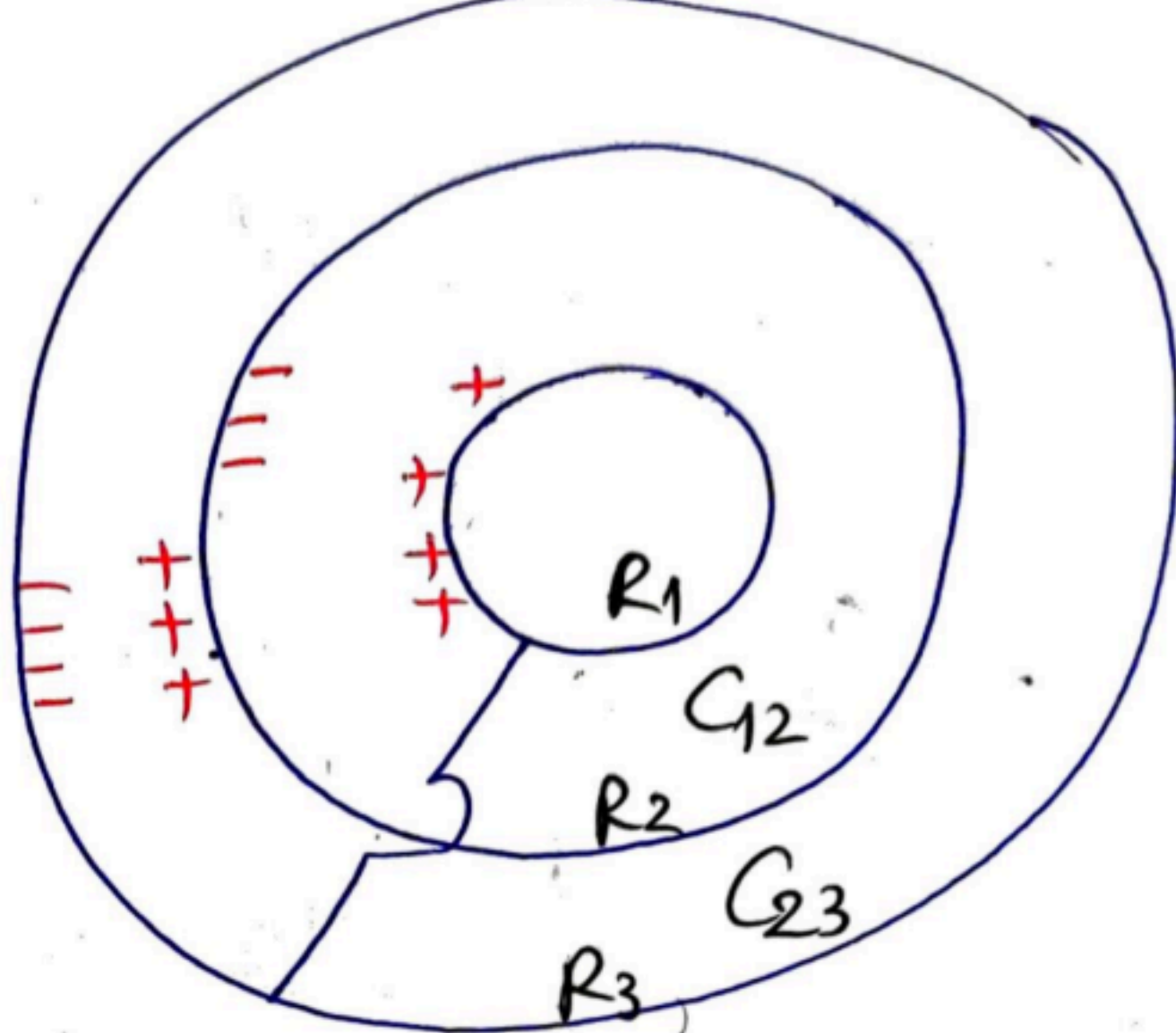
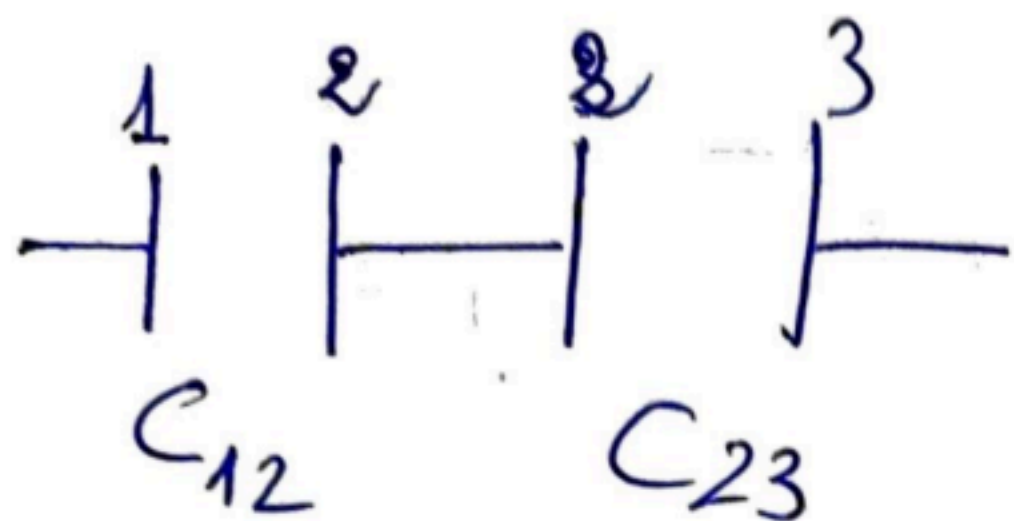
$$\vec{E}(r) = \frac{Q}{4\pi\epsilon_0 r^2} \vec{e}_r = \frac{\sigma_1 R_1^2}{\epsilon_0 r^2} \vec{e}_r = -\frac{dV}{dr} \vec{e}_r$$

$$\int_{V_1}^{V_2} dV(r) = - \int_{R_1}^{R_2} \frac{Q}{4\pi\epsilon_0} \frac{dr}{r^2}$$

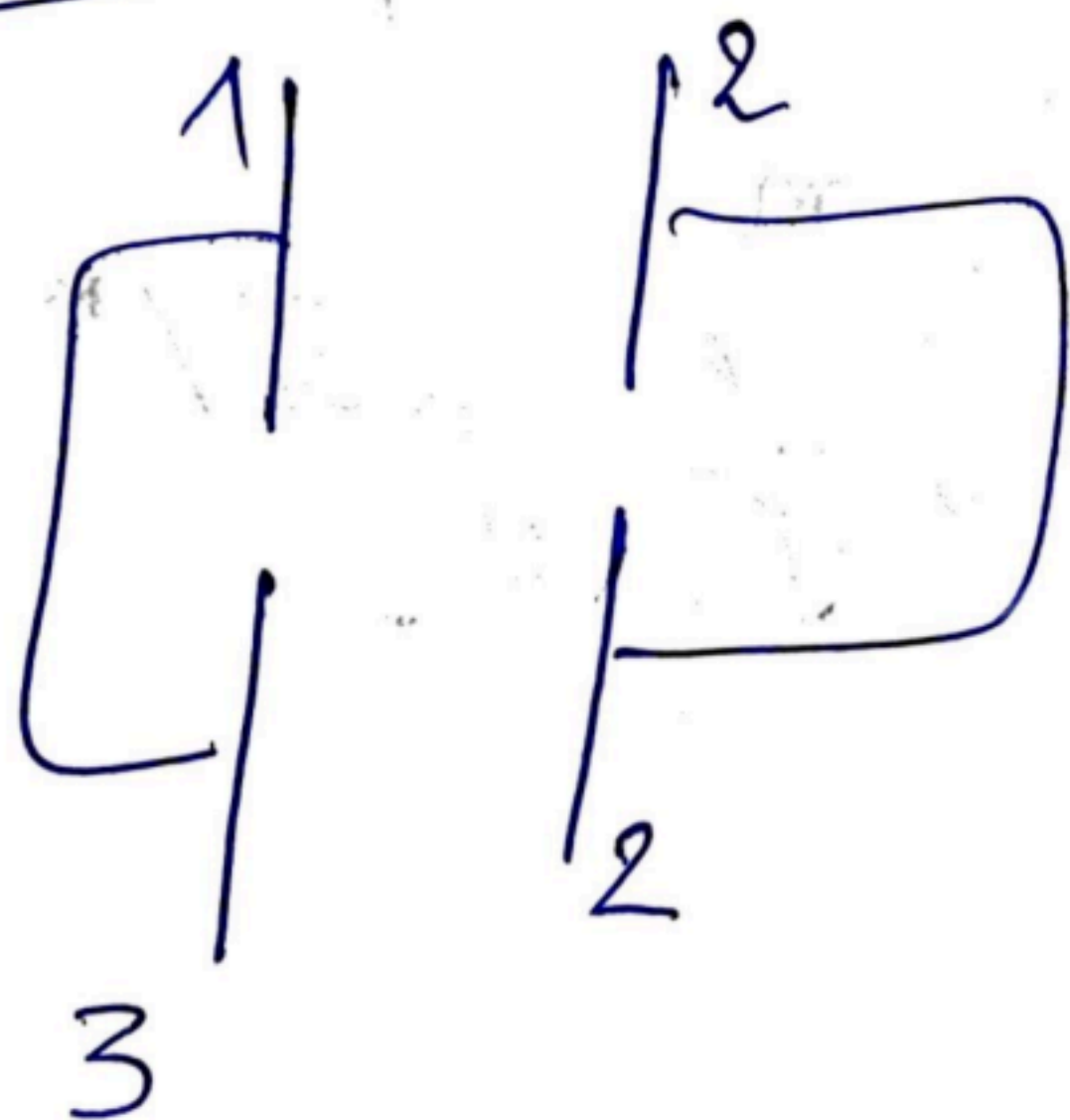
$$V_1 - V_2 = U = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= \frac{Q}{\left(\frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1} \right)}$$

$$C = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$



en //, — mm

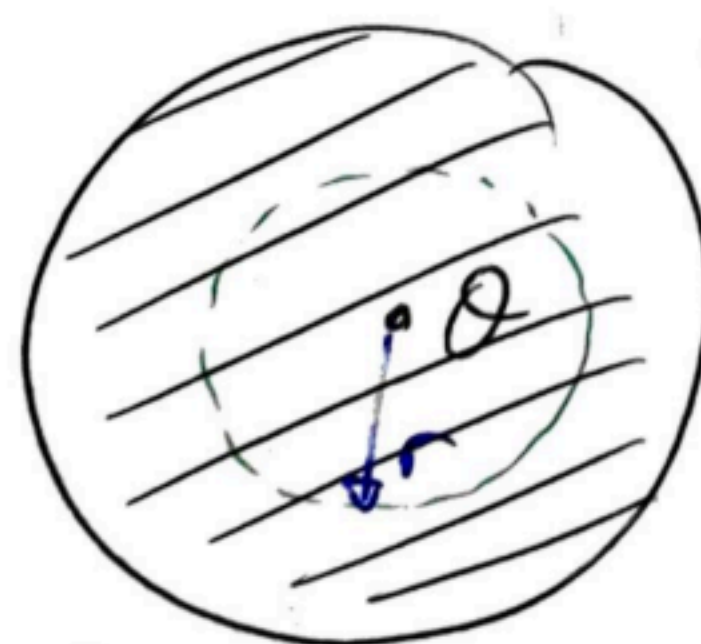


sphère pleine ρ

$$\oint \vec{E} \cdot d\vec{S} = 4\pi r^2 E(r)$$

$$r \leq R$$

$$Q_{\text{int}} = \rho \cdot \frac{4}{3} \pi r^3$$



$$r \geq R$$

$$Q_{\text{int}} = Q = \rho \frac{4}{3} \pi R^3$$

$$V(r) = - \int E(r) dr$$

$$\vec{F}_{ele} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \vec{r}_{12}$$

$$= q_1 \vec{E}_2$$

$$\vec{E}_2 = \frac{q_2 \vec{r}_{12}}{4\pi\epsilon_0 r^2}$$

$$\vec{F}_{Grav} = -\frac{G m_1 m_2}{r^2} \vec{r}_{12}$$

$$= m_1 \vec{g}$$

$$\vec{g} = -\frac{G m_2}{r^2} \vec{r}_{12}$$

$$\begin{array}{ccc} q & \longrightarrow & m \\ \vec{E} & \longrightarrow & \vec{g} = -\vec{g} \rho / V^* \end{array}$$

$$\boxed{\frac{1}{\epsilon_0} \longrightarrow -4\pi G}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{\sum Q_{int}}{\epsilon_0}$$

$$\oint \vec{g} \cdot d\vec{S} = -4\pi G \left(\sum M_{int} \right)$$

$$Q_{int} = \iiint \rho \, d\tau$$

$$M_{int} = \iiint \rho \, d\tau \quad \hookrightarrow \text{kg/m}^3$$



$$\oint \vec{E} \cdot d\vec{S} = \iiint (\text{div } \vec{E}) d\tau = \frac{\sum Q_{int}}{\epsilon_0}$$

$$= \frac{\iiint \rho d\tau}{\epsilon_0}$$

$\text{div } \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0}$

1 - Gauss

or $\vec{E} = -\vec{\nabla} V$

$$-\text{div}(\vec{\nabla} V) = -\frac{\rho}{\epsilon_0} = -\Delta V$$

$$\Delta V(\vec{r}) + \frac{\rho(\vec{r})}{\epsilon_0} = 0$$

$\nwarrow$
m-2

equation de Poisson

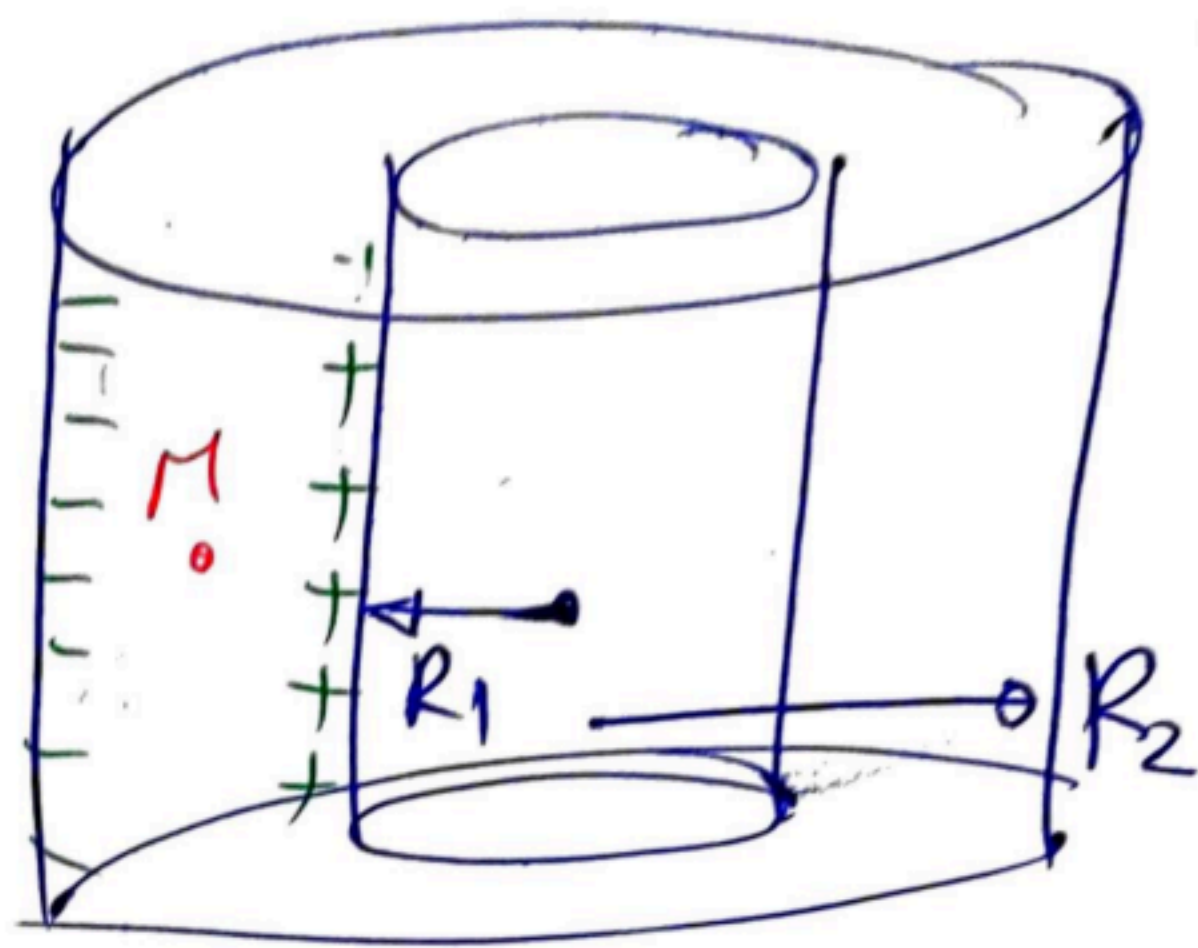
$$\Delta V(\vec{r}) = 0$$

Laplace

Déterminer : C
méthode locale

$$r \in [R_1, R_2]$$

$$\Delta V = 0$$



$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dV}{dr} \right) = 0 \Rightarrow r \frac{dV}{dr} = C_1$$

$$\frac{dV}{dr} = \frac{C_1}{r} \Rightarrow V(r) = C_1 \ln r + C_2$$

$$4 = C_1 \ln \left(\frac{R_1}{R_2} \right) = V(R_1) - V(R_2)$$

$$\vec{E}(R_1) = \frac{\sigma_1}{\epsilon_0} \vec{u}_r = \frac{Q}{2\pi R_1 h \epsilon_0} \vec{u}_r$$

$$= - \left(\frac{dV}{dr} \right)_{R_1} \vec{u}_r = - \frac{C_1}{R_1} \vec{u}_r$$

$$C_1 = \frac{-Q}{2\pi \epsilon_0 h}$$

$$U = \frac{Q}{2\pi\epsilon h} \ln\left(\frac{R_2}{R_1}\right) = \frac{Q}{C}$$

$$\frac{C}{h} = \frac{2\pi\epsilon}{\ln(R_2/R_1)} = \Gamma = \text{capacit\'e' lin\'eique (F.m}^{-1}\text{)}$$

Pq: $R_1 + e = R_2$

$$\ln\left(\frac{R_2}{R_1}\right) = \ln\left(1 + \frac{e}{R_1}\right) \approx \frac{e}{R_1}$$

$$\Gamma \approx \frac{2\pi\epsilon R_1}{e} = \frac{C}{h}$$

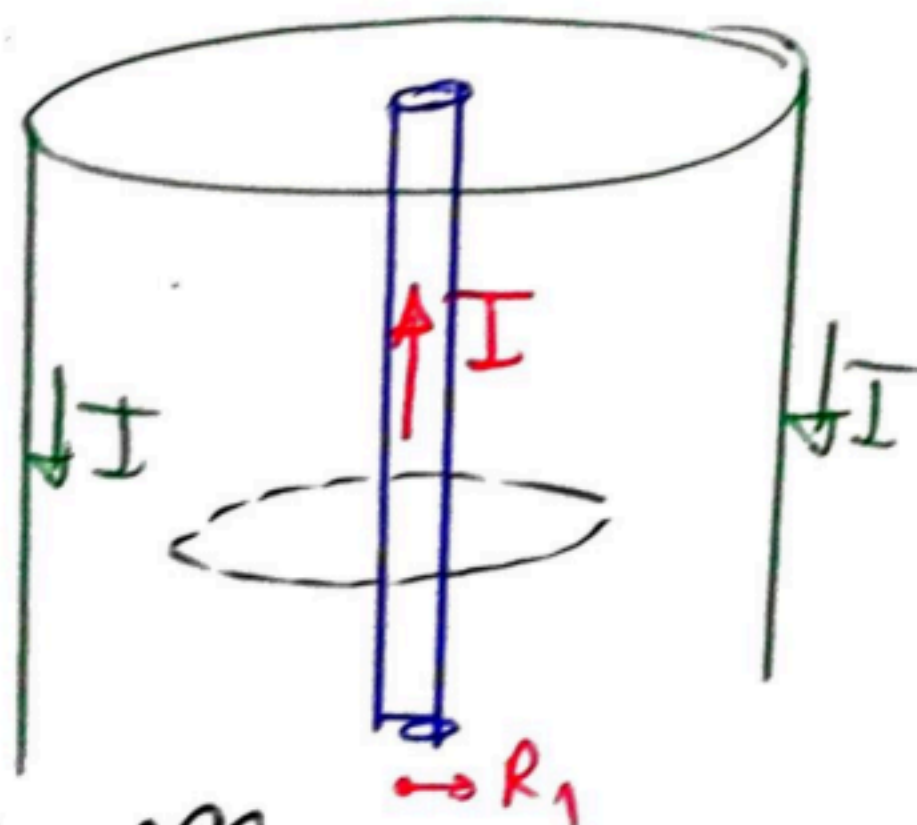
$$C \approx \frac{2\pi R_1 h \epsilon}{e} \approx \frac{\epsilon_0 S_{\text{ext}}}{e}$$

$$I = \int j_{s1} d\vec{n}$$

$$I = j_{s1} 2\pi R_1$$

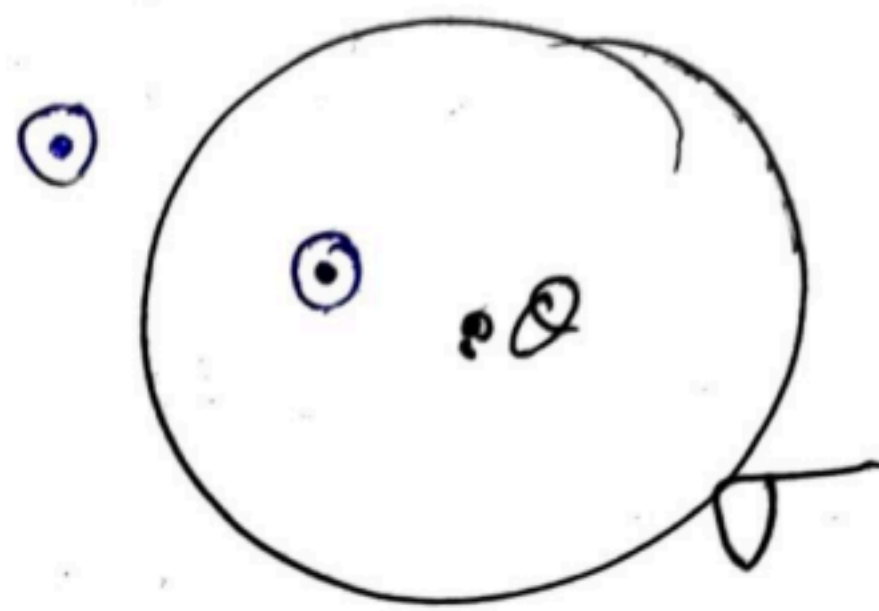
$$\vec{B} = \frac{\mu_0 I}{2\pi r} \vec{e}_\phi$$

$$W_{\text{mag}} = \frac{1}{2} L I^2 = \iint \frac{B^2}{2\mu_0} d\tau$$



sphère creuse : Déterminer le $V(r)$
 $\forall r$

$$\Delta V(r) = 0.$$



$$\Rightarrow \frac{1}{r} \frac{\partial^2}{\partial r^2} (r V(r)) = 0$$

$$\frac{d}{dr} (r V(r)) = C_1 \Rightarrow r V(r) = C_1 r + C_2$$

$$\boxed{V_{\text{int}}(r) = C_2 + \frac{C_1}{r}}$$

$$\boxed{V_{\text{ext}}(r) = C_3 + \frac{C_4}{r}}$$

• $r \rightarrow 0$; $V_{\text{int}}(r) = \infty \Rightarrow C_2 = 0.$

• $r \rightarrow \infty$; $V_{\text{ext}}(r) = C_3 + \frac{C_4}{\infty} = 0.$

$$\left. \begin{array}{l} V_{\text{int}} = C_1 \\ V_{\text{ext}} = \frac{C_4}{r} \end{array} \right\}$$

$$\Rightarrow \boxed{C_1 = \frac{C_4}{R}}$$

• $E_{\text{ext}}(R^+) - E_{\text{int}}(R^-) = \frac{\sigma}{\epsilon_0}$

$$-\frac{dV_{\text{ext}}(r)}{dr} - \left(-\frac{dV_{\text{int}}(r)}{dr} \right) = \frac{\sigma}{\epsilon_0}$$

$$\left[\frac{d}{dr} \left(\frac{C_4}{r} \right) \right] - 0 = \frac{\sigma}{\epsilon_0}$$

$$+\frac{C_4}{R^2} = \frac{\sigma}{\epsilon_0} \Rightarrow C_4 = \frac{\sigma R^2}{\epsilon_0}$$

$$\rho(r) = \frac{a r}{R}$$

$$\text{div } \vec{E} = \frac{\rho}{\epsilon_0} = \text{div} (E(r) \vec{u}_r)$$

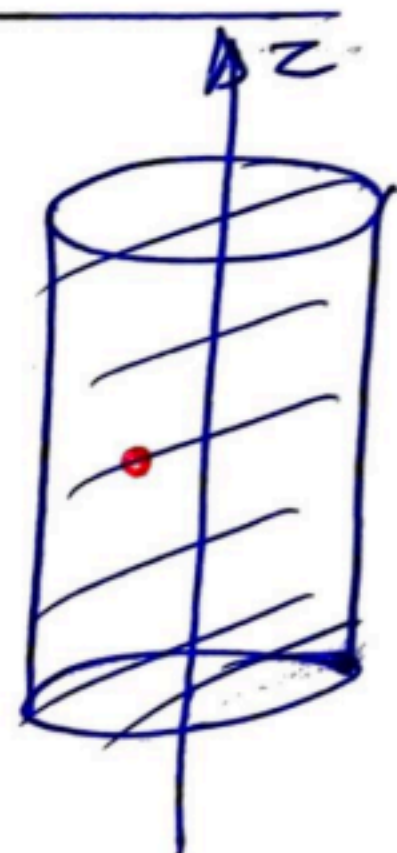
$$\frac{1}{r} \frac{d}{dr} (r E(r)) = \frac{\rho(r)}{\epsilon_0}$$

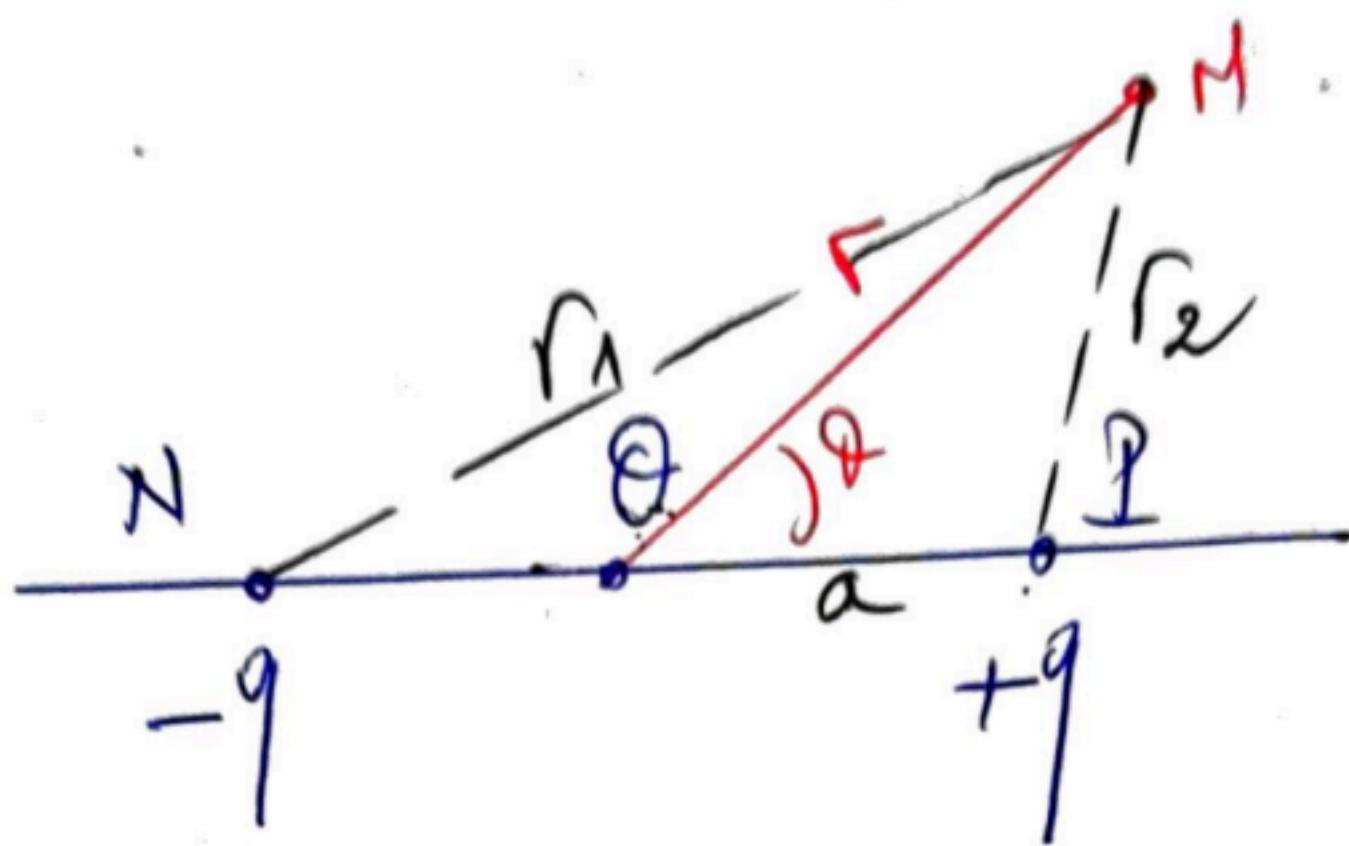
$$r \geq R: \rho(r) = 0 \Rightarrow r E(r) = C_1$$

$$E(r) = \frac{C_1}{r}$$

$$r \leq R \quad \frac{1}{r} \frac{d}{dr} (r E(r)) = \frac{a r}{R \epsilon_0}$$

$$\frac{d}{dr} (r E(r)) = \frac{a r^2}{\epsilon_0 R} \quad \left| \begin{array}{l} r E(r) = \frac{a r^3}{3 \epsilon_0 R} + C_2 \\ E(r) = \frac{a r^2}{3 \epsilon_0 R} + \frac{C_2}{r} \end{array} \right.$$





$$NP = 2a \ll OM$$

$$V(M) = \frac{q}{4\pi\epsilon_0} \frac{1}{r_2} - \frac{q}{4\pi\epsilon_0} \frac{1}{r_1}$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$r_2^2 = a^2 + r^2 - 2ar \cos \theta$$

$$r_2^2 = r^2 \left[1 + \frac{a^2}{r^2} - \frac{2a}{r} \cos \theta \right]$$

$$r_2 = r \left[1 + \frac{a^2}{2r^2} - \frac{2a}{r} \cos \theta \right]$$

$$r_2 \approx r \left[1 - \frac{a}{r} \cos \theta \right]; \quad \frac{a}{r} \approx 10^{-6}$$

$$\frac{1}{r_2} \approx \frac{1}{r} \left(1 + \frac{a}{r} \cos \theta \right)$$

$$\frac{1}{r_1} \approx \frac{1}{r} \left(1 - \frac{a}{r} \cos \theta \right)$$

$$V(r) = \frac{1}{r} \frac{q}{4\pi\epsilon_0} \frac{(2a) \cos \theta}{r}$$

$$= \frac{(2aq) \cos \theta}{4\pi\epsilon_0 r^2}$$

$$V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

$$\vec{p} = 2aq \vec{u}_x$$

$\perp$ to C.M

$$\vec{E} = -\frac{\partial V}{\partial r} \vec{e}_r - \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{e}_\theta - \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \vec{e}_\phi$$

$$E_r = \frac{2p \cos \theta}{4\pi\epsilon_0 r^3}$$

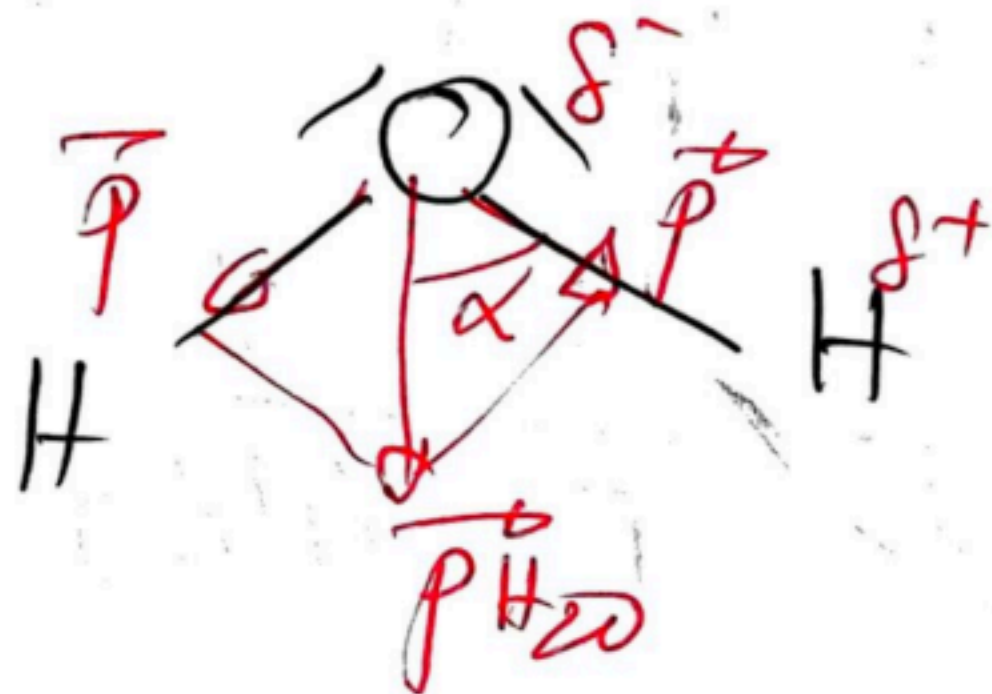
$$\|\vec{E}\|: \frac{V}{m}$$

$$E_\theta = \frac{p \sin \theta}{4\pi\epsilon_0 r^3}$$

$$\vec{E} = \frac{P}{4\pi\epsilon_0 r^3} (2\cos\theta \vec{e}_r + \sin\theta \vec{e}_\theta)$$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3} (2\cos\theta \vec{e}_r + \sin\theta \vec{e}_\theta)$$

dipôle électrique



Les lignes de $\vec{E}$: $d\vec{l} \perp \vec{E} = 0$

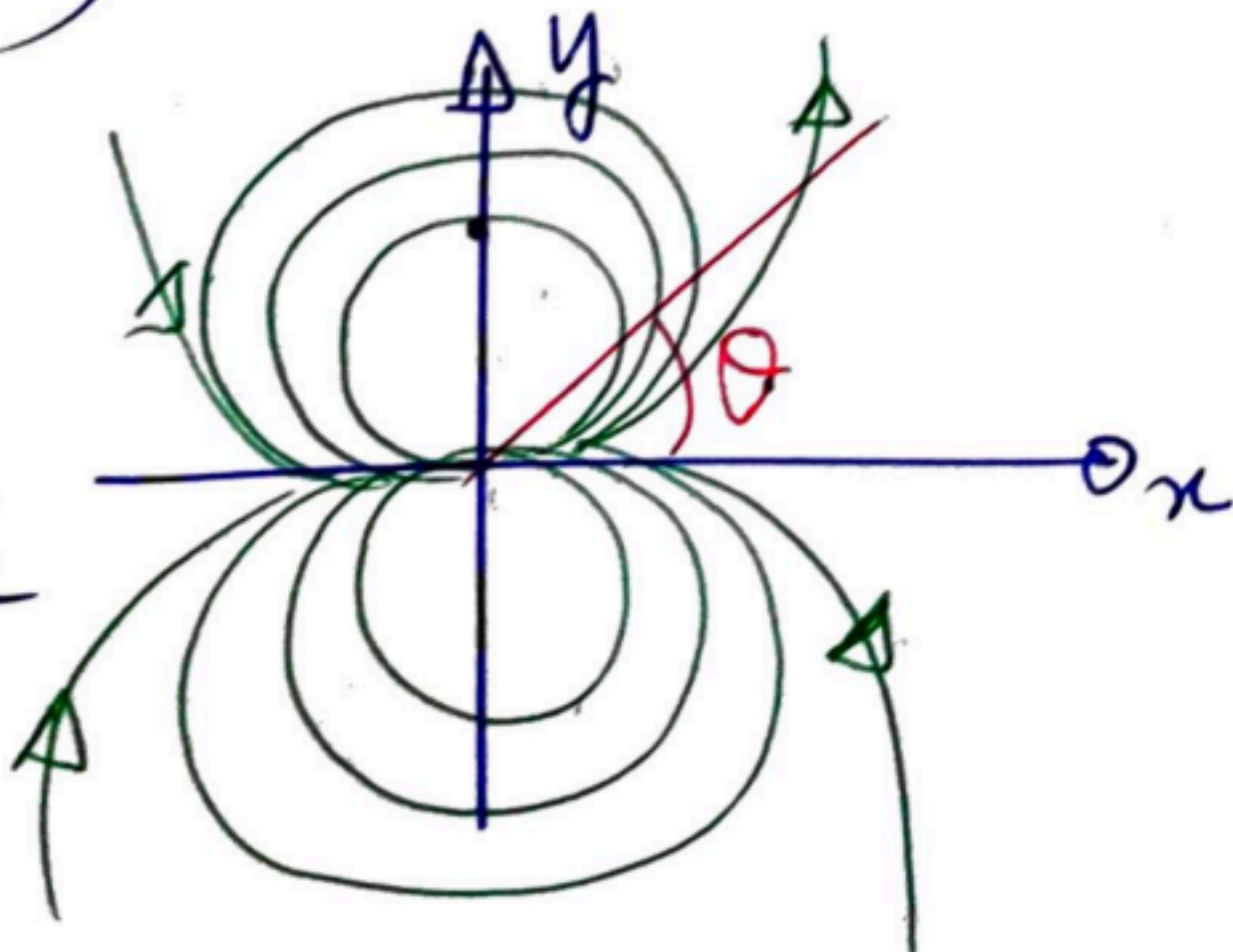
$$\frac{dr}{E_r} = \frac{r d\theta}{E_\theta}$$

$$\frac{dr}{r} = \frac{E_r}{E_\theta} d\theta = \frac{2\cos\theta}{\sin\theta} d\theta$$

$$\ln r = \ln(\sin^2\theta) + \text{cte}$$

$$r = r_0 \sin^2\theta$$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin^2\theta$	0	$1/4$	$1/2$	$3/4$	1



à dire d'un champ E_{ext} sur le dipôle

$$\vec{E}_{ext} = E_0 \vec{u}_x$$

$$V(M) = V_0 + V_{dipôle}$$

$$\vec{E}_0 = - \frac{dV_0}{dx} \vec{u}_x$$

$$V_0 = - E_0 x$$

$$V_{tot} = - E_0 r \cos \theta + \frac{p \cos \theta}{4\pi\epsilon r^2}$$

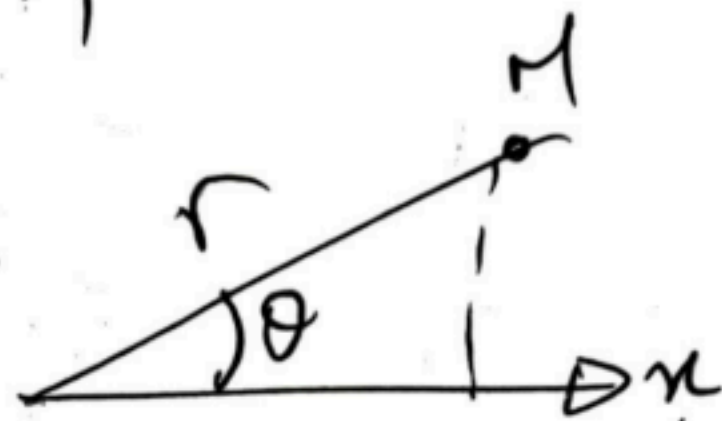
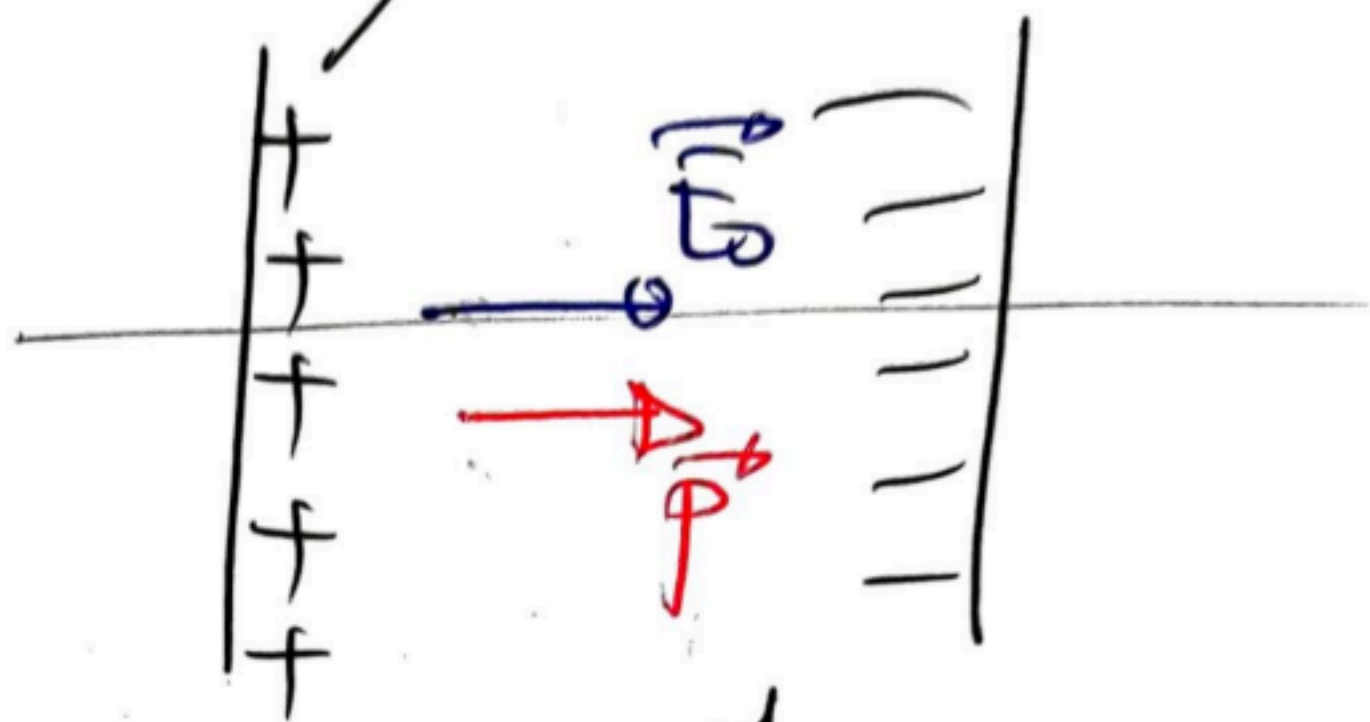
$$V_{tot} = \left(- E_0 r + \frac{p}{4\pi\epsilon r^2} \right) \cos \theta$$

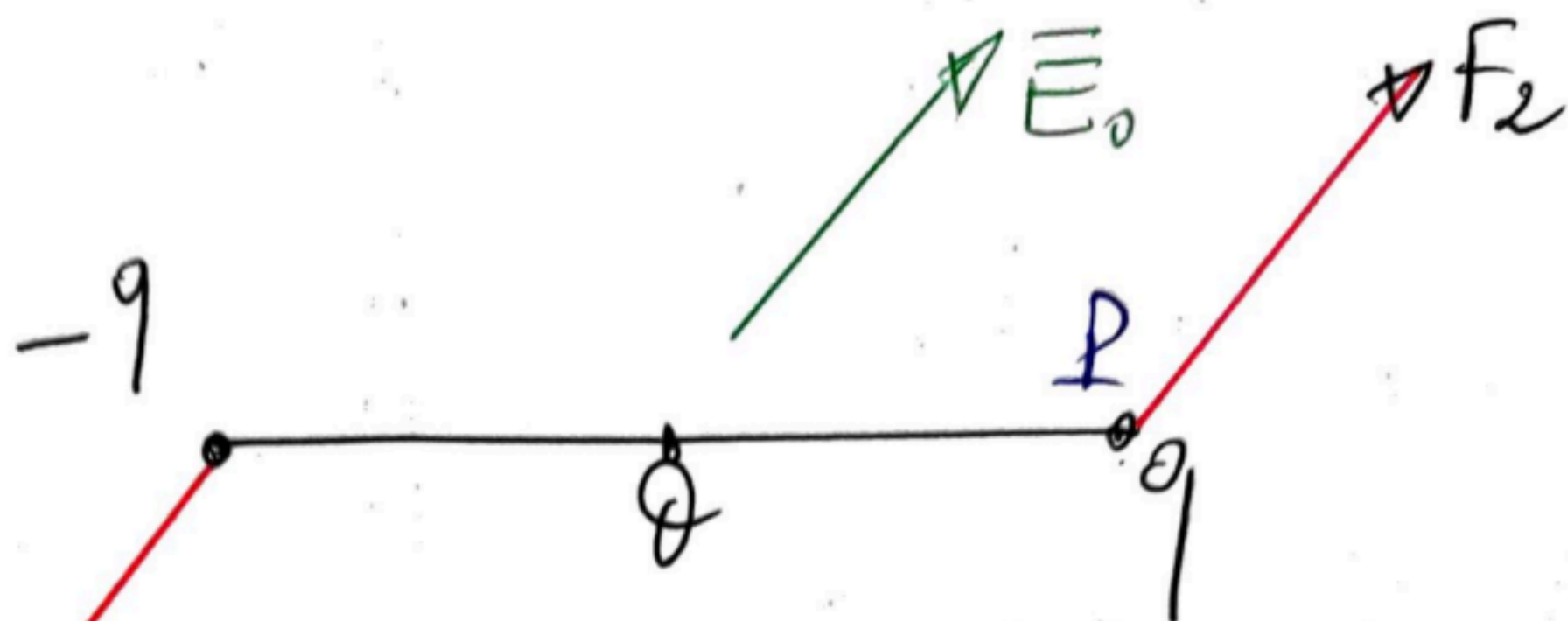
$V_{tot} \rightarrow 0$ $\theta = \frac{\pi}{2} \rightarrow y \text{ ou } z$

$$\Rightarrow r_0^3 E_0 = \frac{p}{4\pi\epsilon}$$

$$r_0 = \left(\frac{p}{4\pi\epsilon E_0} \right)^{1/3};$$

sphere





$$\Sigma \vec{F}_{ext} = \vec{0}; \quad \vec{M}_O(\vec{F}_2) = \vec{OP} \wedge q \vec{E}_0$$

$$\vec{M}_O(\vec{F}_1) = \vec{ON} \wedge (-q \vec{E}_0)$$

$$\vec{\Gamma} = q [\vec{OP} + \vec{NO}] \wedge \vec{E}_0$$

$$= (q \vec{NP}) \wedge \vec{E}_0 = \vec{p} \wedge \vec{E}_0$$

$$\vec{\Gamma} = \vec{m} \wedge \vec{B}_{ext}$$