

$$\oint \vec{B} \cdot d\vec{S} = \iiint \text{div } \vec{B} \, d\tau = 0 \Rightarrow \text{div } \vec{B} = 0$$

$$e = -\frac{d\Phi}{dt} \rightarrow \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \left(\iint \vec{B} \cdot d\vec{S} \right)$$

$$\oint \vec{E} \cdot d\vec{S} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\boxed{\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \text{1-F}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{\sum Q_{\text{int}}}{\epsilon_0} \Rightarrow \text{div } \vec{E} = \frac{\rho}{\epsilon_0}$$

$$J_{\text{tot}} = J_{\text{conduction}} + J_{\text{displacement}} \Rightarrow$$

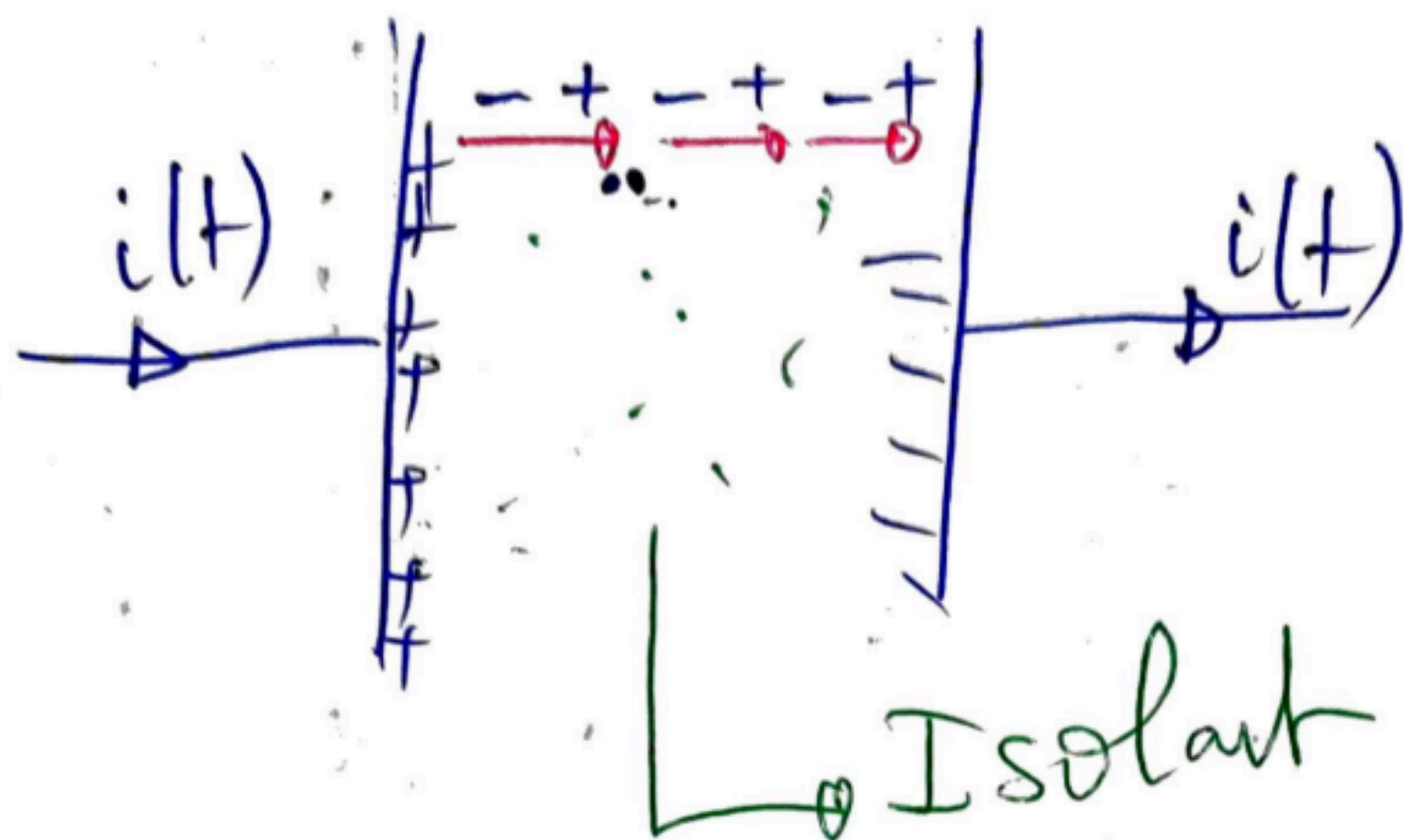
localement neutre $\Rightarrow J_{\text{tot}} = 0$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left[\iint \vec{J}_c \cdot d\vec{S} + \iint \epsilon_0 \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S} \right]$$

$$\Rightarrow \text{rot } \vec{B} = \mu_0 \left[\vec{J}_c + \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \right]$$

$\vec{J}_c$ = courant de conduction des e^- de valence.

$\vec{J}_D$ = " " déplacement des dipôles



$$\left| \epsilon \frac{\partial \vec{E}}{\partial t} \right| \gg \|\vec{j}_{cl}\| : \text{Dielectric}$$

$$\|\vec{j}_{cl}\| \gg \left| \epsilon \frac{\partial \vec{E}}{\partial t} \right| : \text{conductor}$$

$$\text{div} \vec{j} + \frac{\partial \rho}{\partial t} = 0$$

$$\text{div}(\text{rot} \vec{X}) = 0$$

$$\text{div}(\text{rot} \vec{B}) = 0 \Rightarrow \mu_0 \left[\text{div} \vec{j} + \frac{\partial}{\partial t} (\epsilon \text{div} \vec{E}) \right] = 0$$

$$\text{div} \vec{j}_c + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{dU_{\text{em}}}{dt} = - \oint \vec{T} \cdot d\vec{S} - \iiint (\vec{f}_c \cdot \vec{E}) d\tau$$

$$\frac{d}{dt} \iiint u_e(\vec{r}, t) d\tau = - \iiint \text{div}(\vec{T}) d\tau - \iiint (\vec{f}_c \cdot \vec{E}) d\tau$$

$$\boxed{\frac{\partial U_{\text{em}}}{\partial t} + \text{div} \vec{T} + \vec{f} \cdot \vec{E} = 0}$$

$$\vec{T} = \frac{1}{\mu_0} (\vec{E} \otimes \vec{B}) = \vec{R} = \vec{S} = \vec{N}$$

$$\langle \|\vec{T}\| \rangle = \frac{\langle dP_{\text{rayonnée}} \rangle}{dS} \quad \text{watt/m}^2$$

une Intensité

$$u_{\text{em}} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$

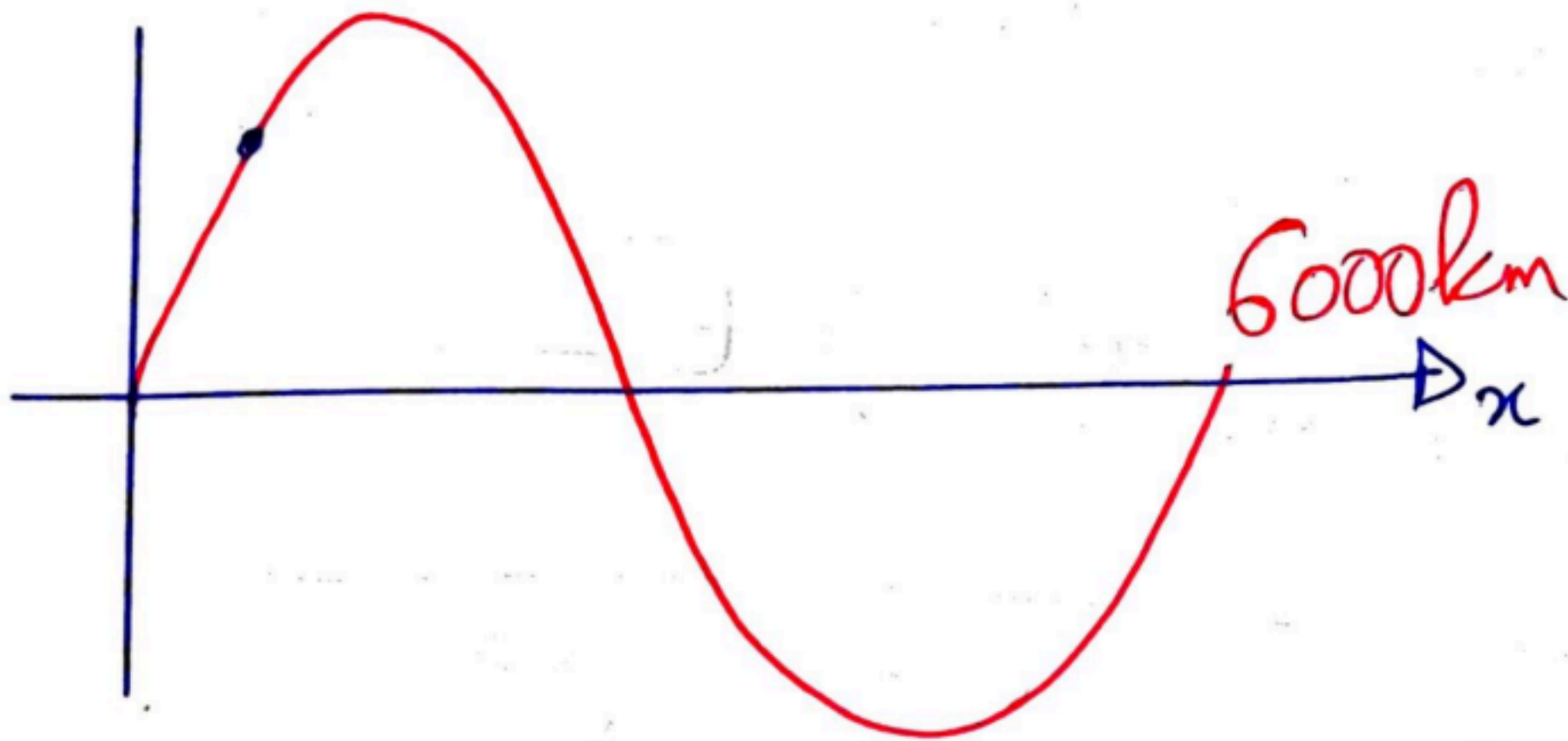
A.R.Q.S.: $\tau_{\text{prop}} \ll T$

$$\frac{d}{c} \ll T \Rightarrow d \ll cT = \lambda$$

$$\tau_{\text{prop}} \ll T^*$$

$$f_{\text{STEG}} = 50 \text{ Hz}; \quad \lambda_{\text{STEG}} = \frac{c}{f} = \frac{3 \cdot 10^8}{50}$$

$$\lambda_{\text{STEG}} = 6000 \text{ km}$$



Serie N°1

Ex 1

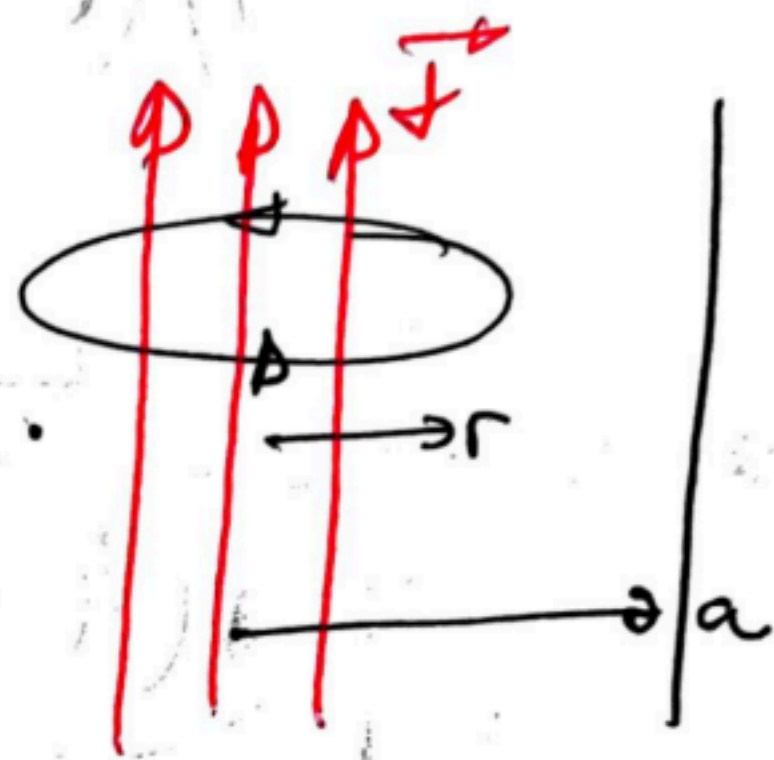
$$\vec{J} = \sigma \vec{E}$$



$$I = \int \vec{J} \cdot d\vec{S} = J \cdot \pi a^2$$

$$1) \vec{E} = \frac{\vec{J}}{\sigma} = \frac{I}{\pi a^2 \sigma} \vec{u}_z$$

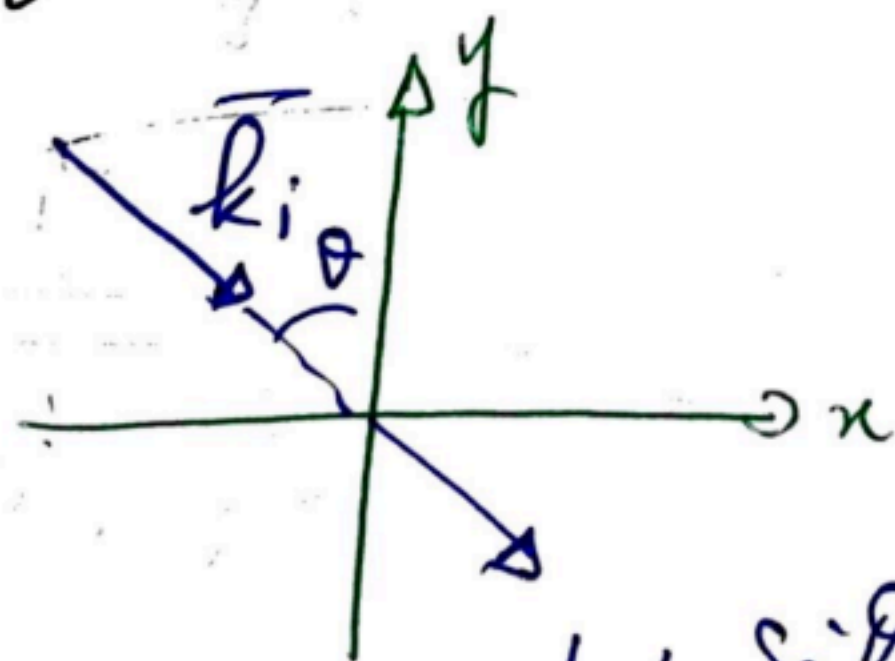
($\vec{r} \cdot \vec{e}_z$) $\perp$ $d\vec{S} \perp$ $\text{pmr } I$



$$\Rightarrow \vec{B} \perp \Rightarrow \oint \vec{B} \cdot d\vec{l} = 2\pi r B(r) = \mu_0 I'$$

$$I' = J \pi r^2 \Rightarrow \vec{B}_\theta = \frac{\mu_0 J r}{2} \vec{e}_\theta$$

$$\boxed{\vec{B} = \frac{\mu_0 I r}{2\pi a^2} \vec{e}_\theta}$$



$$2) L_{em} = \int_0^L \int_0^{2\pi} \int_0^a \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) r dr d\theta dz$$

$$\vec{k}_i = \frac{\omega}{c} \begin{pmatrix} + \sin\theta \\ -\cos\theta \\ 0 \end{pmatrix}$$

$$\frac{dL_{em}}{dt} = 0$$

$$3) \mathcal{L}W_{Joule} = \left[\iiint (\vec{J}_c \cdot \vec{E}) d\tau \right] dt$$

$$P_{Joule} = \iiint \left(\vec{J}_c \cdot \frac{\vec{J}_c}{\sigma} \right) d\tau = \left(\frac{I}{\pi a^2} \right)^2 \frac{1}{\sigma} L (\pi a^2)$$

$$P_{Joule} = \left(\frac{1}{\sigma} \frac{L}{\pi a^2} \right) I^2 = R_c I^2$$

$$4) P_{rayonnée} = \oint \vec{\kappa} \cdot d\vec{S}_{ext}$$

$$\vec{\kappa} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{E \vec{u}_z \wedge B_0 \vec{e}_\phi}{\mu_0} = \frac{E B_0}{\mu_0} (-\vec{e}_r)$$

$$\vec{\kappa} = - \frac{I^2}{2(\pi a^2)^2 \sigma} \vec{e}_r$$

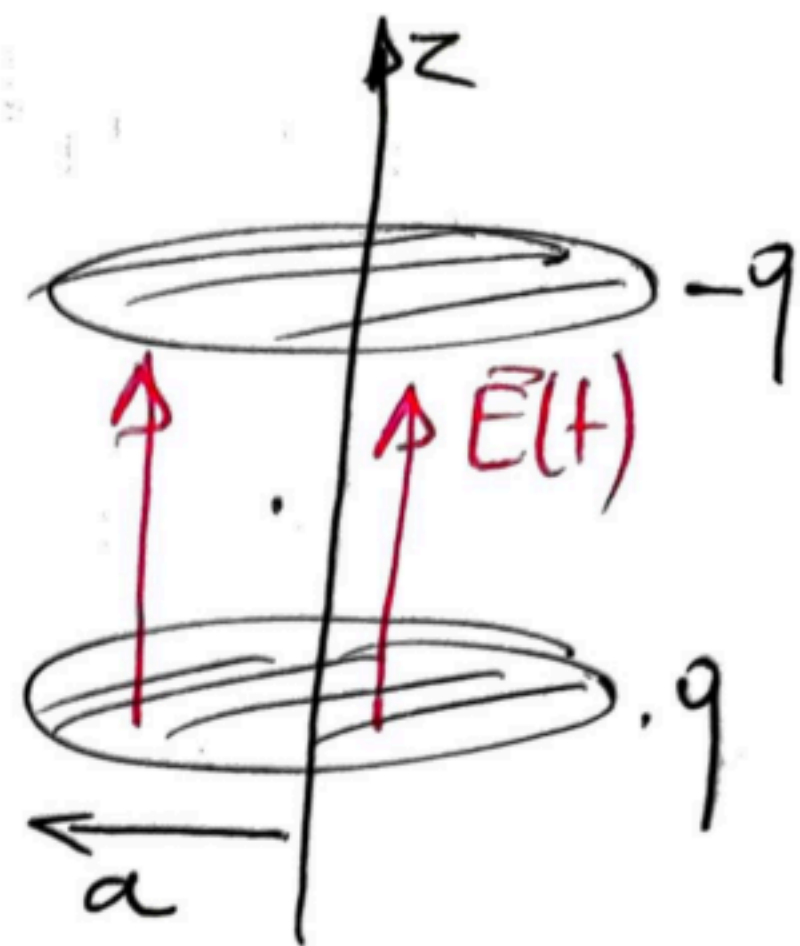
$$P_{rayonnée} = - \frac{I^2}{2\sigma(\pi a^2)^2} \int a(a d\theta dz) \quad \text{latéran}$$

$$= - \frac{2\pi a^2 L}{2\sigma(\pi a^2)^2} I^2 = - R_e I^2$$

$$\mathcal{L}W_{Rms} = (-R_e I^2) dt$$

Ex2 PT2 2006

$$\vec{E}(t) = \frac{\nabla V}{\epsilon_0} = \frac{Q(t)}{S \epsilon_0} \vec{u}_z$$



$$\vec{E}(t) = E_0 e^{-t/\tau} \vec{u}_z$$

$$V = (2h) (\pi a^2)$$

$$\vec{E}(t) \longrightarrow \vec{B}(r, t)$$

$$\nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}(t)}{\partial t}$$

$$\vec{J}_C = \vec{0}$$

$$\vec{J}_D = \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\boxed{\mu_0 \epsilon_0 c^2 = 1}$$

(M $\vec{u} \vec{e}_z$) $\perp$ $\vec{S} \perp$ pour la cause $\left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$

$\Rightarrow$ l'effet $\vec{B}$ $\perp$ à ce plan.

$$\vec{B} = B_0(r, t) \vec{u}_\theta$$

$$\nabla \times [B_0(r, t) \vec{u}_\theta] = \left(\frac{1}{c^2} \frac{\partial E(t)}{\partial t} \right) \vec{u}_z$$

$$\left[\frac{1}{r} \frac{\partial}{\partial r} (r B_0(r, t)) \right] = \frac{1}{c^2} \frac{\partial E(t)}{\partial t}$$

$$\frac{\partial}{\partial r} (r B_\theta(r, t)) = \frac{1}{c^2} r \frac{dE(t)}{dt}$$

$$\cancel{r} B_\theta(r, t) = \frac{\cancel{r}}{2c^2} \frac{dE(t)}{dt} + \frac{A_1}{r}$$

r diverge
 si $r \rightarrow \infty$
 $\Rightarrow A_1 = 0$

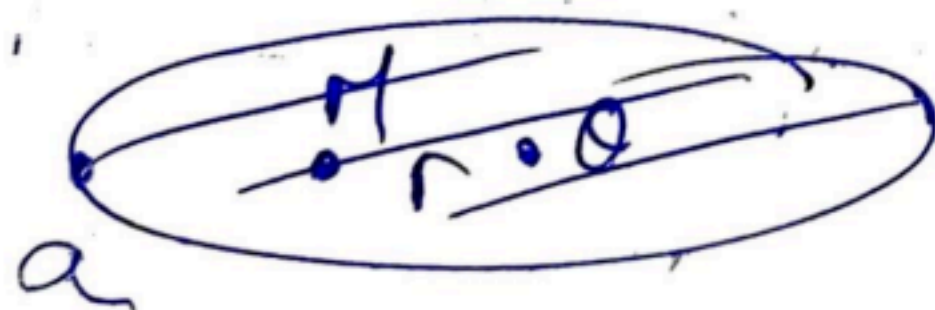
$$\vec{B} = \frac{r}{2c^2} \frac{dE(t)}{dt} \vec{U}_\theta$$

$$E(t) = E_0 e^{-t/\tau}$$

$$\vec{B} = -\frac{r}{2\tau c^2} E(t) \vec{U}_\theta$$

$$2) \frac{u_e}{u_m} = \frac{\epsilon_0 E^2}{B^2} \mu_0 = \left(\frac{E}{B} \right)^2 \frac{1}{c^2}$$

$$= \left(\frac{2\tau c}{r} \right)^2$$



$$r \leq a$$

$$\frac{u_e}{u_m} \geq \left(\frac{2\lambda}{a} \right)^2$$

$$3) \text{ AR Q.S. : } a \ll \lambda \Rightarrow$$

$$u_e \gg u_m \Rightarrow L_{em} \approx u_e = \frac{1}{2} \frac{Q^2}{C}$$

$$4) \vec{\pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{EB}{\mu_0} (\vec{u}_z \wedge \vec{u}_\theta)$$

$$\vec{\pi} = \frac{EB}{\mu_0} (-\vec{u}_r) = + \frac{E^2(t)}{\mu_0 2\pi c^2} \vec{u}_r$$

$$P_{rayonnée} = \oint \vec{\pi}(a) \cdot d\vec{S}$$

$$= \iint \frac{E^2(t) a}{2\pi c^2 \mu_0} \cdot a d\theta dz$$

$$= \frac{2\pi (2h) a^2}{2\pi} \epsilon_0 E^2(t)$$

$$= \frac{\epsilon_0 V E^2(t)}{2}$$

$$\frac{dU_e}{dt} = \frac{d}{dt} \iiint \left(\frac{\epsilon_0 E^2(t)}{2} \right) d\tau \quad \hookrightarrow r dr d\theta dz$$

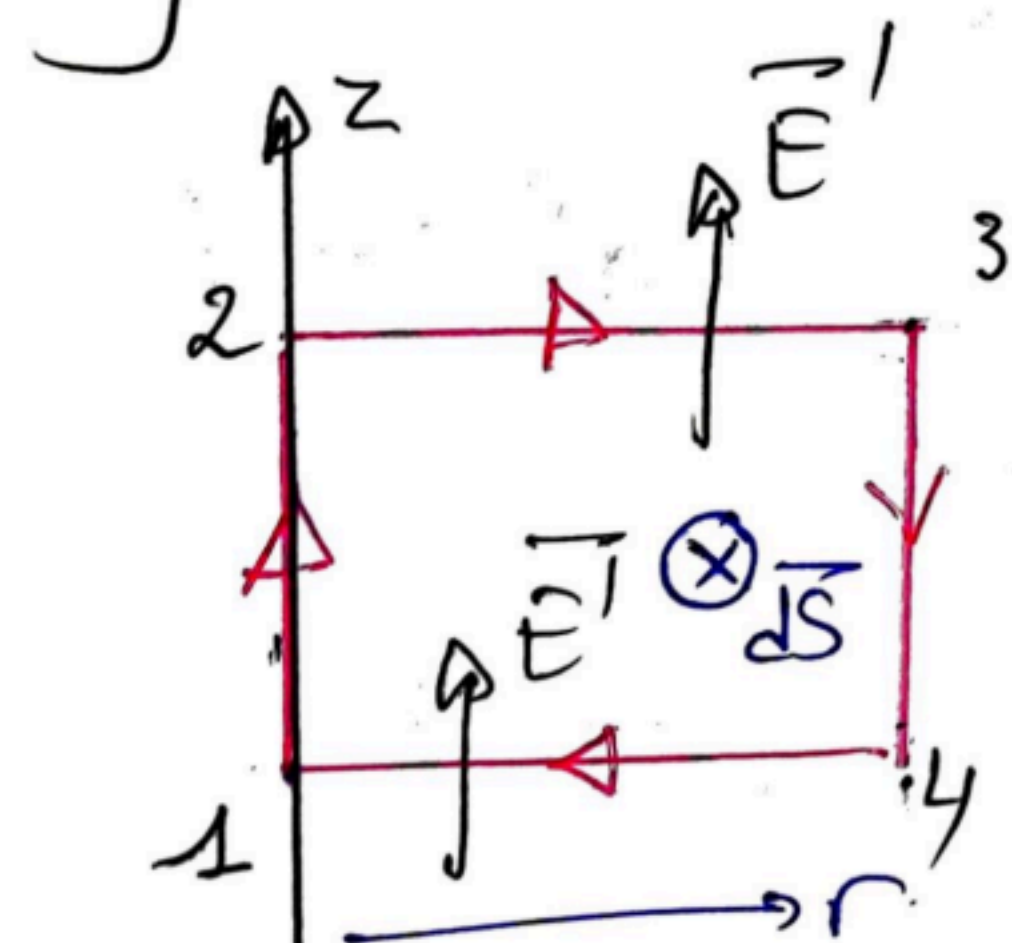
$$= \frac{\epsilon_0}{2} \frac{dE^2(t)}{dt} \cdot \cancel{V} = \epsilon_0 E(t) V \frac{dE(t)}{dt}$$

$$= -\epsilon_0 V \frac{d^2 E(t)}{dt^2}$$

$$\vec{E}(t) \rightarrow \vec{B}(r, t) \rightarrow \vec{E}'(r, t)$$

$$\nabla \times \vec{E}' = -\frac{\partial \vec{B}}{\partial t} = \nabla \times [\vec{E}'(r, t) \vec{u}_z]$$

$$\oint \vec{E}' \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} \quad \hookrightarrow d\vec{S} \vec{e}_z$$



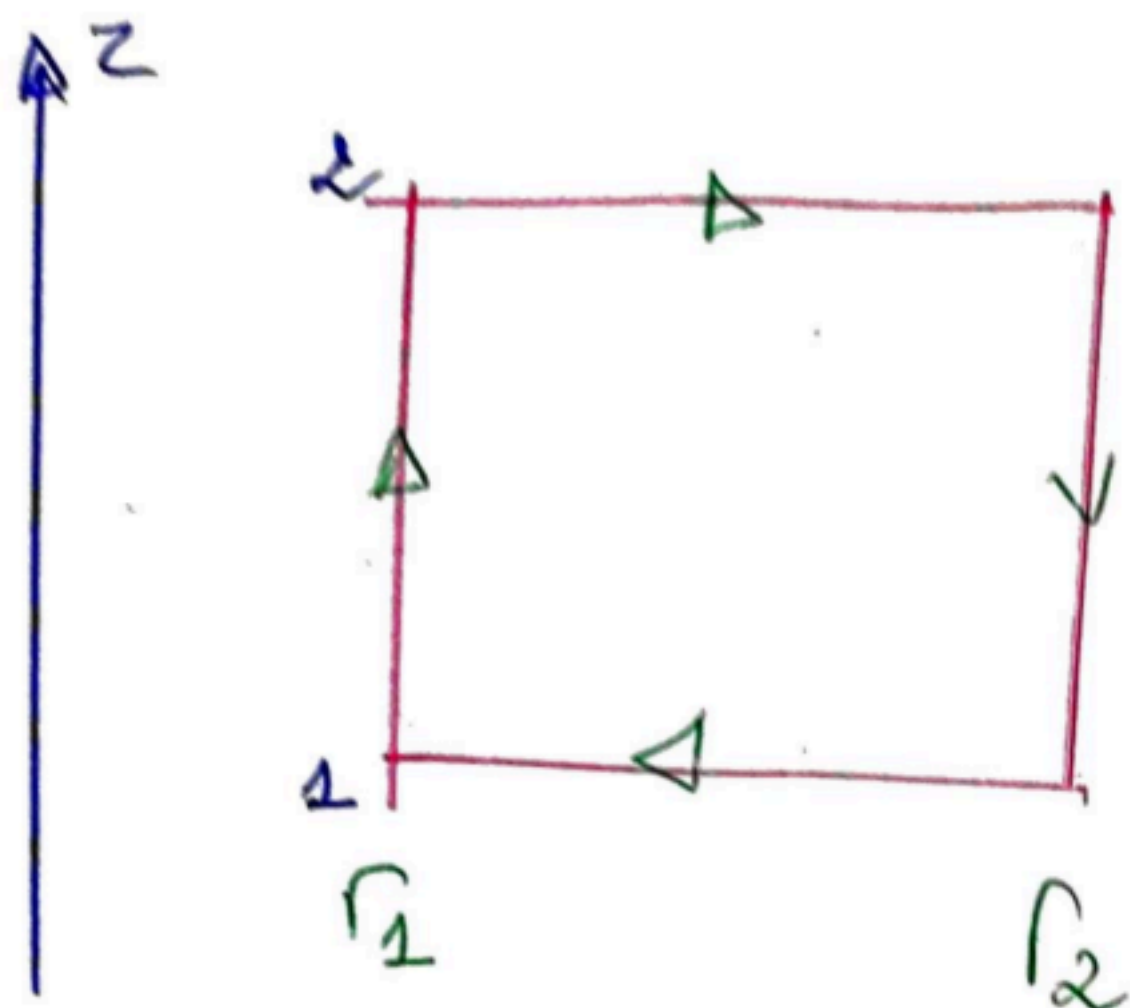
$$\oint \vec{E}' \cdot d\vec{l} = \int_3^4 \vec{E}' \cdot dz \vec{u}_z$$

$$= -l E'(r, t)$$

$$E'(0, t) = 0 \quad ; \quad = - \int_0^r \int_0^l \frac{\Gamma'}{2\pi c^2} \frac{dE(t)}{dt} (dr' dz)$$

$$-l E' = -l \left[\frac{\frac{dE(t)}{dt}}{2\pi c^2} \frac{r^2}{2} \right]$$

$$E' = \frac{\frac{dE(t)}{dt}}{4\pi c^2} r^2$$



$$\oint \vec{E} \cdot d\vec{l} = \int_{r_1}^{r_2} E'(r, t) dr - \int_{r_1}^{r_2} E'(r, t) dr$$

$$= - \int_{r_1}^{r_2} \int_{z_1}^{z_2} \frac{r}{2\pi c^2} \frac{dE(t)}{dt} dr dz$$

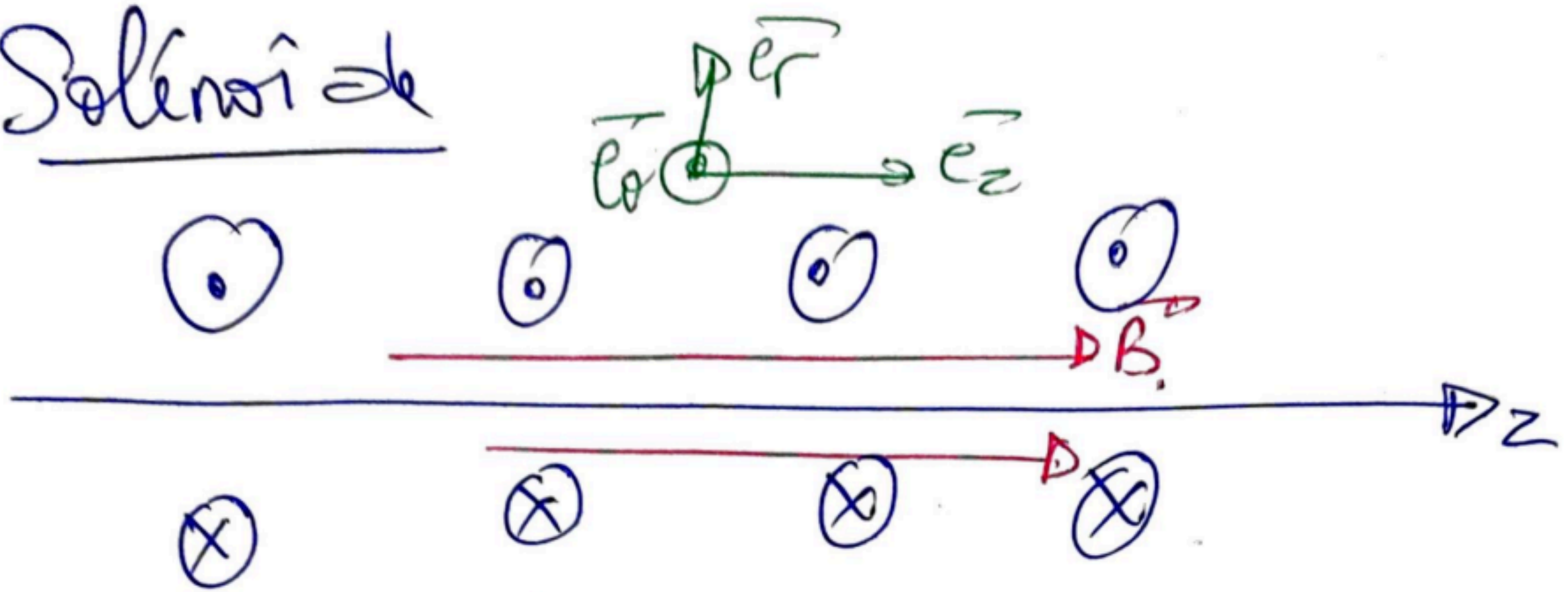
$$= - \left[F(r_1, t) - F(r_2, t) \right] l$$

$$= - l \left[\frac{r_2^2 \frac{dE(t)}{dt}}{2\pi c^2 l} - \frac{r_1^2 \frac{dE(t)}{dt}}{2 \times 2\pi c^2} \right]$$

$$F(r, t) = \frac{\frac{dE(t)}{dt}}{4\pi c^2} r^2 + \text{const.}$$

$$F(0, t) \rightarrow \text{const.}$$

Solénoïde



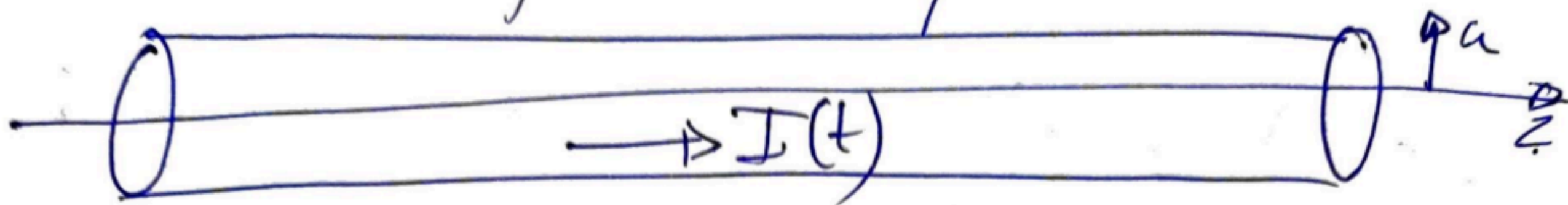
$$\vec{B}(t) = B_0 e^{-t/\tau} \vec{u}_z \rightarrow \vec{E}(r, t)$$

$$(\vec{r} \ \vec{e}_r \ \vec{e}_\theta) \text{ PSI pour } \vec{B}(t)$$

$$(\vec{r} \ \vec{e}_r \ \vec{e}_z) \text{ PSII pour } \vec{B} \rightarrow \text{l'effet } \vec{E}$$

à ce point

Comportement Fréquentiel



1) $\vec{J}(r, t) = \vec{J}(r, t)$: rot + div

$I(t) \longrightarrow \vec{B}(r, t) \longrightarrow \vec{E}(r, t) = \frac{\vec{J}(r, t)}{\nabla}$

$\text{rot } \vec{E} \neq -\frac{\partial \vec{B}}{\partial t} = \vec{0} \quad ??$
 $\Rightarrow \vec{E}(r, t) = \text{oléps}$
in temps.

2) $\omega \leq \omega_{\max} = 2 \cdot 10^4 \text{ rad/s}$

$\Sigma_{\text{prop}} \ll T$ on $l \ll \lambda = \frac{2\pi c}{\omega} = c T$

$l \ll \lambda_{\min} = \frac{2 \times \pi \times 3 \cdot 10^8}{2 \cdot 10^4} = 10.5 \text{ m}$

$l \ll 100 \text{ km} = \lambda_{\min}$

2 $l \approx 100 \text{ m} \ll 100 \text{ km}$

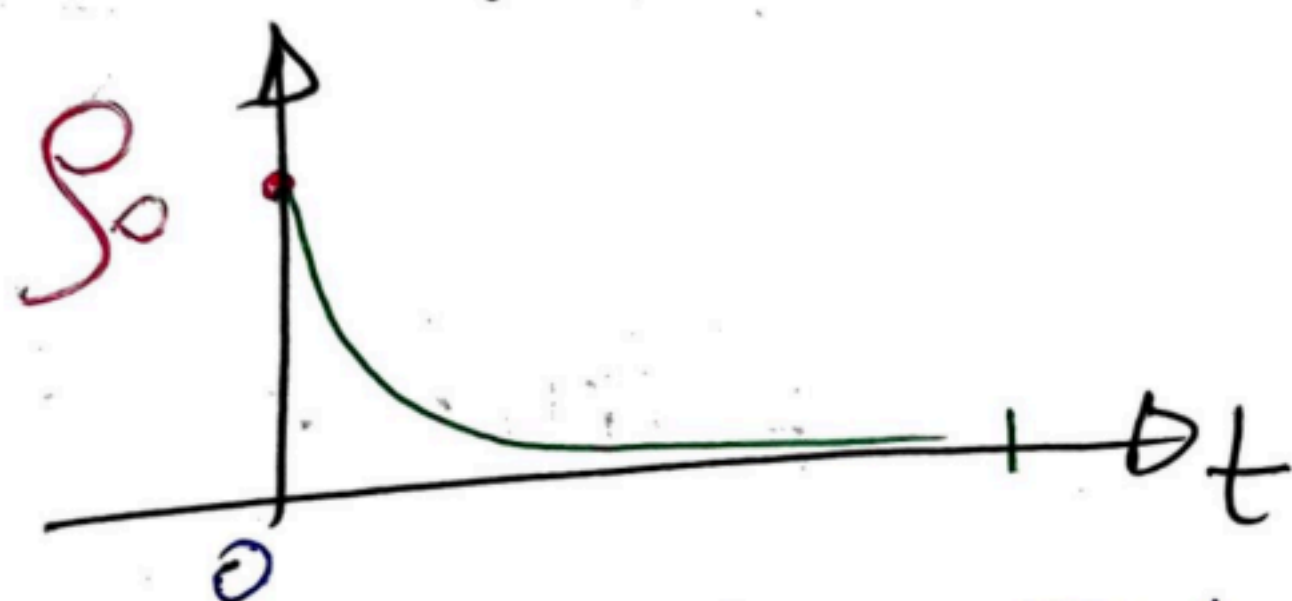
3) $\boxed{\text{div } \vec{E} = \rho / \epsilon_0}$

$\boxed{\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0}$

$$\operatorname{div}(\sigma \vec{E}) + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial t} = 0; \quad \tau = \frac{\epsilon}{\sigma} = 10^{-18} \text{ s}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \rho(t) = \rho_0 e^{-t/\tau}$$



le milieu reste
localement neutre

$$\begin{aligned} \textcircled{a} \frac{\|\vec{J}_D\|}{\|\vec{J}_C\|} &= \frac{\left| \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right|}{|\sigma \vec{E}|} = \left| \frac{i \omega \epsilon}{\sigma} \right| \\ &= \left| \frac{\epsilon \omega}{\sigma} \right| \leq \frac{\epsilon \omega_{\max}}{\sigma} \approx 10^{-2} \ll 1 \end{aligned}$$

$$\boxed{\vec{\nabla} \times \vec{B} \simeq \mu_0 (\sigma \vec{E})}$$

$$\begin{aligned} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) &= \vec{\nabla} (\operatorname{div} \vec{E}) - \Delta \vec{E} \\ &= -\Delta \vec{E} \end{aligned}$$

$$\text{rot}(\text{rot} \vec{E}) = \text{rot} \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\frac{\partial (\text{rot} \vec{B})}{\partial t}$$

$$-\Delta \vec{E} \simeq -\mu_0 \sigma \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\Delta \vec{E} - \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} = \vec{0}}$$

équation différentielle de chaleur

$$\Delta T + \cancel{\frac{q}{\lambda}} = \frac{\rho c}{\lambda} \frac{\partial T}{\partial t}$$

$$\Delta \vec{J}(r) e^{i\omega t} - \mu_0 \sigma \frac{\partial \vec{J}(r) e^{i\omega t}}{\partial t} = \vec{0}$$

$$\left[\Delta \vec{J}(r) - i\omega \mu_0 \sigma \vec{J}(r) \right] e^{i\omega t} = \vec{0}$$

$$\frac{d^2 \vec{J}(r)}{dr^2} + \frac{1}{r} \frac{d\vec{J}(r)}{dr} - i\mu_0 \sigma \omega \vec{J}(r) = 0$$

$$r = \lambda_r \delta$$

$$\vec{J}(r)/\vec{J}(0) = G(\lambda_r)$$

$$\frac{d^2 G(\lambda_r)}{d\lambda_r^2} + \frac{1}{\lambda_r} \frac{dG(\lambda_r)}{d\lambda_r} - 2i G(\lambda_r) = 0$$

$$x^2 + \frac{1}{\lambda_r} x - 2i = 0$$

à la

$$4) G(\lambda r) = \sum_0^{\infty} g_n \lambda r^n$$

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$$\frac{1}{\lambda r} \frac{dG(\lambda r)}{d\lambda r} = \sum_2^{\infty} n g_n \lambda r^{n-1}$$

$$\frac{d^2 G(\lambda r)}{d\lambda r^2} = \sum_2^{\infty} n(n-1) g_n \lambda r^{n-2}$$

$$\sum_2^{\infty} n(n-1) g_n \lambda r^{n-2} + \sum_2^{\infty} n g_n \lambda r^{n-2} = 2i \sum_0^{\infty} g_n \lambda r^n$$

$$\sum_2^{\infty} n^2 g_n \lambda r^{n-2} = 2i \sum_2^{\infty} g_n \lambda r^{n-2}$$

$$n^2 g_n = 2i g_{n-2}$$

$$\boxed{g_n = \frac{2i}{n^2} g_{n-2}}$$

$$\textcircled{*} n = 2p \quad g_{2p} = \frac{i}{2p^2} g_{2p-2}$$

$$g_{2p-2} = \frac{i}{2(p-1)^2} g_{2p-4}$$

$$g_2 = \left(\frac{i}{2p^2} \right) \left(\frac{i}{2(p-1)^2} \right)^2 \dots g_0$$

$$g_2 = \frac{(i)^p}{2^p (p!)^2} g_0$$

$$\underline{J}(\lambda_r) = \underline{J}(0) [g_0 \lambda_r^0 + g_2 \lambda_r^2 + g_4 \lambda_r^4 + \dots]$$

$$g_2 = \frac{2i}{2^4} g_0 = \frac{i}{2}$$

$$\underline{J}(\lambda_r) = \underline{J}_0 \left[1 + \frac{i}{2} \lambda_r^2 - \frac{\lambda_r^4}{16} + \dots \right]$$

$$\underline{E}(r, t) = \frac{\underline{J}_0}{\sigma} \left[\left(1 - \frac{\lambda_r^4}{16} \right) + i \frac{\lambda_r^2}{2} \right] e^{i\omega t} \underline{u}_z$$

$$\underline{E}(r, t) = \frac{\underline{J}_0}{\sigma} \left[\left(1 - \frac{\lambda_r^2}{16} \right) \cos(\omega t) - \frac{\lambda_r^2}{2} \sin(\omega t) \right] \underline{u}_z$$

$$\begin{aligned}
\mathcal{I}(t) &= \iint \text{Re}[\underline{J}(r,t)] dS_{\text{base}}; \quad \lambda_r = \frac{r}{\delta} \\
&= \text{Re} \int_0^a J_0 \left[\left(1 - \frac{\lambda_r^4}{16}\right) + i \frac{\lambda_r^2}{2} \right] r dr dt e^{i\omega t} \\
&= 2\pi J_0 \text{Re} \int_0^a \delta^2 \left(\left(1 - \frac{\lambda_r^5}{16}\right) + \frac{i \lambda_r^3}{2} \right) d\lambda_r e^{i\omega t} \\
&= 2\pi \delta^2 J_0 \text{Re} \int_0^{\lambda_a} \left[\left(\lambda_r - \frac{\lambda_r^5}{16}\right) + i \frac{\lambda_r^3}{2} \right] d\lambda_r e^{i\omega t} \\
&= 2\pi \delta^2 J_0 \text{Re} \left[\left(\frac{\lambda_a^2}{2} - \frac{\lambda_a^6}{6 \times 16} \right) + i \frac{\lambda_a^4}{4 \times 2} \right] e^{i\omega t} \\
&= \pi \delta^2 J_0 \lambda_a^2 \text{Re} \left[\left(1 - \frac{\lambda_a^4}{48}\right) + i \frac{\lambda_a^2}{4} \right] e^{i\omega t} \\
&\quad \underbrace{\hspace{1.5cm}}_{a^2} \\
\langle \mathcal{I}^2(t) \rangle &= (\quad) \left[1 + \frac{1}{48} \lambda_a^4 \right] \\
\mathcal{I}(t) &= \pi a^2 J_0 \left[\left(1 - \frac{\lambda_a^4}{48}\right) \cos(\omega t) - \frac{\lambda_a^2}{4} \sin(\omega t) \right]
\end{aligned}$$

$$\langle I^2(t) \rangle = (\pi a^2 J_0)^2 \left[\left(1 - \frac{\lambda_a^4}{48} \right) \langle \cos^2(\omega t) \rangle + \left(\frac{\lambda_a^2}{4} \right)^2 \langle \sin^2(\omega t) \rangle \right]$$

$$= \frac{1}{2} (\pi a^2 J_0)^2 \left[1 - 2 \frac{\lambda_a^4}{48} + \frac{3 \lambda_a^4}{16 \times 3} \right]$$

$$= \frac{1}{2} (\pi a^2 J_0)^2 \left[1 + \frac{1}{48} \lambda_a^4 \right].$$