onde dans le vide

$$\overrightarrow{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \qquad div\,\vec{B} = 0.$$

$$div\,\vec{E} = 0, \qquad \overrightarrow{rot}\,\vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\overrightarrow{rot}(\overrightarrow{rot}\,\vec{E}) = \overrightarrow{grad}(div\,\vec{E}) - \Delta\vec{E} = -\Delta\vec{E}$$

$$= \overrightarrow{rot}\left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\overrightarrow{rot}\,\vec{B}\right)$$

$$\boxed{\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}}$$

si $E(x,t) = f\left(t - \frac{x}{c}\right) + g\left(t + \frac{x}{c}\right)$

$(ox)^+$ $\qquad$ $(ox)^-$

si $\vec{E} = \vec{E_0}\exp i(\omega t - \vec{k}\vec{r})$

onde plane $\qquad \Phi = \omega t - \vec{k}\vec{r} = cte$

$$\vec{k}.\vec{r} = cte = x k_x + y k_y + z k_z$$

plan

$$\Delta = -k^2 \qquad \frac{\partial^2}{\partial t^2} = -\omega^2$$

$$\frac{\partial}{\partial t} = i\omega \; ; \; div \to -i\vec{k}.$$

$\operatorname{div}\vec{E} = -i\vec{k}\cdot\vec{E} = 0$, $\vec{k} \perp \vec{E}$ ($\perp E$)

$\operatorname{div}\vec{B} = 0 = -i\vec{k}\cdot\vec{B}$; $\vec{k} \perp \vec{B}$ ($\perp M$)

($\perp E.M$)

$\overrightarrow{\operatorname{rot}}\,\vec{E} = -i\vec{k}\wedge\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

$-i\vec{k}\wedge\vec{E} = -i\omega\vec{B}$

$$\boxed{\vec{B} = \frac{\vec{k}\wedge\vec{E}}{\omega}}$$

onde plane

⊙ $\vec{E} = E_0 \cos(\omega t - kx)\,\vec{e}_y$ onde monochromatique, plane ; progressive $\cos(kx - \omega t)$

$(ox)^+$; polarisation rectiligne ($\perp E$)



$$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} \quad , \quad k = \frac{\omega}{c} \text{ vide}$$

$$\vec{B} = \frac{1}{c} \vec{u}_x \wedge E \vec{u}_y = \frac{E}{c} \vec{u}_z$$

$$\boxed{\vec{B} = \frac{E_0}{c} \cos(\omega t - kx) \vec{u}_z}$$

Rq : Relation de dispersion

$$\left(\Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = \vec{0}$$

$$\left[-k^2 - \frac{1}{c^2} (-\omega^2) \right] = 0$$

$$\boxed{k = \frac{\omega}{c}}$$

* $\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{+E_0^2}{\mu_0 c} \cos^2(\omega t - kx) \vec{e}_x$

$$\boxed{\langle \vec{\Pi} \rangle = \frac{\varepsilon_0 c E_0^2}{2} \vec{e}_x}$$

$$\langle \vec{\Pi} \rangle = \frac{\varepsilon_0 c E_0^2}{2} \vec{e}_x$$

$$\langle u_{em} \rangle = \langle u_e \rangle + \langle B|m \rangle$$

$$= \left\langle \frac{\varepsilon E^2}{2} + \frac{B^2}{2\mu} \right\rangle$$

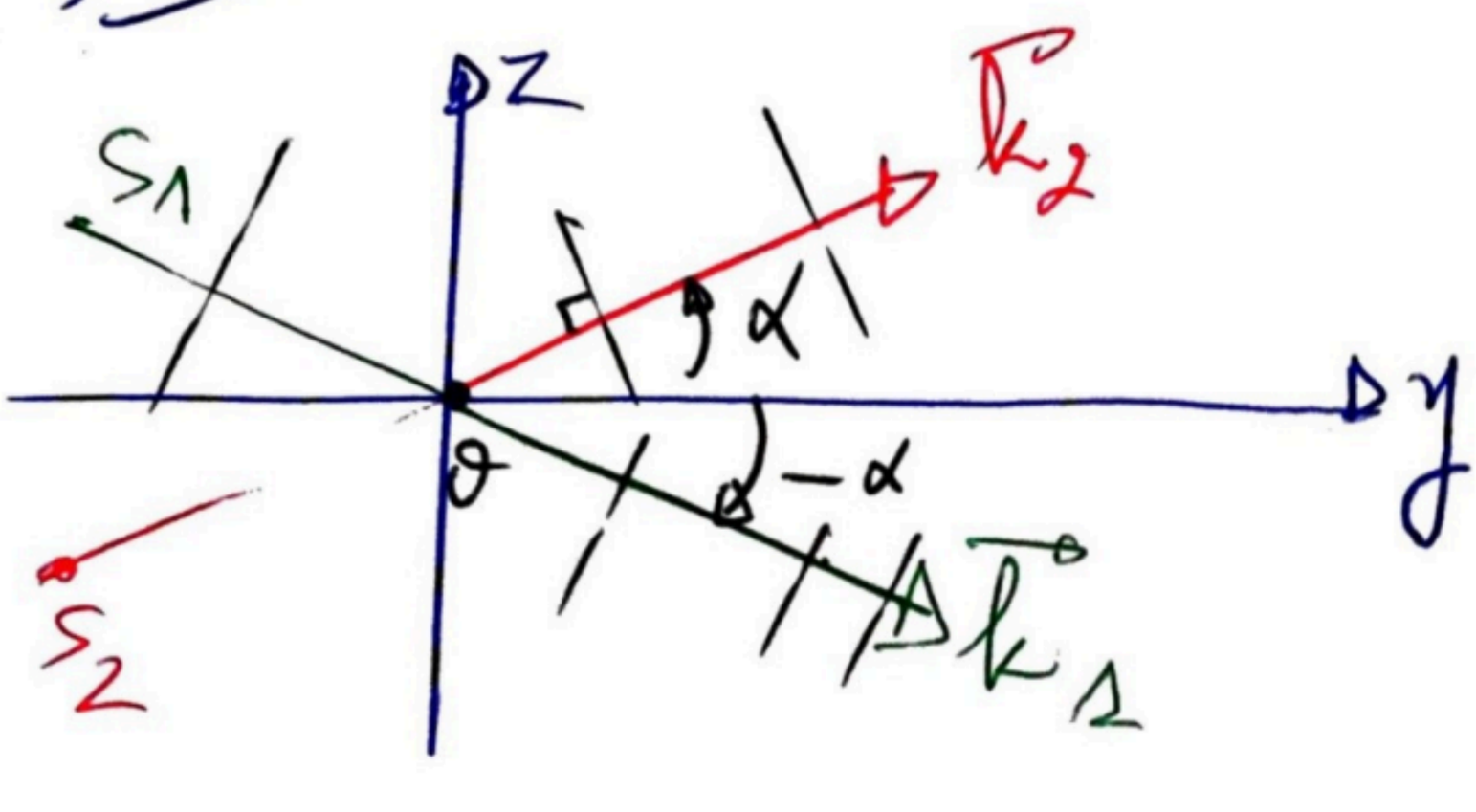
$$= \frac{\varepsilon}{4} E_0^2 + \frac{E_0^2}{4\mu c^2} = \frac{\varepsilon E_0^2}{4} \times 2$$

$$\boxed{\langle u_{em} \rangle = \frac{\varepsilon E_0^2}{2}}$$

$$\langle \vec{\Pi} \rangle = \langle u_{em} \rangle \cdot c \, \vec{u}_x$$

$$\vec{v}_e = \frac{\langle \vec{\Pi} \rangle}{\langle u_{em} \rangle}.$$

MP2 1999 : 4h



$$\underline{\vec{E}_1} = E_0 \exp i(\omega t - \vec{k}_1 \vec{r})\,\vec{e}_x$$

$$\underline{\vec{E}_2} = E_0 \exp i(\omega t - \vec{k}_2 \vec{r})\,\vec{e}_x$$

en O: $\vec{r}\Big|_0^0 \Rightarrow \Phi_1 = \omega t = \Phi_2$

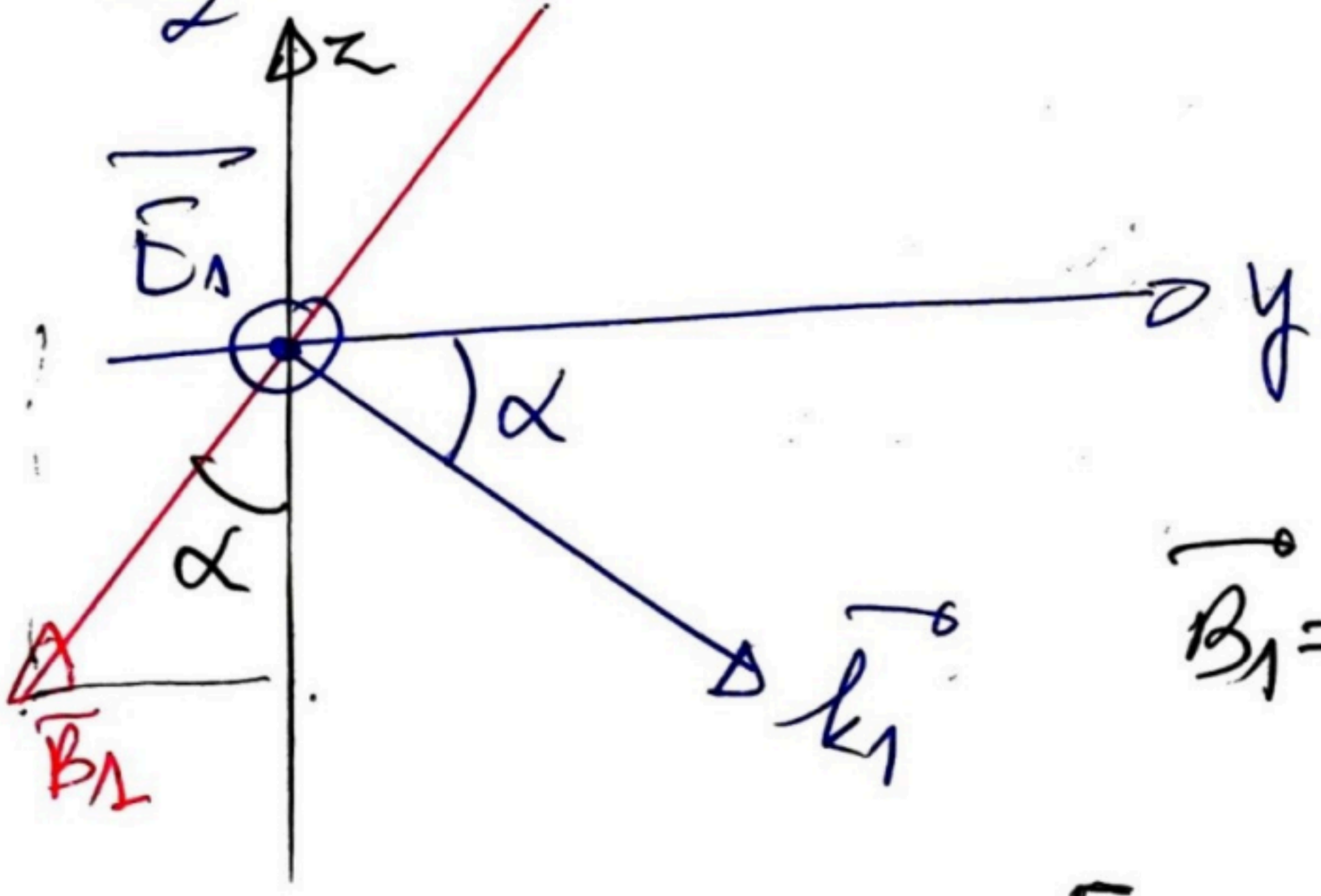
$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0} \Rightarrow \left(-k^2 + \frac{\omega^2}{c^2}\right)\vec{E} = \vec{0}$$

$$\boxed{k = \frac{\omega}{c}}$$

a) $\vec{k}_1 = \frac{\omega}{c}\Big|\begin{matrix}0\\ \cos\alpha\\ -\sin\alpha\end{matrix}$ $\qquad \vec{k}_2 = \frac{\omega}{c}\Big|\begin{matrix}0\\ \cos\alpha\\ \sin\alpha\end{matrix}$

$$\vec{E}_1 = E_0 \exp i(\omega t - k y\cos\alpha + k z\sin\alpha)\,\vec{e}_x$$

$$\vec{E}_2 = E_0\, e^{\,i(\omega t - k y\cos\alpha - k z\sin\alpha)}\,\vec{e}_x$$



$$\vec{B}_1 = \frac{\vec{k}_1 \wedge \vec{E}_1}{\omega}$$

$$\vec{B}_1 = \begin{pmatrix} \\ \\ \end{pmatrix}\Big|\begin{matrix}0\\ -\sin\alpha\\ \cos\alpha\end{matrix}.$$

$$\underline{\vec{E}} = 2E_0 \cos(kz \sin\alpha)\, e^{i(\omega t - ky\cos\alpha)}\, \vec{e}_x$$

onde progressive $(oy)^+$

" stationnaire (oz)

$$\vec{E} = 2E_0 \cos(kz\sin\alpha)\cos(\omega t - ky\cos\alpha)\vec{e}_x$$

$\alpha = 0$; $2E_0 \cos(\omega t - ky)$

$\alpha = \frac{\pi}{2}$; $2E_0 \cos(kz)\cos(\omega t)$

$$\vec{B} = \frac{\vec{k}_1 \wedge \vec{E}_1}{\omega} + \frac{\vec{k}_2 \wedge \vec{E}_2}{\omega}$$

$$\overrightarrow{\text{rot}}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\langle \vec{\Pi} \rangle = \begin{cases} \langle \Pi_x \rangle = 0 \cdot \\ \langle \Pi_y \rangle \neq 0 \cdot \\ \langle \Pi_z \rangle = 0 \cdot \end{cases}$$

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}$$

$\frac{\partial}{\partial t} = -i\omega$ $\qquad$ div $\longrightarrow i\vec{k}$.

$\frac{\partial^2}{\partial t^2} = -\omega^2$ $\qquad$ $\Delta = -k^2$

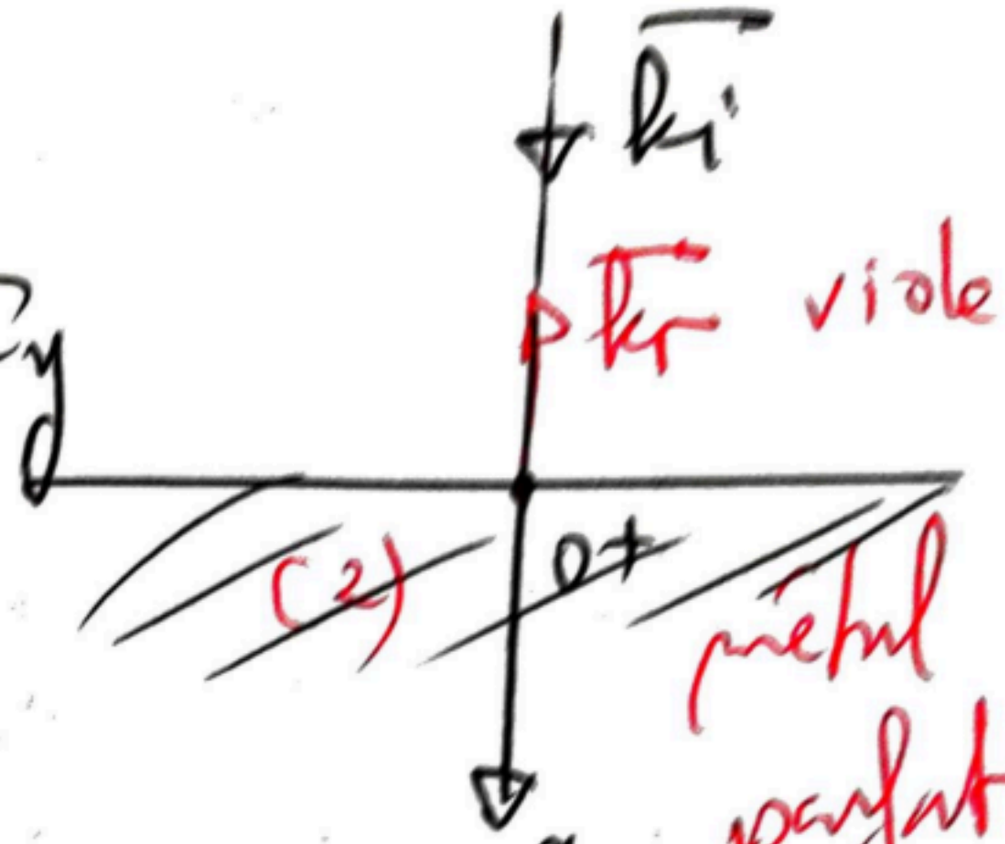
$$\frac{d^2 G(\lambda_R)}{d\lambda_R^2} = \frac{d}{d\lambda_R}\left(\frac{dG}{d\lambda_R}\right)$$

$$\frac{d^2 G}{dr^2} = \frac{d^2 G}{d(\delta\lambda_R)^2} = \frac{1}{\delta^2}\frac{d^2 G}{d\lambda_R^2}.$$

Serie N°1

Ex1

$$\underline{E_i} = E_0 e^{i(\omega t - kx)} \vec{e_y}$$



$\vec{E_2} = \vec{0}$, $\vec{B_2} = \vec{0}$

$\rho = 0$ $\quad \vec{j_c} = \vec{0}$

$\gamma \to \infty$

$\rho_s = 0$ /

$$\underline{E_r} = \underline{E_{0r}} e^{i(\omega t + kx)} \vec{e_y}$$

$$\vec{E_2} - \vec{E_1} = \frac{\sigma}{\varepsilon_0} \vec{n_{12}}$$

$$\vec{0} - (\underline{E_i} + \underline{E_r}) \vec{e_y} = \frac{\sigma}{\varepsilon_0} \vec{e_x}$$

$$\begin{cases} \sigma = 0 \\ \underline{E_i} + \underline{E_r} = 0. \end{cases}$$

$$\left(E_{0i} + \underline{E_{0r}}\right) e^{i\omega t} = 0, \quad \underline{\underline{x = 0}}$$

$$\underline{\underline{\forall t}}$$

$$\frac{E_{0r}}{E_{0i}} = r = -1 = e^{i\pi}, \quad \varphi = \pi$$

1) b) $\vec{J_s} = \frac{\vec{n_{12}} \wedge (\vec{B_2} - \vec{B_1})}{\mu_0}$

$\vec{B_1} = \vec{B_i} + \vec{B_r}$

$\vec{B_i} = \frac{\vec{k_i} \wedge \vec{E_i}}{\omega} = \frac{\vec{E_{0i}}}{c} e^{i(\omega t - kx)} \vec{e_z}$

$\vec{B_r} = \frac{\vec{k_r} \wedge \vec{E_r}}{\omega} = + \frac{\vec{E_{0i}}}{c} e^{i(\omega t + kx)} \vec{e_z}$

$\vec{B} = \frac{\vec{E_{0i}}}{c} e^{i(\omega t)} 2 \cos(kx) \vec{e_z}$

$$\boxed{\vec{B} = \frac{2\vec{E_0}}{c} \cos(kx) e^{i\omega t} \vec{e_z}}$$

$$\vec{J_s} = \frac{\vec{u_n} \wedge \left(\vec{0} - \frac{2\vec{E_0}}{c} \cos(\omega t) \vec{e_z} \right)}{\mu_0}$$

$$\boxed{\vec{J_s} = \frac{2\vec{E_0}}{\mu_0 c} \cos(\omega t) \vec{u_y}}$$

$I = \int \vec{J_s} \, d\vec{l} \wedge \vec{n}$

$\Leftrightarrow$ A/m.

2) $\vec{E} = \vec{E_i} + \vec{E_r}$.

$= E_0 e^{i\omega t} (-2i \sin kx) \vec{e_y}$

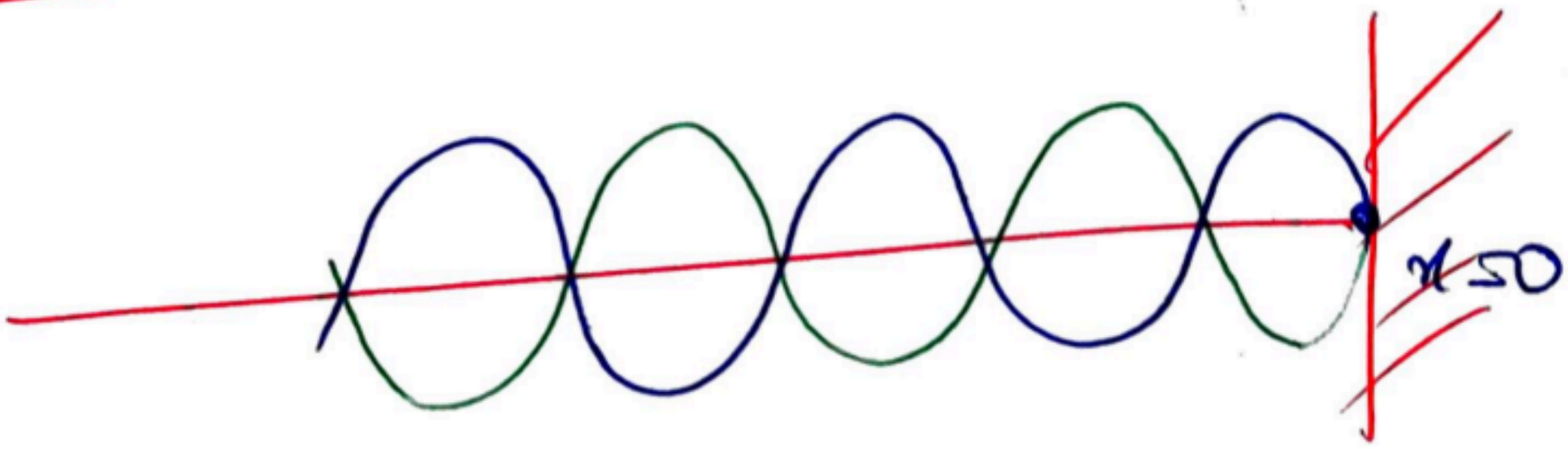
$$\underline{E} = 2\underline{E}_0 \sin(kx) \sin(\omega t)\, \underline{e}_y$$

$$\underline{B} = \frac{2\underline{E}_0}{c} \cos(kx) \cos(\omega t)\, \underline{e}_z$$

b) onde stationnaire (ou), polarisée (oy)

noeuds $\begin{cases} \underline{E}: \sin(kx) = 0 \\ kx = n\pi \end{cases}$

ventres $\underline{E}: |\sin(kx)| = 1$



$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$v = \sqrt{\frac{T}{\mu}}$, μ = masse linéique

c) $\langle \vec{\Pi} \rangle = \frac{1}{2\mu_0} \overrightarrow{Re(\underline{E} \wedge \underline{B}^*)}$

$\vec{\Pi} = \frac{Re(\underline{E}) \wedge Re(\underline{B})}{\mu_0}$

$$\langle \vec{\Pi} \rangle = \frac{1}{2\mu_0} Re\left[-2i E_0 \sin(kx)\, \vec{e}_y \wedge \frac{2\underline{E_0}}{c} \cos(kx)\, \vec{e}_z \right]$$

$\langle \vec{\Pi} \rangle = \vec{0} \Leftrightarrow$ stationnaire

$\vec{} \quad \vec{} \qquad \langle \vec{\Pi_c} \rangle$

$V_e = 0 = \overline{\langle u_{em} \rangle}$:

3) $d\vec{F}_{rad} = \frac{1}{2}\left[\vec{J}^* \vec{E}_{tot} + \vec{J}_s \wedge \vec{B}_{tot} \right] dS$

a) $\frac{d\vec{F}_{rad}}{dS} = \frac{1}{2}\left[\frac{2E_0}{\mu_0 c} \cos(\omega t)\, \vec{e}_y \wedge \frac{2E_0}{c} \cos(\omega t)\, \vec{e}_z \right]$

$\frac{d\vec{F}_{rad}}{dS} = \frac{2E_0^2}{\mu_0 c^2} \cos^2(\omega t)\, \vec{e}_n$

$\langle p_{rad} \rangle = \varepsilon_0 E_0^2 = \langle u_{tot} \rangle = 2\langle u_i \rangle$

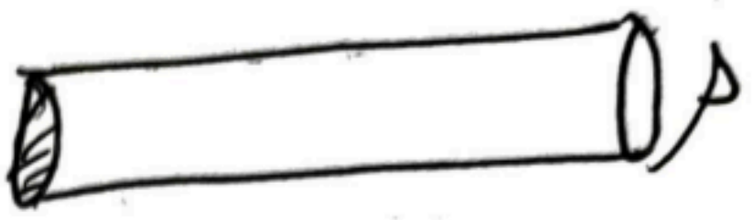
$\langle u_e \rangle = \langle u_m \rangle$: incidente

$= \frac{\varepsilon_0 E_0^2}{4}$

- $\langle u_{incidente} \rangle = \frac{\varepsilon_0 E_0^2}{2} = \frac{\langle p_{rad} \rangle}{2}$
- $\langle u_{ref} \rangle = \frac{\varepsilon_0 E_0^2}{2} \Rightarrow \langle u_{tot} \rangle = 2\langle u_i \rangle = \varepsilon_0 E_0^2$

3) b) $\langle p_{rad} \rangle = ??$ $\langle I \rangle = 3\ mW/m^2$

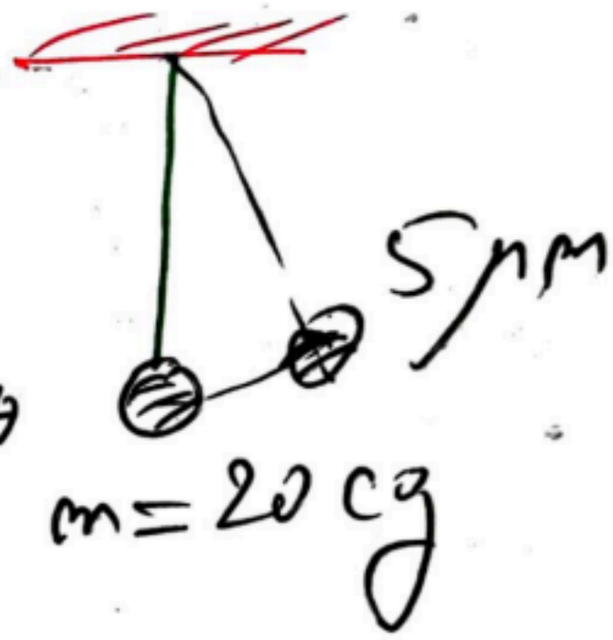
$\langle I \rangle = \langle \|\vec{\Pi}\| \rangle S$;



$= \frac{\varepsilon_0 c E_0^2}{2} S = \frac{S c}{2} \langle p_{rad} \rangle$

$\langle p_{rad} \rangle = \frac{2 \langle I \rangle}{S c} = 2 \cdot 10^{-5}\ Pa$

$P_{atm} = 10^{+5}\ Pa$



4) $\langle p_{rad} \rangle = \varepsilon_0 E_0^2$

$E = \hbar \omega$ $\quad \hbar = \frac{h}{2\pi}$

$\vec{p} = \hbar \vec{k}$

$E = \hbar \omega = h\nu$

$\vec{p} = \hbar \vec{k} = \frac{h}{2\pi} \frac{2\pi}{\lambda} \vec{u} = \frac{h}{\lambda_{DB}} \vec{u} = \frac{h\nu}{c} \vec{u}$

- 5 - De Broglie

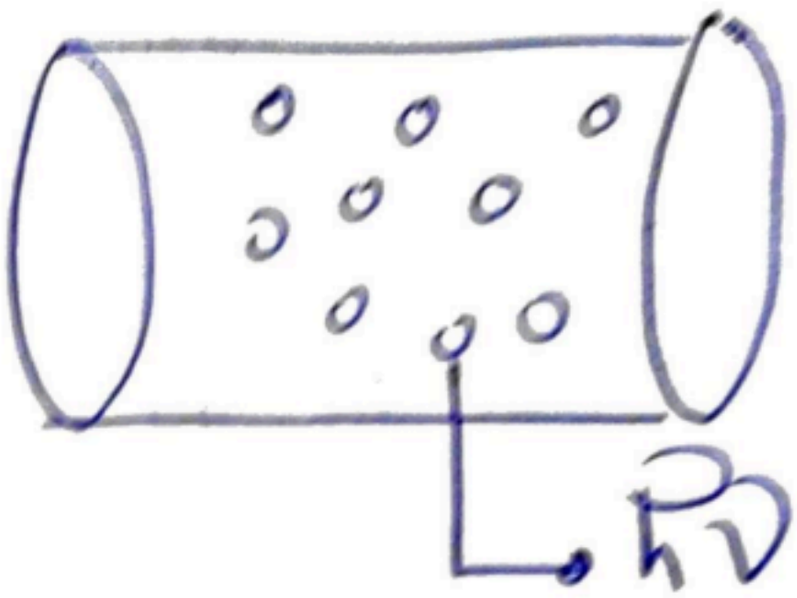
2T2 2011

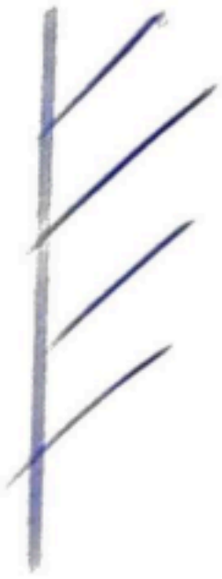
a)

$$n_{photons} = \frac{dN}{dV} \quad J/m^3$$

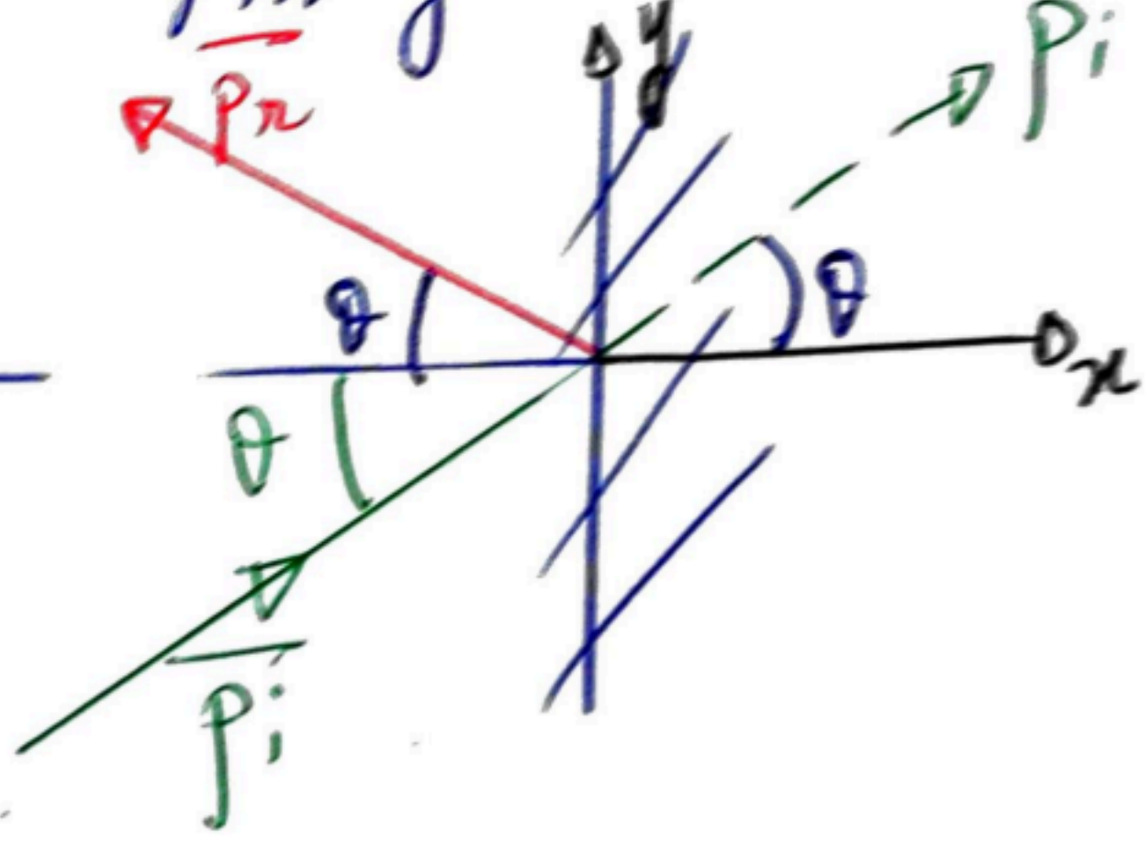
$$= \frac{\langle u_{em} \rangle}{(h\nu)_J}$$

$$\left(n_{ph} = \frac{\varepsilon_0 E_0^2}{2 h\nu} \right)$$





b) $\vec{dp}_{transférée}$



$$\vec{dp} = (\vec{p_i} - \vec{p_r})$$

$$= \frac{h\nu}{c} \cos\theta \, \vec{e}_x - \left(-\frac{h\nu}{c} \cos\theta \right) \vec{e}_x$$

$$\vec{dp} = \frac{2h\nu}{c} \cos\theta \, \vec{e}_x$$

c) R.F.D. : $\langle \vec{dF}_{rad} \rangle = \frac{\vec{dp}}{dt}$

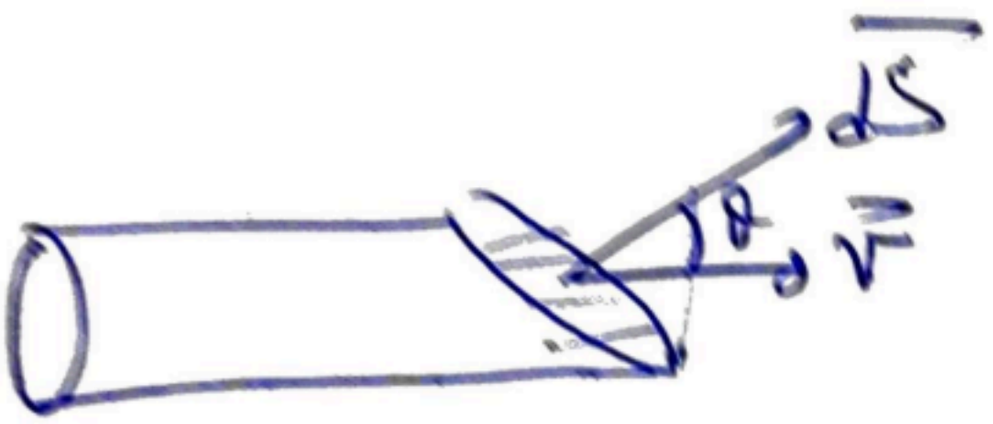
$$m(t) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0$$

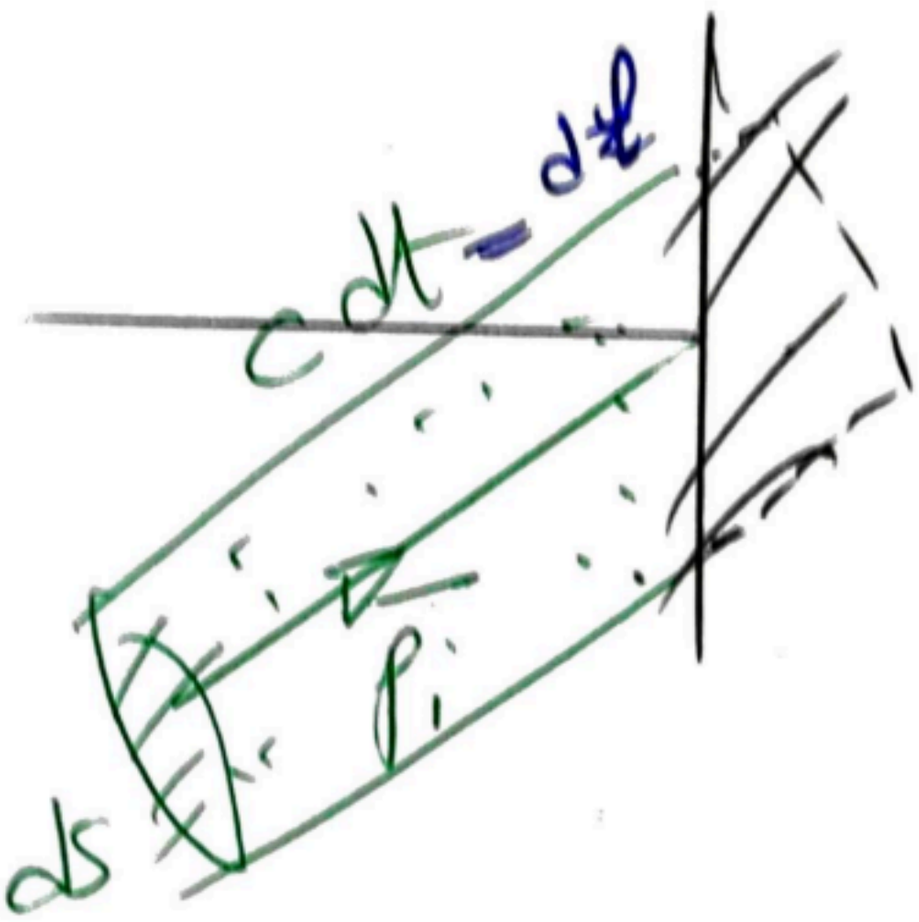
photon $m = 0$

$$d\vec{p} = \frac{2\pi\nu}{c}\cos\theta\,\vec{e_n}$$

$$d\vec{p}_{tot} = \frac{2\pi\nu}{c}\cos\theta \;\; \text{no. photons} \cdot dC \;\; \vec{e_n}$$

$$= \frac{2\pi\nu}{c}\cos\theta \cdot \frac{\varepsilon E_0^2}{2\pi\nu}\cdot\left(dS\,c\,dt\cos\theta\right)\vec{e_n}$$





$$\frac{d\vec{p}_{tot}}{dt} = \left(\varepsilon E_0^2\cos^2\theta\,\vec{dS}\right)ds$$

$$\langle p_{rad}\rangle = \varepsilon E_0^2\cos^2\theta$$

$$= \frac{2I}{c}\cos^2\theta$$

$$I = \langle\|\vec{\Gamma}\|\rangle = \frac{\varepsilon E_0^2}{2}\,c$$

5) a) 400 watt = $\langle \mathcal{P} \rangle$



$\vec{E} = \vec{E}_0 \cos(\omega t - kx)$

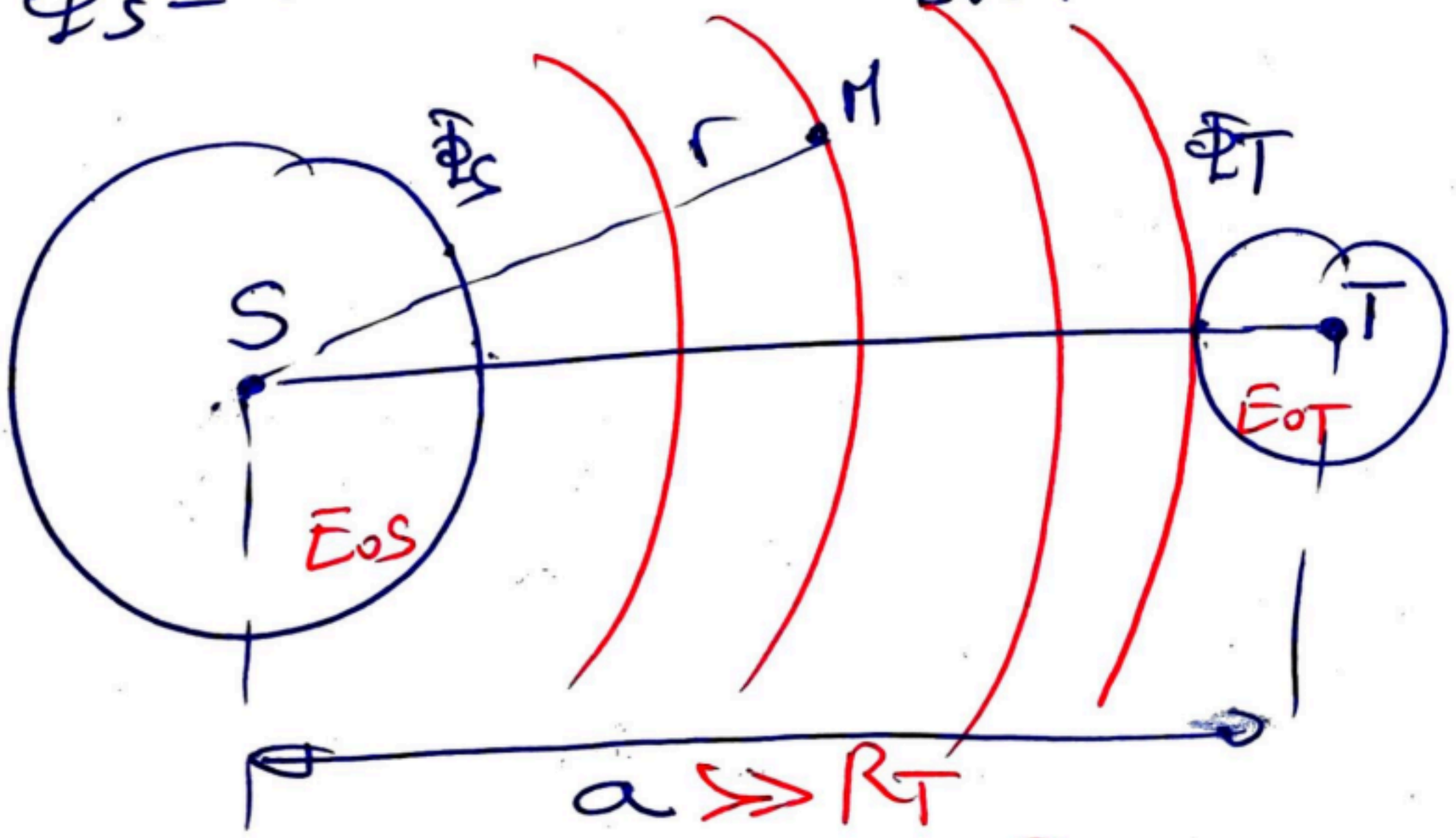
$\langle \mathcal{P} \rangle = \frac{\varepsilon_0 c \vec{E}_0^2}{2} S = \frac{\varepsilon_0 c E_0^2}{2} \frac{\pi d^2}{4}$

$L = 2R$; $E_0 = 2.10^5$ V/m

b) $\mathcal{L}_S = 10^{26}$ watts

$\Phi_T = 1,4$ kwatt/m^2 = $\langle \| \vec{\Pi} \| \rangle_{Terre}$

$\Phi_S = ?? = \langle \| \vec{\Pi} \| \rangle_{Soleil}$



$\langle \| \vec{\Pi}(r) \| \rangle \; 4\pi r^2 = \mathcal{P}_S$.

r ↗ ; $\langle \| \vec{\Pi}(r) \| \rangle$ ↘ ; Surface ↗

$$P_S = \Phi_S \cdot 4\pi R_S^2 \simeq \Phi_T \, 4\pi a^2$$

$$\boxed{\Phi_S = \Phi_T \, \frac{a^2}{R_S^2}}$$

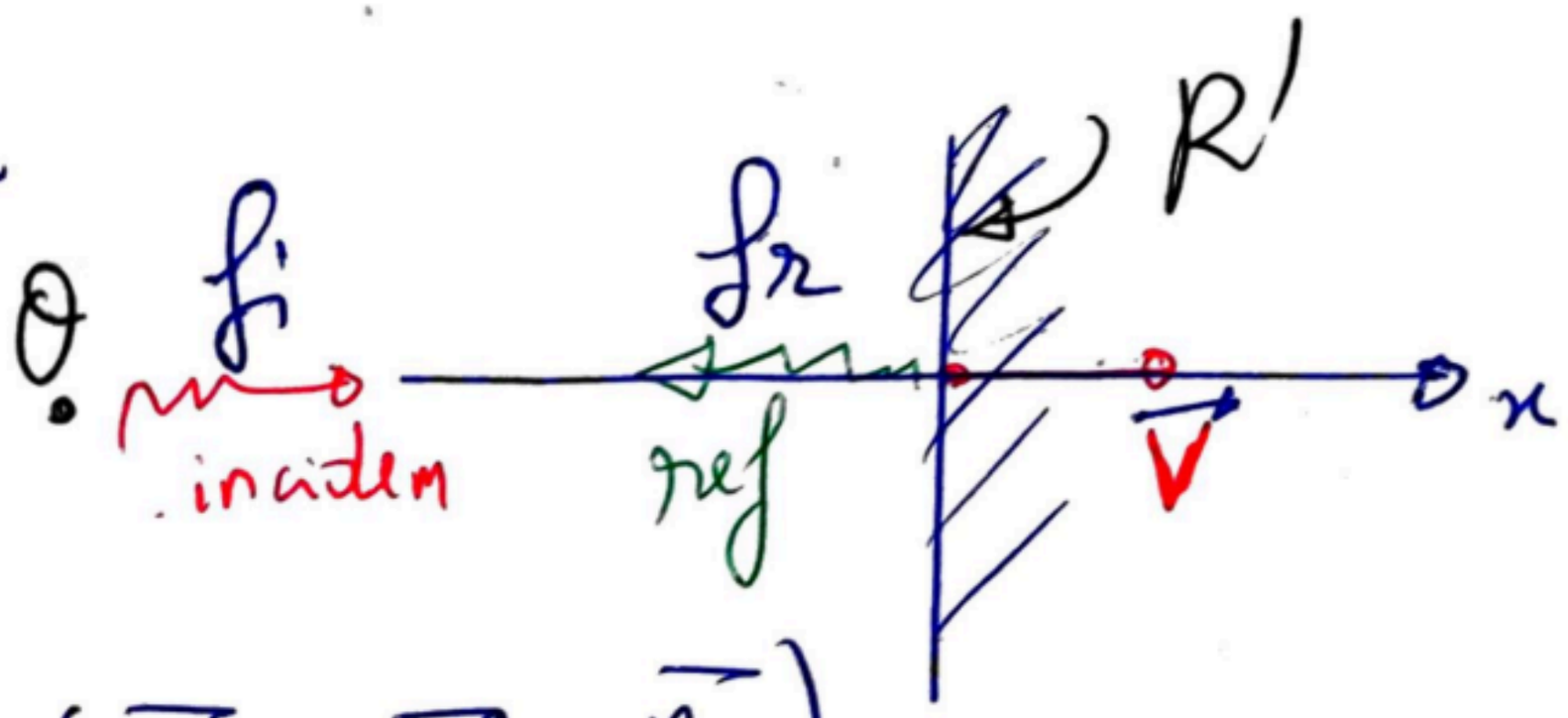
$$\Phi_S = \frac{\varepsilon c E_{0S}^2}{2} \quad , \quad \Phi_T = \frac{\varepsilon c E_{0T}^2}{2}$$

$$E_{0S} = \sqrt{\frac{2\Phi_S}{\varepsilon c}} = 2\,10^5 \ V/m.$$

Rq : soleil corps noir

$\Phi_S = \sigma T_S^4$: stéfann

Effet Doppler



$V \ll c$

Non relativiste

a) $\vec{F}_{Lorentz} = q\left(\vec{E} + \vec{v}_a \wedge \vec{B}\right)$

$$\vec{F}_{Lorentz} = q\left(\vec{E}' + \vec{v}_r \wedge \vec{B}'\right)$$

$$\vec{v}_a = \vec{v}_r + \vec{v}_e = \vec{v}_r + \vec{V}$$

b) $$q\left(\vec{E} + (\vec{v}_r + \vec{V}) \wedge \vec{B}\right) = q\left(\vec{E}' + \vec{v}_r \wedge \vec{B}'\right)$$

$$\vec{E}' = \vec{E} + \vec{V} \wedge \vec{B}$$

$$\vec{B}' = \vec{B}$$

MP2 2022

c) $$\vec{E}_i' = \vec{E}_i + V\vec{u}_x \wedge \vec{B}_i$$

$$\vec{B}_i = \frac{\vec{k}_i \wedge \vec{E}_i}{\omega} = \frac{1}{c}\vec{u}_x \wedge E_i \vec{u}_y = \frac{E_i}{c}\vec{u}_z$$

$$\vec{E}_i' = \vec{E}_i + \frac{V}{c} E_i \left(\vec{u}_x \wedge \vec{u}_z\right)$$

$$\vec{E}_i' = \left(1 - \frac{V}{c}\right) E_0 \cos\left(\omega t - \frac{\omega x}{c}\right) \vec{e}_y$$

$$\vec{E}_r' = \left(1 + \frac{V}{c}\right) E_0 r \cos\left(\omega t + \frac{\omega x}{c}\right) \vec{e}_y$$

$$\vec{B}_r = \frac{\vec{k}_r \wedge \vec{E}_r}{\omega}.$$

$$\vec{E}_i' + \vec{E}_r' = \vec{0} \qquad x = Vt$$

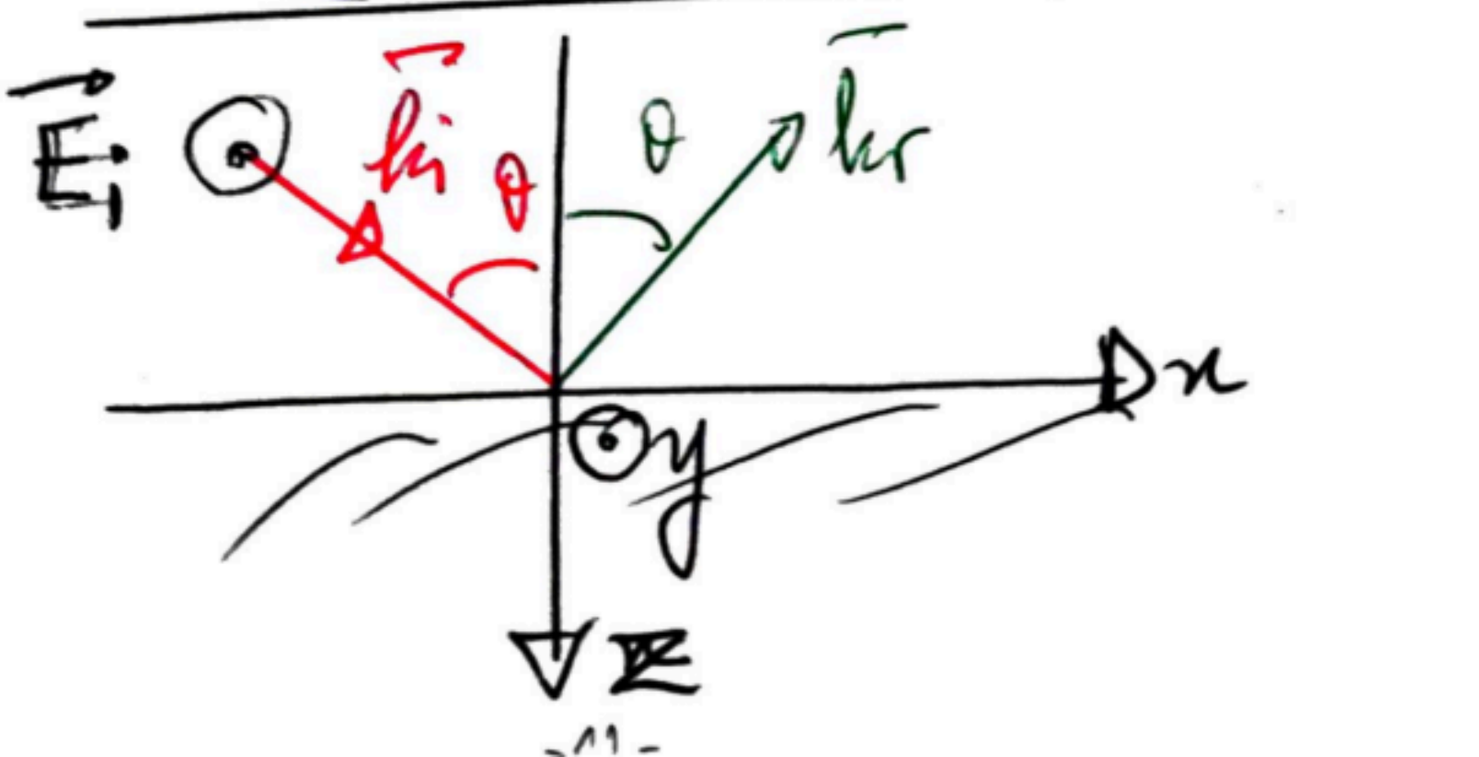
$$\left(1-\frac{v}{c}\right)E_0\cos\left[\omega_i\left(1-\frac{v}{c}\right)t\right]$$

$$+\,r\,E_0\left(1+\frac{v}{c}\right)\cos\left[\omega_r\left(1+\frac{v}{c}\right)t\right]=0 \qquad \forall t$$

$$\begin{cases}\omega_i\left(1-\frac{v}{c}\right)=\omega_r\left(1+\frac{v}{c}\right)\\ \left(1-\frac{v}{c}\right)+r\left(1+\frac{v}{c}\right)=0\,.\end{cases}$$

$$r=-\frac{1-v/c}{1+v/c}\simeq-\left(1-\frac{2v}{c}\right)$$

$$f_r=f_i\frac{\left(1-\frac{v}{c}\right)}{\left(1+\frac{v}{c}\right)}\simeq f_i\left(1-\frac{2v}{c}\right)$$



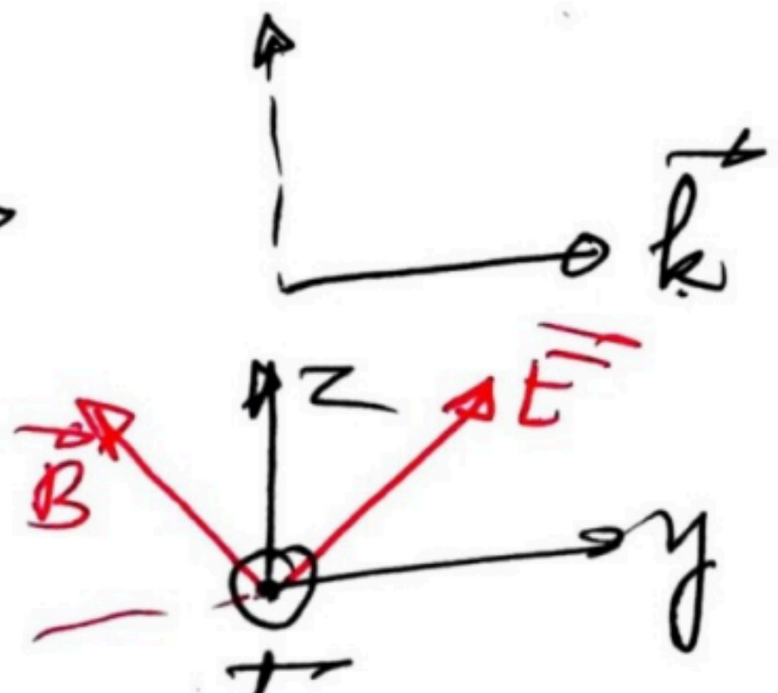
Serie N°1

Q62

$$\vec{E} = \begin{cases} 0 \\ E_y = E_{oy}\cos(\omega t - kx) \\ E_z = E_{oz}\cos(\omega t - kx - \varphi) \end{cases}$$

$$\varphi \in [0, 2\pi]$$

1) $\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} = \frac{1}{c}\vec{u_x} \wedge \vec{E}$



2) a) une courbe fermée décrite par la projection de l'extrémité de $\vec{E}$ sur un plan d'onde

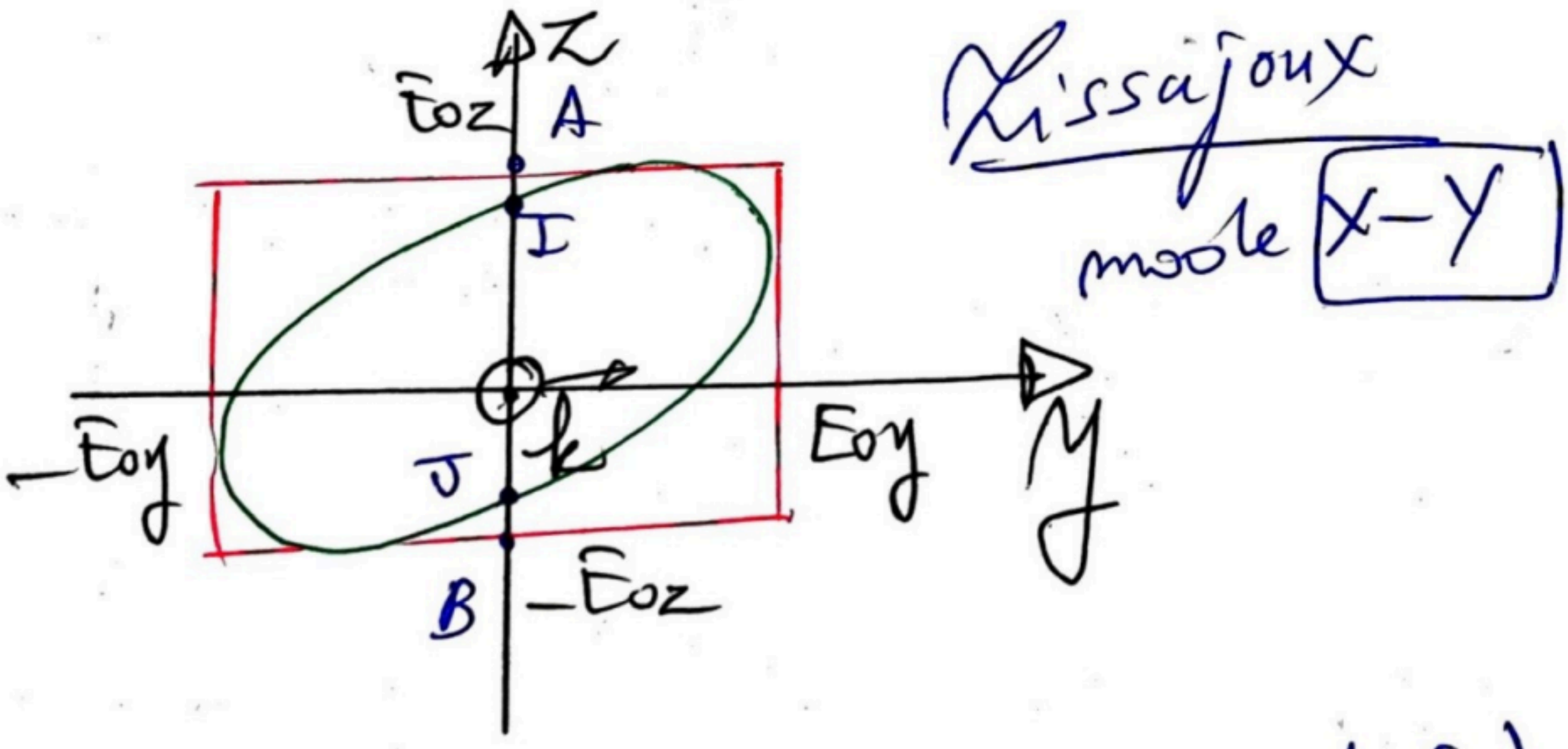
b) $\frac{E_z}{E_{oz}} = \cos(\omega t - kx)\cos\varphi + \sin\varphi \sin(\omega t - kx)$

$\frac{E_z}{E_{oz}} = \left(\frac{E_y}{E_{oy}}\right)\cos\varphi + \sin\varphi \sin(\omega t - kx)$

$\left[\frac{E_z}{E_{oz}} - \frac{E_y}{E_{oy}}\cos\varphi\right]^2 = \sin^2(\omega t - kx)\sin^2\varphi$

$$\left(\frac{E_y}{E_{oy}}\right)^2 \sin^2\varphi = \cos^2(\omega t - kx)\sin^2\varphi$$

$$\left(\frac{E_y}{E_{oy}}\right)^2 + \left(\frac{E_z}{E_{oz}}\right)^2 - \frac{2E_y E_z}{E_{oy} E_{oz}}\cos\varphi = \sin^2\varphi$$

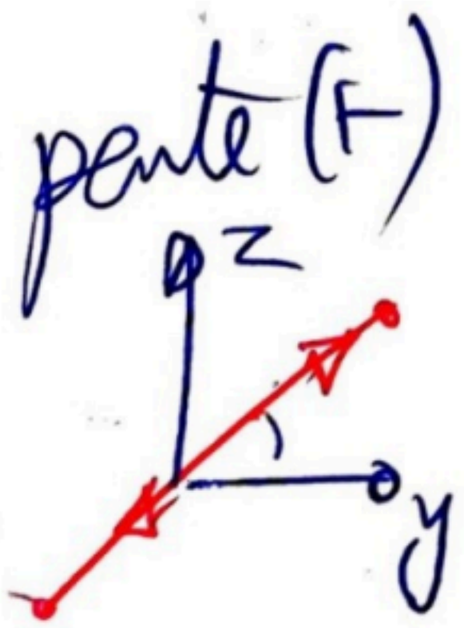


Lissajoux

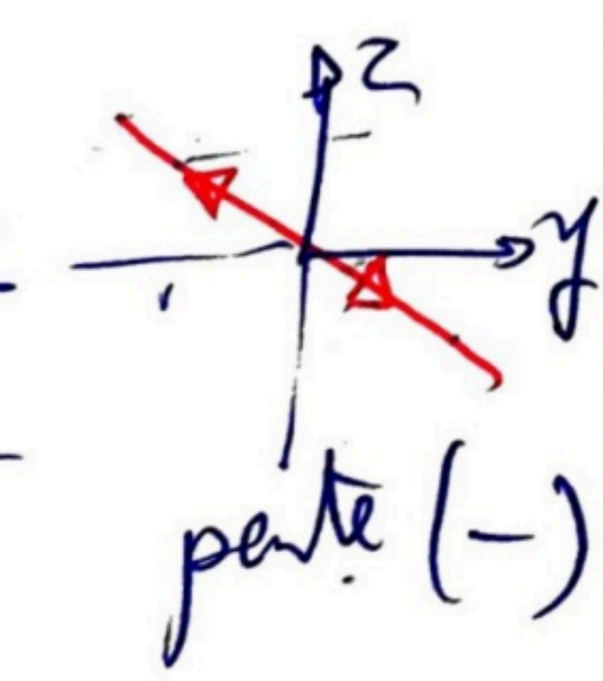
mode X-Y

$$\frac{IJ}{AB} = \sin\varphi$$

$\varphi = 0 \Rightarrow \frac{E_z}{E_y} = -\frac{E_{oz}}{E_{oy}}$

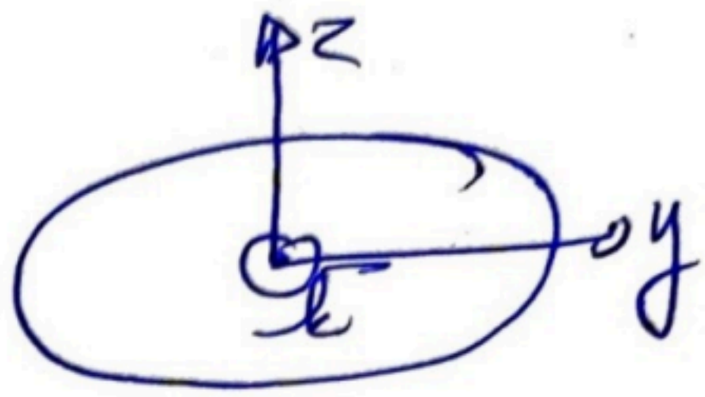


$\varphi = \pi \Rightarrow \frac{E_z}{E_y} = -\frac{E_{oz}}{E_{oy}}$



$\varphi = \frac{\pi}{2}$; $\frac{3\pi}{2}$, $\sin^2\varphi = (\pm 1)^2 = 1$

$$\left(\frac{E_z}{E_{oz}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 = 1$$



ellipse

en outre $E_{oy} = E_{oz} = E_0$ cercle

3) Sens de rotation

① $S = \left(\vec{E} \wedge \frac{d\vec{E}}{dt}\right) \cdot \vec{k} > 0$; gauche

② $\frac{dE_z}{dt} = -E_{oz}\omega \sin(-\varphi)$

$(\omega t - kx) = 0$

$= +E_{oz}\omega \sin\varphi$

si $\varphi \in]0, \pi[\Rightarrow \sin\varphi > 0$

$\vec{E}_z(t)$



$$\underline{\underline{E}} = E_0 \begin{vmatrix} 0 \\ e^{i(\omega t - kx)} \\ e^{i(\omega t - kx - \frac{\pi}{2})} \end{vmatrix}$$

$$\vec{E} = E_0 \begin{vmatrix} 0 \\ \cos(\omega t - kx) \\ \sin(\omega t - kx) \end{vmatrix} \rightarrow \cos\left(\omega t - kx - \frac{\pi}{2}\right)$$

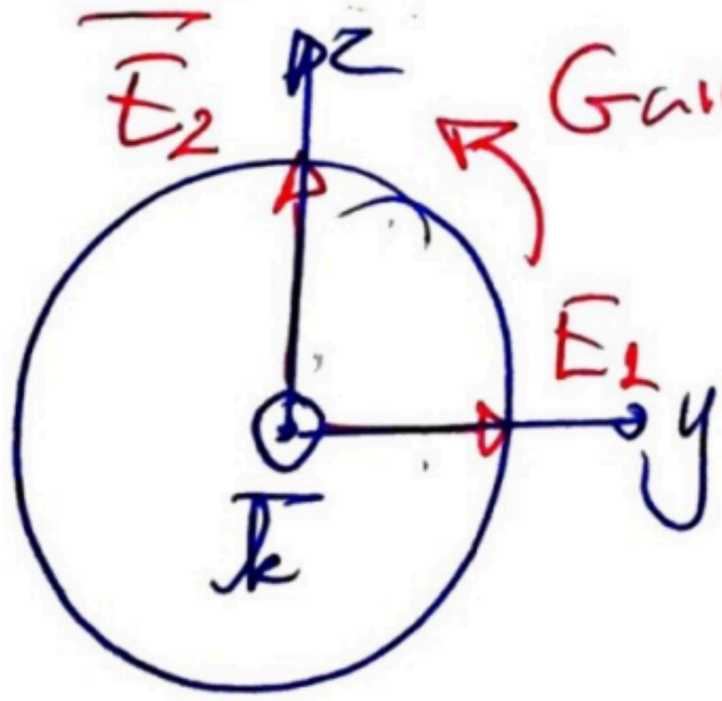
$$\underline{\underline{E}} = E_0 \, e^{i(\omega t - kx)} \left[\vec{u}_y - i \vec{u}_z \right]$$

$$\left(\frac{dE_z}{dt} \right)_{\substack{t=0 \\ x=0}} = + E_0 \omega > 0, \quad \text{Circulaire gauche}$$

3ème méthode $\quad \Phi = (\omega t - kx)$

$$\underline{\underline{E}}_1 (\Phi = 0) = E_0 \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$$

$$\underline{\underline{E}}_2 \left(\Phi = \frac{\pi}{2}\right) = E_0 \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix}$$



Exples

$$\vec{E} = \begin{vmatrix} 0 \\ E_0 \cos\alpha \cos(\omega t - kx) = E_y \\ E_0 \sin\alpha \cos(\omega t - kx) = E_z \end{vmatrix}$$

$\frac{E_z}{E_y} = \tan\alpha = cte$ · <u>rectiligne</u>

$$\vec{E} = \begin{vmatrix} 0 \\ (E_0 \cos\alpha) \cos(\omega t - kx) \\ E_0 \sin\alpha \sin(\omega t - kx) \end{vmatrix}$$

$$\left(\frac{E_y}{E_0 \cos\alpha}\right)^2 + \left(\frac{E_z}{E_0 \sin\alpha}\right)^2 = 1$$

<u>ellipse</u>

$\alpha = \frac{\pi}{4}$: <u>cercle</u>

<u>cercle</u> : $\vec{E}_{circulaire} = E_0 \begin{vmatrix} 0 \\ 1 \\ -i \end{vmatrix} e^{i(\omega t - kx)}$

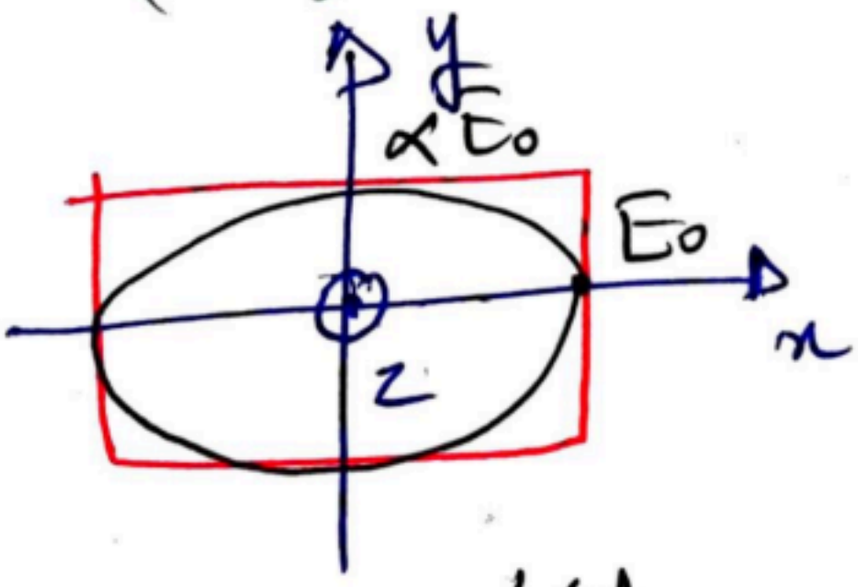
$-i \hookrightarrow e^{-i\frac{\pi}{2}}$

Farhe B

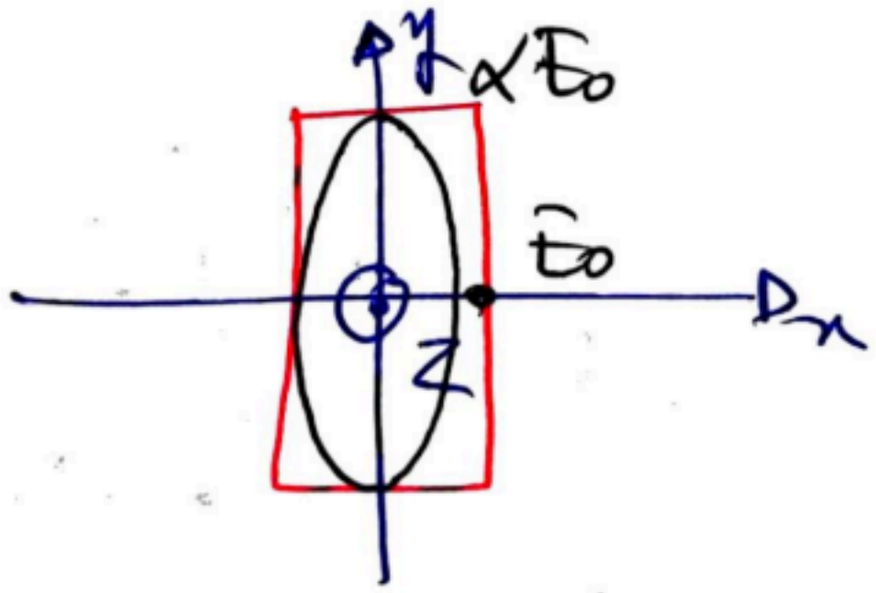
$$\underline{\vec{E}} = E_0 e^{i(\omega t - kz)} \begin{vmatrix} 1 \\ i\alpha \\ 0 \end{vmatrix} \qquad \alpha \geq 0$$

$$\vec{E} = E_0 \left(\cos(\omega t - kz)\, \vec{e}_x - \alpha \sin(\omega t - kz)\, \vec{e}_y \right)$$

- $\alpha = 0$: rectiligne (ox)
- $\left(\frac{E_x}{E_0}\right)^2 + \left(\frac{E_y}{\alpha E_0}\right)^2 = 1$



$\alpha < 1$



$\alpha > 1$

$\left(\frac{dE_y}{dt}\right)_{\omega t - kz = 0} = -\alpha E_0 \omega < 0$

droite

- $\alpha = 1$ circulaire droite

PT2 2015

2) reflexion

$$\vec{E}_r = -E_0 e^{i(\omega t + kz)} \begin{vmatrix} 1 \\ i\alpha \\ 0 \end{vmatrix}$$

$$\vec{E}_i = E_0 e^{i(\omega t - kz)} \begin{vmatrix} 1 \\ i\alpha \\ 0 \end{vmatrix}$$

$$\underline{\vec{E}} = E_0 e^{i(\omega t)} \begin{vmatrix} (-e^{ikz} + e^{-ikz}) \\ i\alpha(-e^{ikz} + e^{-ikz}) \\ 0 \end{vmatrix}$$

$$= E_0 e^{i(\omega t)} \begin{vmatrix} -2i\sin(kz) \\ +2\alpha\sin(kz) \\ 0 \end{vmatrix}$$

$$\vec{E} = 2E_0 \sin(kz)\, e^{i\omega t} \begin{vmatrix} -i \\ \alpha \\ 0 \end{vmatrix}$$

Partie C

La lumière naturelle est non polarisée :

$$\vec{E} = \begin{cases} E_y = E_0 \cos(\omega t - kx) \\ E_z = E_0 \cos(\omega t - kx - \varphi(t)) \end{cases}$$

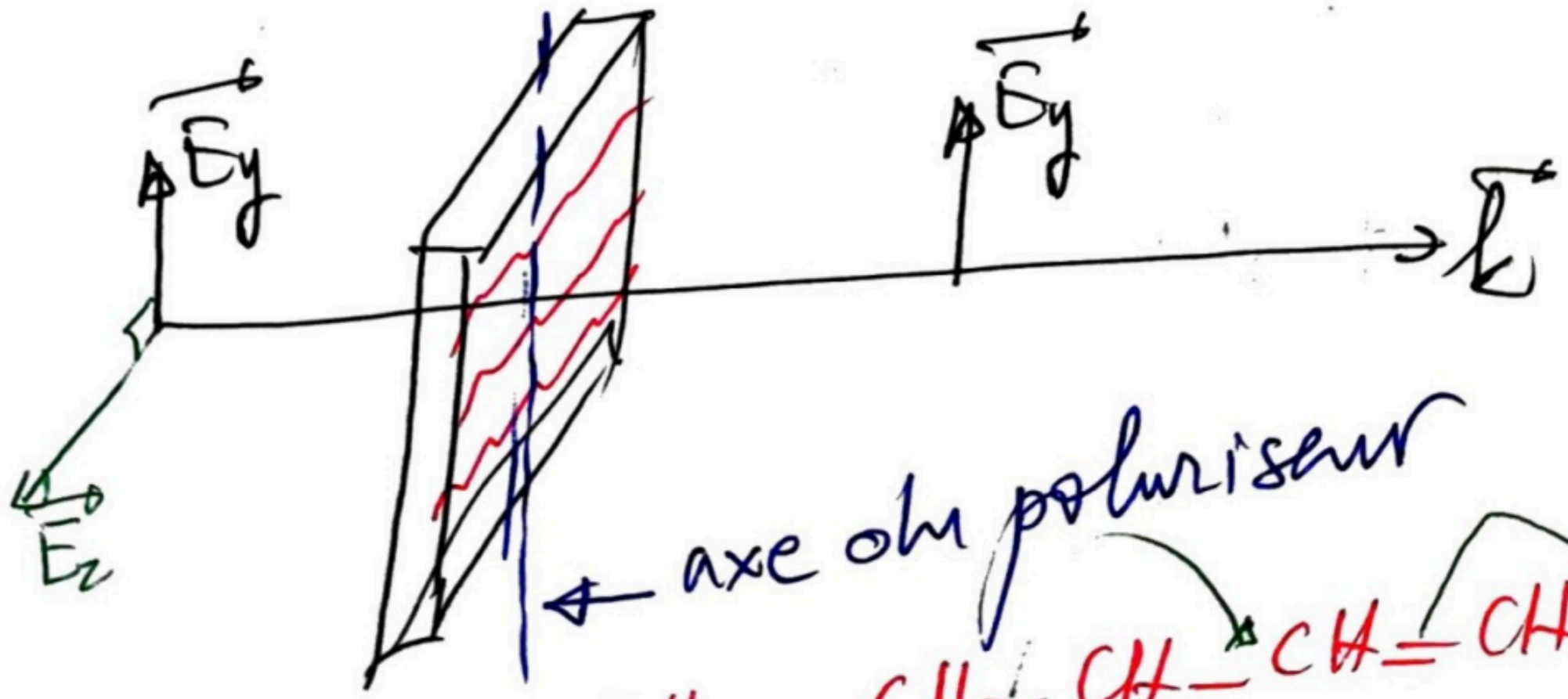
$\varphi(t)$ = aléatoire

exple soleil

Lampe spectrale

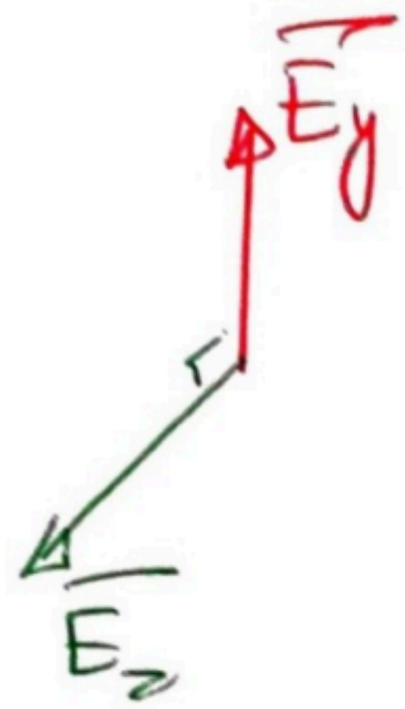
Source thermique (اللؤلؤة ..)

2) Polariseur = Polaroïd



$CH_3 - CH = CH - CH = CH \cdots CH_3$

2)



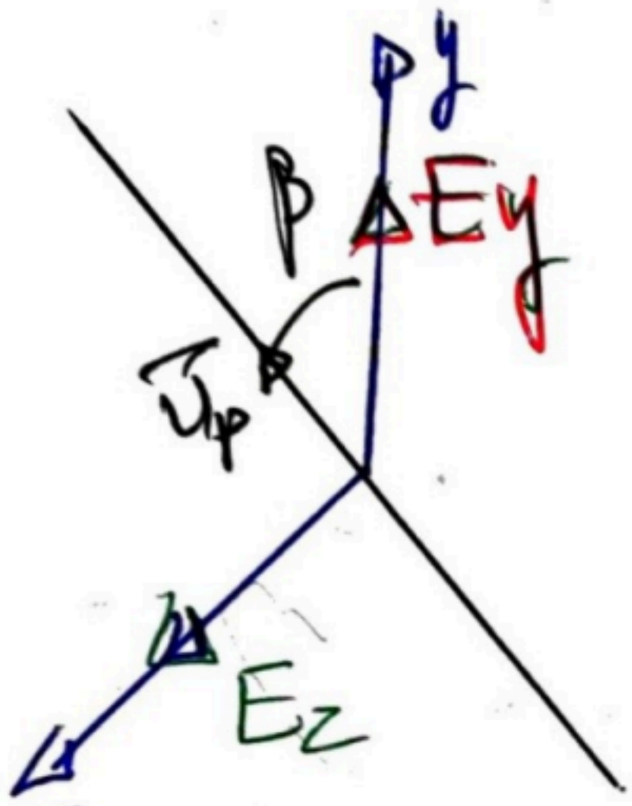






3)a)

nature lle



$P_{émissaire} = \langle \| \vec{\Pi} \| \rangle S$

$P = K \langle \| \vec{E} \|^2 \rangle$

z

- Avant le polariseur :

$$P_0 = \frac{K \bar{E}_0^2}{2} + \frac{K E_0^2}{2} = K \bar{E}_0^2$$

- Après le polariseur

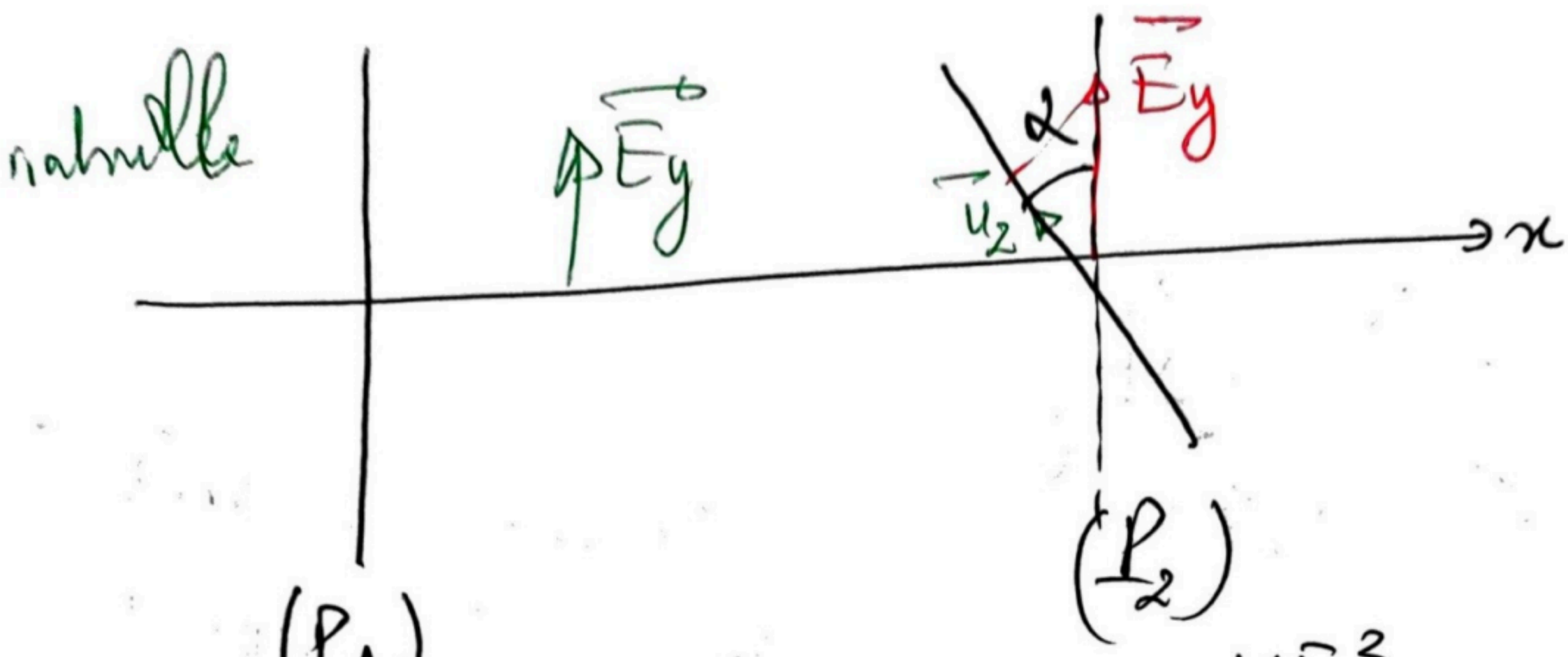
$$\vec{E}_1 = E_0 \cos\beta \cos(\omega t - kz)\, \vec{u}_I + E_0 \sin\beta \cos(\omega t - kz - \varphi)\, \vec{u}_p$$

$$P_1 = \frac{K E_0^2}{2}\left(\cos^2\beta + \sin^2\beta\right) = \frac{P_0}{2}$$

$\forall \beta$

3) b) La loi de Malus

P_1 , $P_2 = P_1 \cos^2\alpha$



après (P_1) : la puissance $\frac{KE_0^2}{2}$

après (P_2) : $\vec{E_2} = E_0 \cos\alpha \cos(\omega t - kx)\vec{u_2}$

$$\frac{P}{2} = \frac{K E_0^2}{2} \cos^2\alpha$$

$$\boxed{P_2 = P_1 \cos^2\alpha}$$

$\alpha = [\alpha]_\lambda l C$

La loi de Biot

Partie B

Lame à retard de phase

(oz) : n_0 = indice ordinaire

(oy) : n_e : " extraordinaire

Lame anisotrope

1) Les lignes neutres (oy) et (oz).

$n_e > n_0 \Rightarrow n_{oy} > n_{oz}$

$v_\varphi = \frac{c}{n}$; $\frac{1}{n_{oy}} < \frac{1}{n_{oz}}$

$v_{oy} < v_{oz}$: (oy) : axe lent

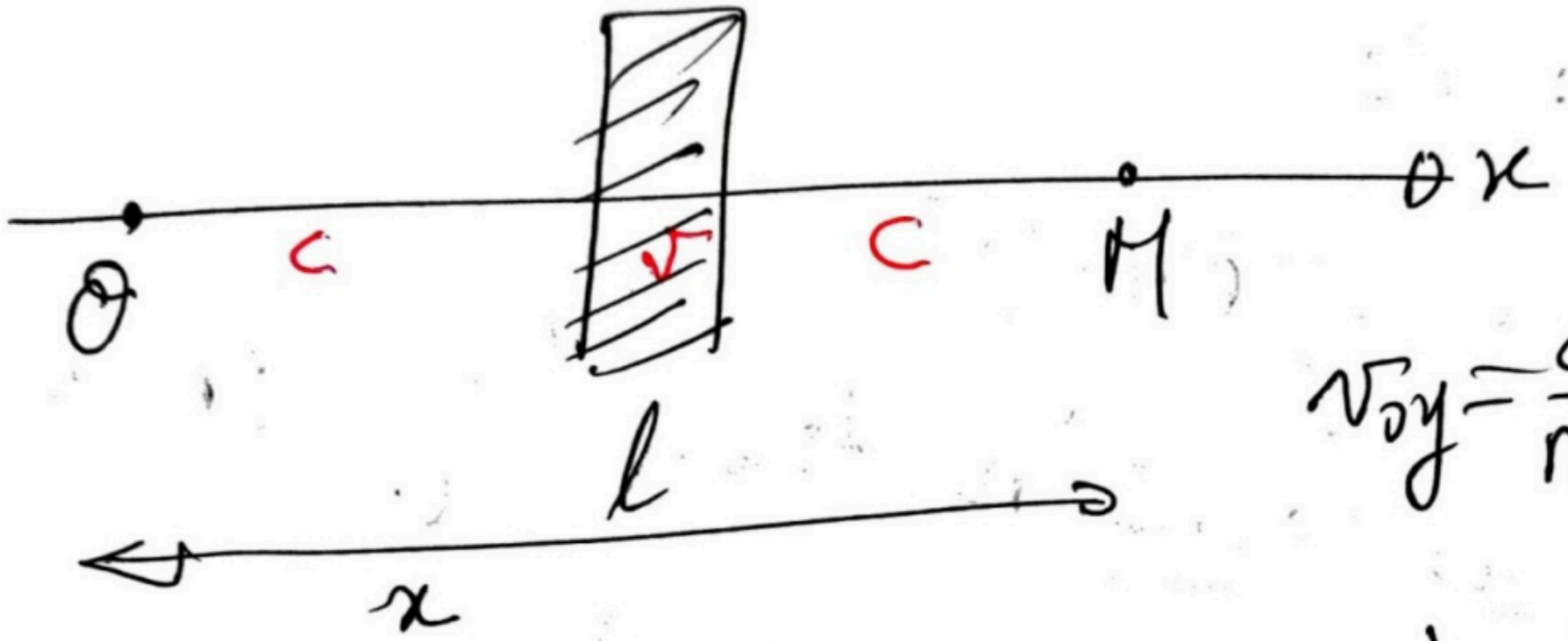
(oz) : axe rapide

2) $\vec{E_i} = E_0 \cos(\omega t - kx)\left(\cos\alpha\,\vec{e_y} + \sin\alpha\,\vec{e_z}\right)$

$$\frac{E_{iz}}{E_{iy}} = +\operatorname{tg}\alpha = \underline{\alpha}$$

$$E_{iy} = E_0 \cos\alpha \cos(\omega t - kx)$$

$$E_{ty} = (E_0 \cos\alpha) \cos\left[\omega\left(t - \frac{x-l}{c} - \frac{l}{v_{oy}}\right)\right]$$



$$v_{oy} = \frac{c}{n_{oy}}$$

$$E_{ty} = (E_0 \cos\alpha) \cos\left[\omega t - k(x-l) - \frac{kl n_e}{}\right)$$

$$E_{ty} = (E_0 \cos\alpha) \cos(\omega t - k(x-l) - kl n_o)$$

$$E_{tz} = (E_0 \sin\alpha) \cos(\omega t - k(x-l) - kl n_o)$$

$$\Delta\varphi = kl(n_e - n_o)$$

$$= \frac{2\pi}{\lambda}(l(n_e - n_o))$$

$$\left(\delta = d(\sin\theta - \sin\theta_0) = m\lambda\right)$$

$$\Delta\varphi = \frac{2\pi}{\lambda}\delta$$

$$\delta = l(n_e - n_o)$$

3) demi-onde $\delta = \frac{\lambda}{2}$; $\Delta\varphi = \pi$

quart onde $\delta = \frac{\lambda}{4}$; $\Delta\varphi = \frac{\pi}{2}$

" dépende de λ "

4) $\boxed{x' = x + n_e l}$

$$E_{ty} = E_0 \cos(\alpha) \cos\left[\omega t + kl - k(x + l n_e)\right]$$

$$E_{ty} = E_0 \cos\alpha \cos\left(\omega t + kl - kx'\right)$$

$$E_{tz} = \left(E_0 \sin\alpha\right) \cos\left(\omega t + kl - k(x + l n_o)\right)$$

$$= \left(E_0 \sin\alpha\right) \cos\left(\omega t + kl - k(x' - n_e l + n_o l)\right)$$

$$= \left(E_0 \sin\alpha\right) \cos\left(\omega t + kl - kx' + kl(n_e - n_o)\right)$$

$$\boxed{E_{tz} = \left(E_0 \sin\alpha\right) \cos\left(\omega t + kl - kx' + \Delta\varphi\right)}$$

a) $\Delta\varphi = \pi$; $E_{tz} = -\left(E_0 \sin\alpha\right) \cos\left(\omega t + kl - kx'\right)$

4) b) $\Delta\varphi = \frac{\pi}{2}$ $\boxed{\lambda/4 = 8}$

$$\underline{E}_{tz} = -\left(E_0 \sin\alpha\right) \sin\left(\omega t - kx' + kl\right)$$

$$\underline{E}_{ty} = +\left(E_0 \cos\alpha\right) \cos\left(\omega t - kx' + kl\right)$$

$$\left(\frac{\underline{E}_{tz}}{E_0 \sin\alpha}\right)^2 + \left(\frac{\underline{E}_{ty}}{E_0 \cos\alpha}\right)^2 = 1$$

ellipse

$\boxed{\alpha = \frac{\pi}{4}}$ cercle

5) $\underline{E}_{iCG} = E_0 \left[\cos\left(\omega t - kx\right) \vec{u}_y + \sin\left(\omega t - kx\right) \vec{u}_z\right]$

$$\underline{E}_{iy} = E_0 \cos\left(\omega t - kx\right)$$

$$\underline{E}_{ty} = E_0 \cos\left(\omega t + kl - kx'\right)$$

$$\underline{E}_{tz} = E_0 \cos\left(\omega t + kl - kx' + \Delta\varphi - \frac{\pi}{2}\right)$$

a) lame demi onde

$$\Delta\varphi = \frac{\pi}{2}$$

$$\vec{E}_{tz} = E_0 \cos\left(\omega t + kl - kn' + \frac{\pi}{2}\right)$$

$\vec{E}_{CG}$ → $\lambda/2$ → $\vec{E}_{CD}$

$\Delta\varphi = \frac{\pi}{2}$ $\quad \vec{E}_t = \begin{cases} 0 \\ E_{yt} = E_0 \cos(\quad) \\ E_{zt} = E_0 \cos(\quad) \end{cases}$

rectiligne → $\lambda/4$ → rectiligne

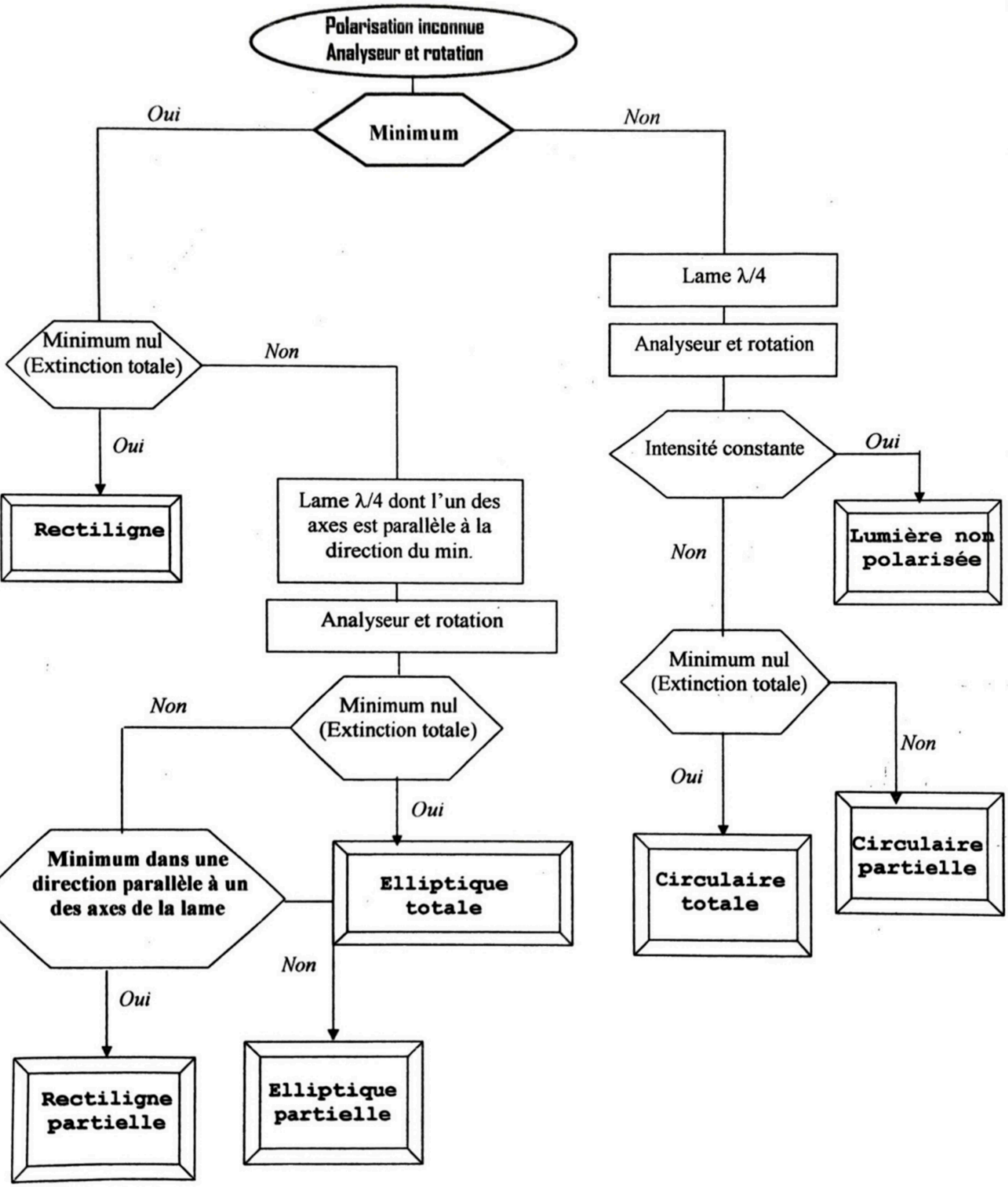
circulaire

Polarisation rectiligne

$\vec{E}_{CD} = E_0 \begin{vmatrix} 0 \\ \cos(\omega t - kx) \\ -\sin(\omega t - kx) \end{vmatrix}$; $\vec{E}_{CG} = E_0 \begin{vmatrix} \cos(\omega t - kx) \\ +\sin(\omega t - kx) \end{vmatrix}$

$\vec{E}_{CD} + \vec{E}_{CG} = 2E_0 \cos(\omega t - kx)\, \vec{u}_y$

rectiligne

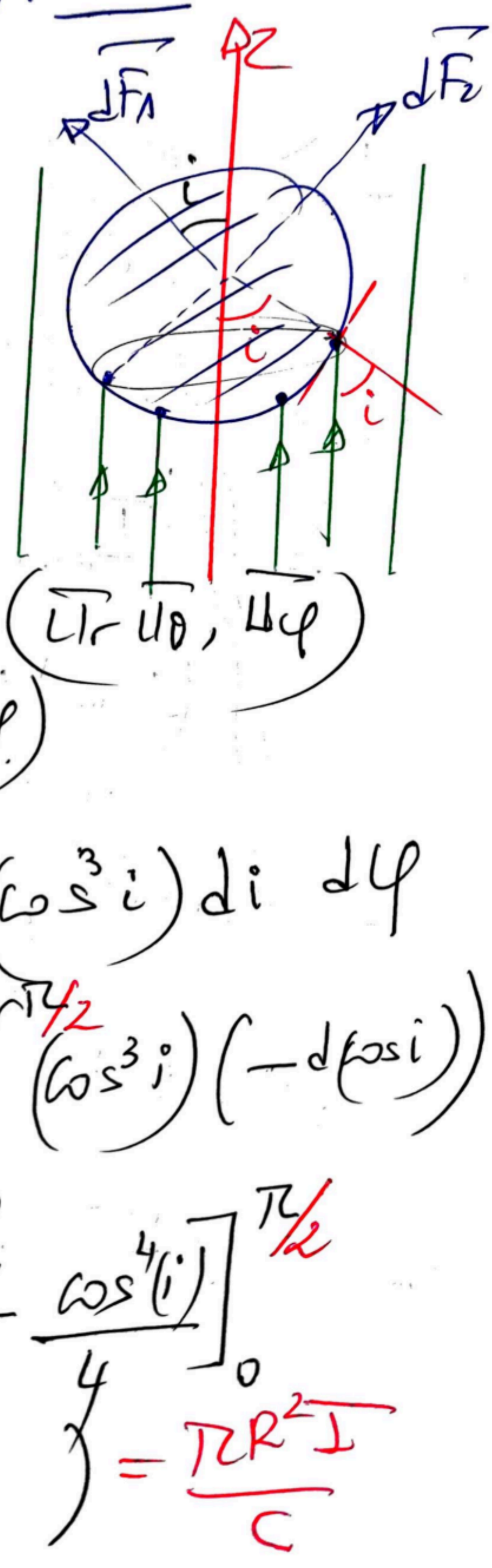

Polarisation inconnue
Analyseur et rotation
Minimum
Oui
Non
Minimum nul
(Extinction totale)
Oui
Non
Rectiligne
Lame λ/4 dont l'un des axes est parallèle à la direction du min.
Analyseur et rotation
Minimum nul
(Extinction totale)
Non
Oui
Minimum dans une direction parallèle à un des axes de la lame
Elliptique totale
Non
Oui
Rectiligne partielle
Elliptique partielle
Lame λ/4
Analyseur et rotation
Intensité constante
Oui
Non
Lumière non polarisée
Minimum nul
(Extinction totale)
Oui
Non
Circulaire totale
Circulaire partielle

$\frac{dI_{b3}}{}$

$$I_{mol} = \frac{2I}{C}\cos^2 i$$

$p(7)$: en bas

$\overrightarrow{dF_{mol}} \cdot \overrightarrow{u_z} = \overrightarrow{dF_z}$ $\frac{dI_{z}}{}$ $\frac{2\pi 1}{}$



$$dF_z = \left(\frac{2I}{C}\cos^2 i\right)\left(\vec{u}_r \, dS\right)\vec{u}_z$$

$$= \frac{2I}{C} dS \cos^2 i \cos i$$

$$= \frac{2I}{C}(\cos^3 i)(R^2 \sin i \, di \, d\varphi)$$

$$dF_z = \frac{2I R^2}{C} \sin i (\cos^3 i)\, di \, d\varphi$$

$$F_z = \frac{2I R^2}{C} \int_0^{2\pi} d\varphi \int_0^{\pi/2} (\cos^3 i)(-d(\cos i))$$

$$= \frac{2I R^2}{C} 2\pi \times \left[-\frac{\cos^4(i)}{4}\right]_0^{\pi/2}$$

$$= \frac{\pi R^2 I}{C}\left(1\right) = \frac{\pi R^2 I}{C}$$

$$F_z = \frac{\pi R^2 I}{c} \geq mg$$



$$I \geq \frac{mgc}{\pi R^2} = I_{min}$$

$$P_m = I_{min} \cdot (\pi R^2)$$

$$= \iint \vec{I}_{min} \, d\vec{S}$$

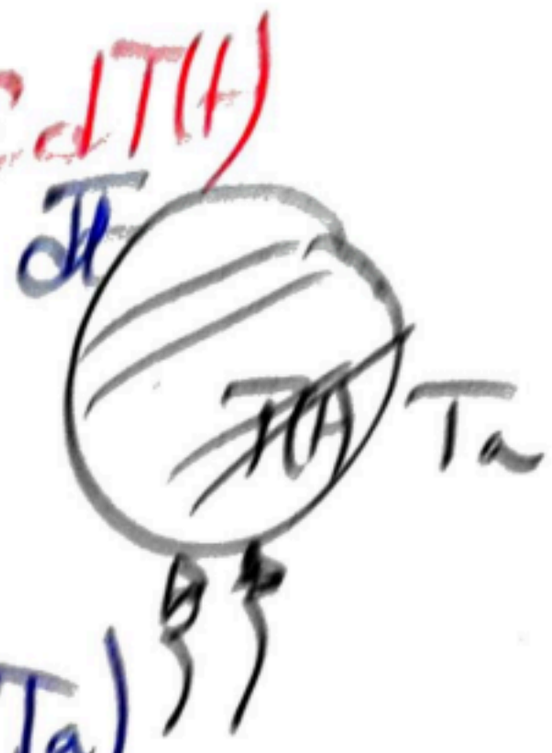
$$= \iint (I_{min}) \vec{u}_z \cdot R^2 \sin i \, di \, d\varphi \vec{u}_r$$

$$= I_{min} R^2 \int_0^{\pi/2} \cos i \sin i \, di \int_0^{2\pi} d\varphi$$

$$= 2\pi R^2 I_{min} \left(\frac{\sin^2 i}{2} \right)_0^{\pi/2}$$

$$\boxed{P_m = \pi R^2 I_{min}}$$

3) $\alpha P_m = K\,4\pi R^2 (T - T_a) + C\frac{dT(t)}{dt}$



$dU = \delta Q_{\text{рассе}}$

$\frac{dU}{dt} = \alpha P_m - K\,4\pi R^2 (T - T_a)$

$dU = C\,dT$

$C\frac{dT(t)}{dt} + \frac{K\,4\pi R^2 (T(t) - T_a)}{C} = \frac{\alpha P_m}{C}$

опрose $u(t) = T(t) - T_a$

$\frac{du}{dt} + \frac{u(t)}{\tau} = \frac{\alpha P_m}{C}$

$u(t) = A e^{-t/\tau} + \frac{\alpha P_m \tau}{C}$

$u(0) = T(0) - T_a = 0$

$A + \frac{\alpha P_m \tau}{C} = 0$

$\lim_{t \to \infty} T(t) = T_a + \frac{\alpha P_m \tau}{C} > T_{\text{плав}}$

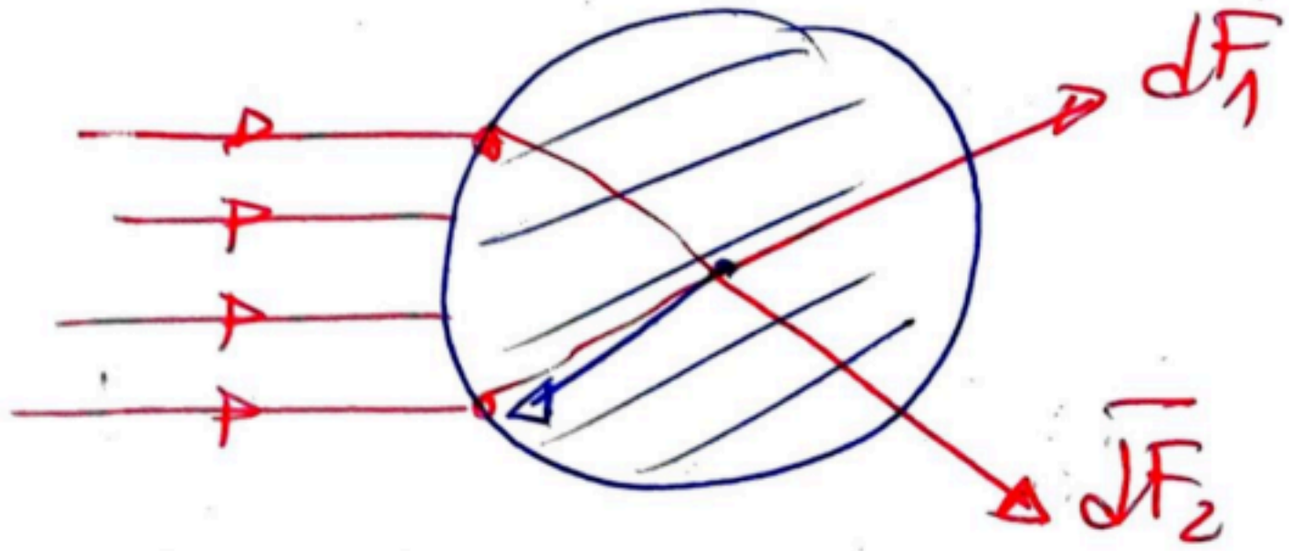
Pb4 :

$$p = \varepsilon E_0^2 \cos^2\theta = \frac{2I}{c}\cos^2\theta$$

1) $P_S = \left(\frac{\varepsilon c E_0^2}{2}\right) \cdot 4\pi r^2$ $\qquad I = \frac{\varepsilon c E_0^2}{2}$

$$E_0 = \sqrt{\frac{P_S}{2\pi\varepsilon c}} \qquad \frac{1}{r} = \frac{ds}{r}$$

$$\langle \|\vec{\pi}\| \rangle \sim \frac{1}{r^2} \qquad S = 4\pi r^2$$



2) $d = 2R$

$$F_{incl} = \frac{\pi R^2 I}{c}$$

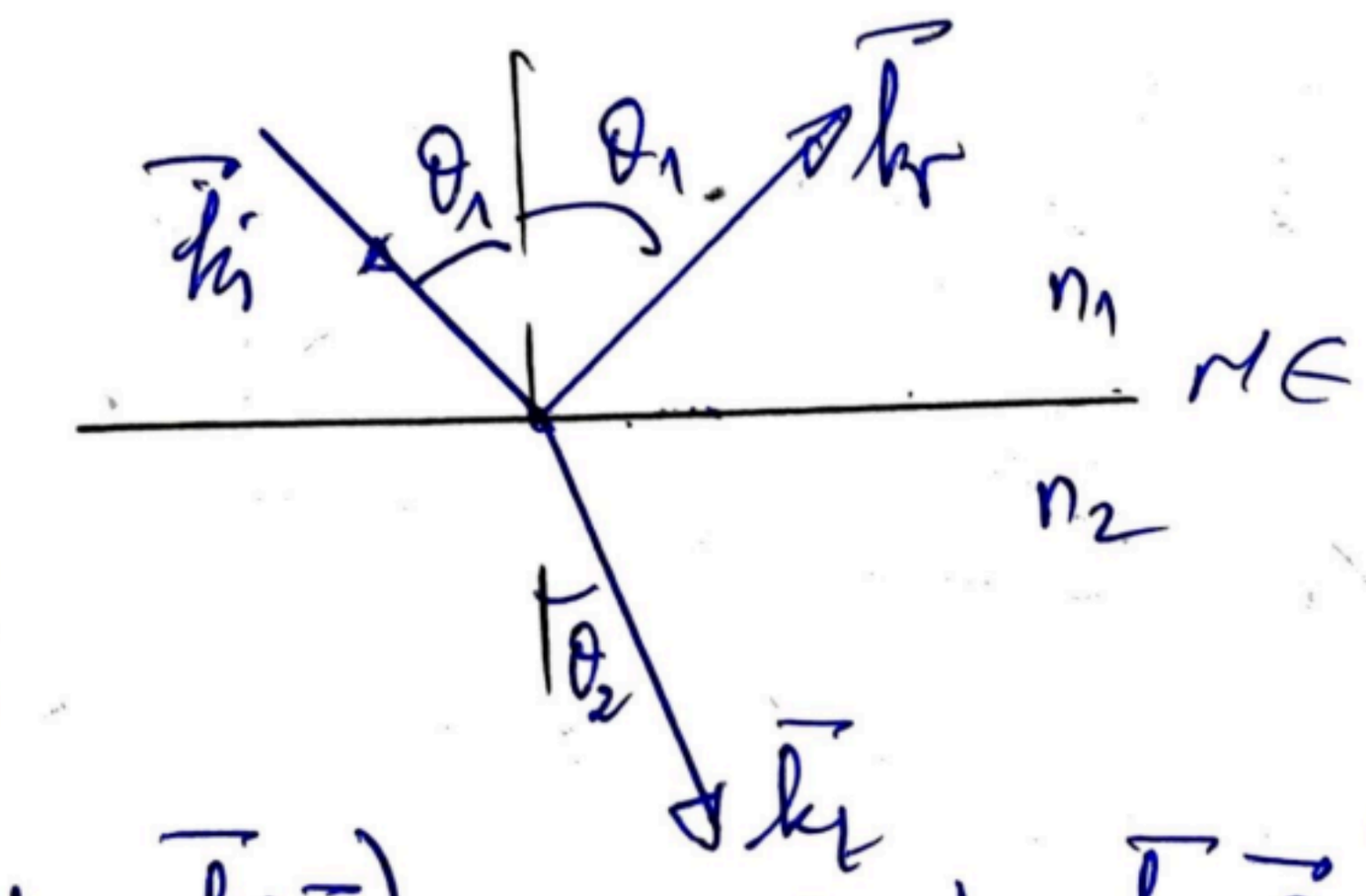
$$= \frac{\pi}{c}\,\frac{\varepsilon c E_0^2}{2}\left(\frac{d}{2}\right)^2$$

$$F_{ind} = \frac{\pi\varepsilon E_0^2}{8} d^2 = \frac{\pi d^2 \varepsilon}{8}\,\frac{P_S}{2\pi\varepsilon c}\,\frac{1}{r^2}$$

$$F_{incl} = \frac{d^2 P_S}{16 c}\,\frac{1}{r^2} \geq |F_G| = \frac{G M_S m}{r^2}$$

$$\frac{d^2 P_s}{16C} \geq \frac{G\eta_s}{} \; m \; L_0 \rho_0 \cdot \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$d \leq d_0 = - - -$$



$$v_\varphi = \frac{c}{n}$$

$$k_2 = \frac{\omega}{v_\varphi}$$

$$\vec{E}_{oi}\, e^{i(\omega_i t - \vec{k_i}\vec{r})} + \vec{E}_{or}\, e^{i(\omega_r t - \vec{k_r}\vec{r})} = \vec{E}_{ot}\, e^{i(\omega_t t - \vec{k_t}\vec{r})}$$

en $\vec{r} = \vec{0}$

$$\vec{E}_{oi} + \vec{E}_{or}\, e^{i(\omega_r - \omega_i)t} = \vec{E}_{ot}\, e^{i(\omega_t - \omega_i)t} \quad \forall t$$

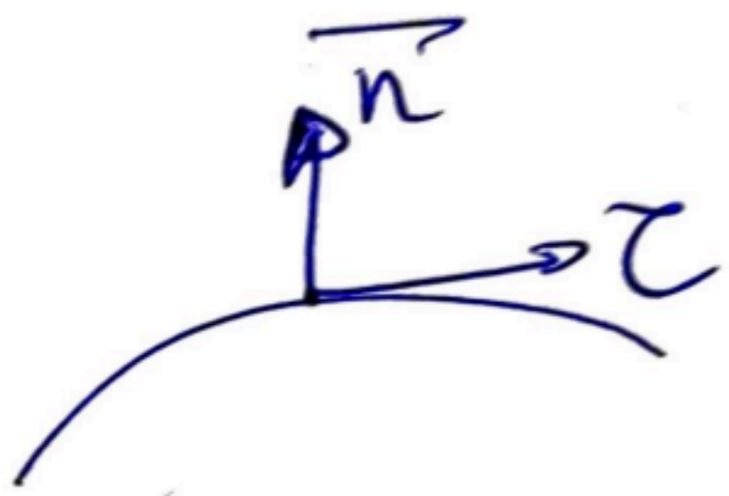
$$\left.\begin{array}{l} \omega_r = \omega_i \\ \omega_t = \omega_i \end{array}\right\} \Rightarrow E_{oi} + E_{or} = E_{ot}$$

$$\vec{E}_{oi} + \vec{E}_{or}\, e^{-i(\vec{k}_r - \vec{k}_i)\vec{r}} = \vec{E}_{ot}\, e^{-i(\vec{k}_t - \vec{k}_i)\vec{r}}$$

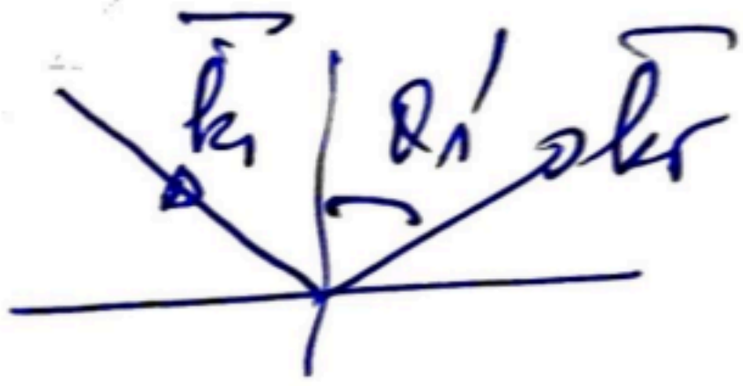
$$\left(\vec{k}_r - \vec{k}_i\right)\cdot\vec{r}^{\,\circ} = 0 \qquad \underline{\forall r}$$

$$\left(\vec{k}_t - \vec{k}_i\right)\vec{r} = 0$$

$$\begin{cases} \vec{k}_r - \vec{k}_i = \alpha\, \vec{n} \\ \left(\vec{k}_t - \vec{k}_i\right) = \beta\, \vec{n} \end{cases}$$



$$\left(\vec{k}_t - \vec{k}_i\right)\vec{\tau}^{\,\circ} = 0$$



$$\|\vec{k}_{2t}\| \sin\theta_1 = \|\vec{k}_r\| \sin\theta_1' \qquad \theta_1 = \theta_1'$$

$$\left(\frac{\omega}{c} n_2\right) \sin\theta_2 = \left(\frac{\omega}{c} n_1\right) \sin\theta_1$$

$$\boxed{n_2 \sin\theta_2 = n_1 \sin\theta_1}$$

$$\underline{\vec{E}} = E_0 \exp i(\omega t - \underline{k} z)\, \vec{e}_x$$

$$\underline{k} = k' - i k''$$

$k' = Réel(\underline{k})$: responsable à la prop

$k'' = Im(\underline{k})$: " à l'atténuat°

$$\underline{\vec{E}} = E_0 \exp i(\omega t - (k' - i k'')z)\, \vec{e}_x$$

$$= E_0 e^{-k''z}\, e^{i(\omega t - k' z)}\, \vec{e}_x$$

atténuée ; progressive

$\underline{\text{Cas}}$ où $k^2 < 0$ $\quad \underline{k} = -i k''$ évanescente

$$\underline{\vec{E}} = E_0 \exp i(\omega t - (-i k'')z)\, \vec{e}_x$$

$$= E_0 e^{-k''z}\, e^{i(\omega t)}\, \vec{e}_x$$

évanescente : $\underline{\text{réflexion totale}}$

$$v_\varphi = \frac{\omega}{Réel(k)}, \qquad v_g = \frac{d\omega}{d(Réel(k))} = 0$$

$$\overrightarrow{\text{rot}}\,\vec{B} = \mu_0 \left[\gamma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right] \quad \text{Exo serie 2}$$

1)a)

b) $$\Delta \vec{E} - \mu_0 \gamma \frac{\partial \vec{E}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$$

$$\underline{\vec{E}} = E_0 \exp i(\omega t - \underline{k} z)\, \vec{e}_y$$

$$\left(-\underline{k}^2 - \mu_0 \gamma i\omega + \frac{\omega^2}{c^2} \right) \underline{\vec{E}} = \vec{0}$$

$$\underline{k}^2 = \frac{\omega^2}{c^2} - i\mu_0 \gamma \omega$$

$$\underline{k}^2 = \frac{\omega^2}{c^2} \left[1 - i \left(\frac{\gamma}{\omega \varepsilon} \right) \right]$$

- $\gamma \ll \varepsilon\omega \Rightarrow \underline{k} \simeq \frac{\omega}{c} \left(1 - \frac{i}{2} \frac{\gamma}{\varepsilon\omega} \right)$ — d.l

- $\gamma \gg \varepsilon\omega \quad \underline{k}^2 \simeq \frac{\omega^2}{c^2} \left[-i \frac{\gamma}{\varepsilon\omega} \right]$

$$\underline{k}^2 \simeq -i \left(\gamma \omega \mu_0 \right)$$

$$= \frac{(1-i)^2}{\left(\frac{2}{\mu_0 \gamma \omega} \right)} = \left(\frac{1-i}{\delta} \right)^2$$

$$k^2 = \frac{\omega^2}{c^2} - i\mu_0\gamma\omega$$

$$k = k' - ik''$$

$$|k|^2 = k'^2 + k''^2$$

$$k^2 = (k' - ik'')^2 = k'^2 - k''^2 - 2ik'k''$$

$$\begin{cases} k'^2 - k''^2 = \frac{\omega^2}{c^2} \\ 2k'k'' = (\mu_0\gamma\omega) > 0. \end{cases}$$

$$|\underline{k}|^2 = |\underline{k}^2| = \sqrt{\left(\frac{\omega^2}{c^2}\right)^2 + (\mu_0\gamma\omega)^2}$$

$$= k'^2 + k''^2$$

$$\begin{cases} k'^2 + k''^2 = \sqrt{(\quad)} \\ k'^2 - k''^2 = \frac{\omega^2}{c^2} \end{cases}$$

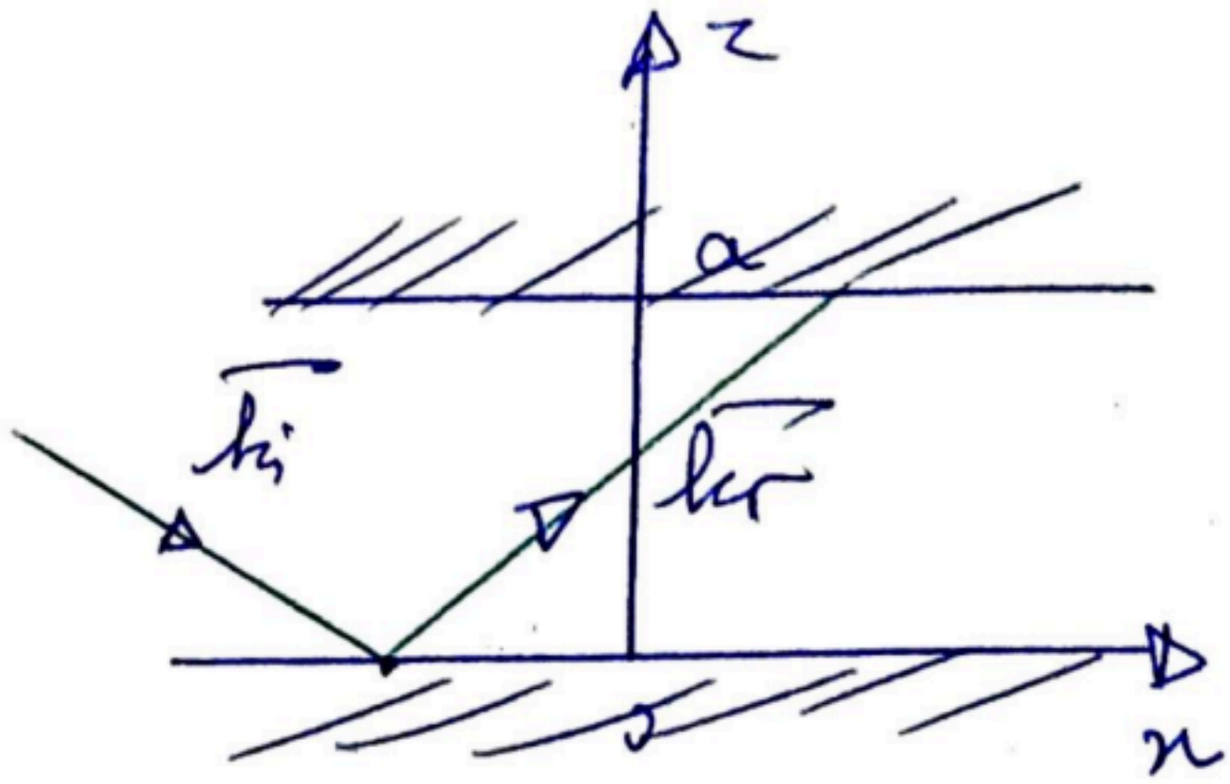
$$2k'^2 = \frac{\omega^2}{c^2} + \sqrt{\frac{\omega^4}{c^4} + (\mu_0\gamma\omega)^2}$$

PT2 2011

MP2 2004

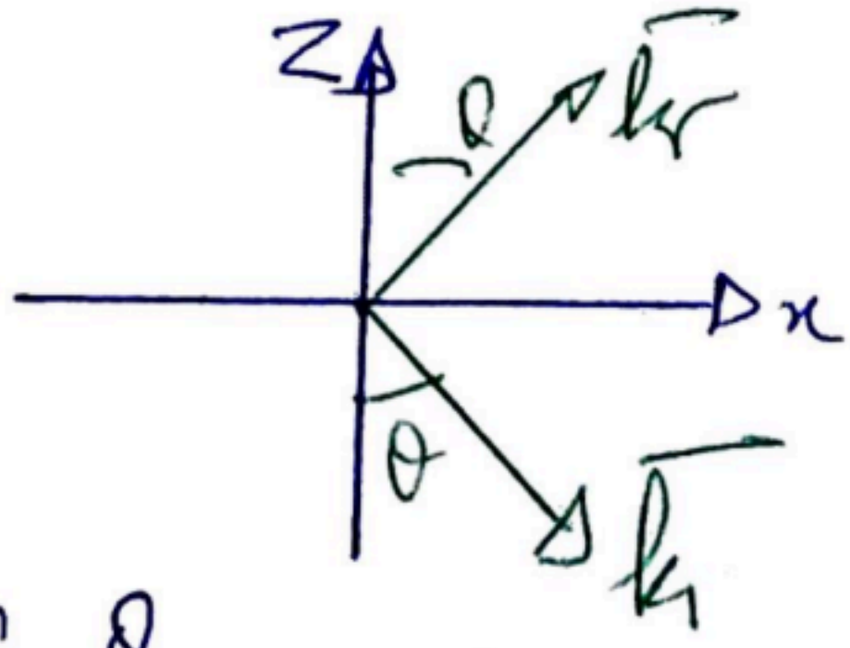
PT2 2018

12/20



$$\vec{E_j} = E_{0i}\, e^{i(\omega t - \vec{k_i}\vec{r})}\, \vec{u_y}$$

$$\underline{E_r} = \underline{E_{0r}}\, e^{i(\omega t - \vec{k_r}\vec{r})}\, \vec{u_y}$$



$$\vec{k_i} = \frac{\omega}{c}\begin{vmatrix} \sin\theta \\ 0 \\ -\cos\theta \end{vmatrix} \qquad \vec{k_r} = \frac{\omega}{c}\begin{vmatrix} \sin\theta \\ 0 \\ +\cos\theta \end{vmatrix}$$

$$\underline{\vec{E_i}} = E_{0i}\, e^{i(\omega t - k x \sin\theta + k z \cos\theta)}\, \vec{u_y}$$

$$\underline{\vec{E_r}} = \underline{E_{0r}}\, e^{i(\omega t - k x \sin\theta - k z \cos\theta)}\, \vec{u_y}$$

$\underline{z = 0}$; $\left(\underline{E_{0r}} + \underline{E_{0i}}\right) e^{i(\omega t - k x \sin\theta)}\, \vec{u_y} = 0$

$\forall x, \forall t$, $\underline{E_{0r}} = -\underline{E_{0i}}$

$\vec{E_{tot}} = E_{0i}\left(2i \sin(kz\cos\theta)\right) e^{i(\omega t - kx\sin\theta)}\, \vec{u_y}$

stationnaire (oz) ; progrut

$$\vec{E}(x,z,t) = E_0(z)\cos(\omega t - kx)\vec{u_y}$$

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \vec{0}$$

$$\left[-k^2 E_0(z) + \frac{d^2E_0(z)}{dz^2} + \frac{\omega^2}{c^2}E_0(z)\right] = 0.$$

$$\frac{d^2E_0(z)}{dz^2} + \left(\frac{\omega^2}{c^2} - k^2\right)E_0(z) = 0$$

$$r^2 + \left(\frac{\omega^2}{c^2} - k^2\right) = 0$$

2) a) $\underline{\omega < kc}$: $\frac{\omega^2}{c^2} - k^2 = -\alpha^2$

$r = \pm\alpha$, $E_0(z) = C_1 e^{\alpha z} + C_2 e^{-\alpha z}$

$z \in [0, a]$

borné = borthen

$A_1 ch(\alpha z) + A_2 sh(\alpha z) = E_0(z)$

$E_0(z=0) = 0 \quad A_1 = 0$

$E_0(z=a) = 0 \quad A_2 = 0 \not\Rightarrow sh(\alpha a) \neq 0$

$\underline{\omega = kc}$, $E_0(z) = C_1 z + C_2 \Rightarrow$

$z=0 \quad C_2 = 0$

$z=a \quad C_1 a = 0 \Rightarrow C_1 = 0$ ~~✗~~

$\underline{\omega > kc}$: $\frac{\omega^2}{c^2} - k^2 = +\gamma^2$ R

$$r^2 + \gamma^2 = 0 \quad , \quad r = \pm i\gamma$$

$$E_0(z) = C_1 e^{i\gamma z} + C_2 e^{-i\gamma z}$$

$$= A_1 \cos(\gamma z) + A_2 \sin(\gamma z)$$

$$= B_1 \sin(\gamma z + \varphi)$$

$z=0$; $\sin\varphi = 0$; $\varphi = 0$

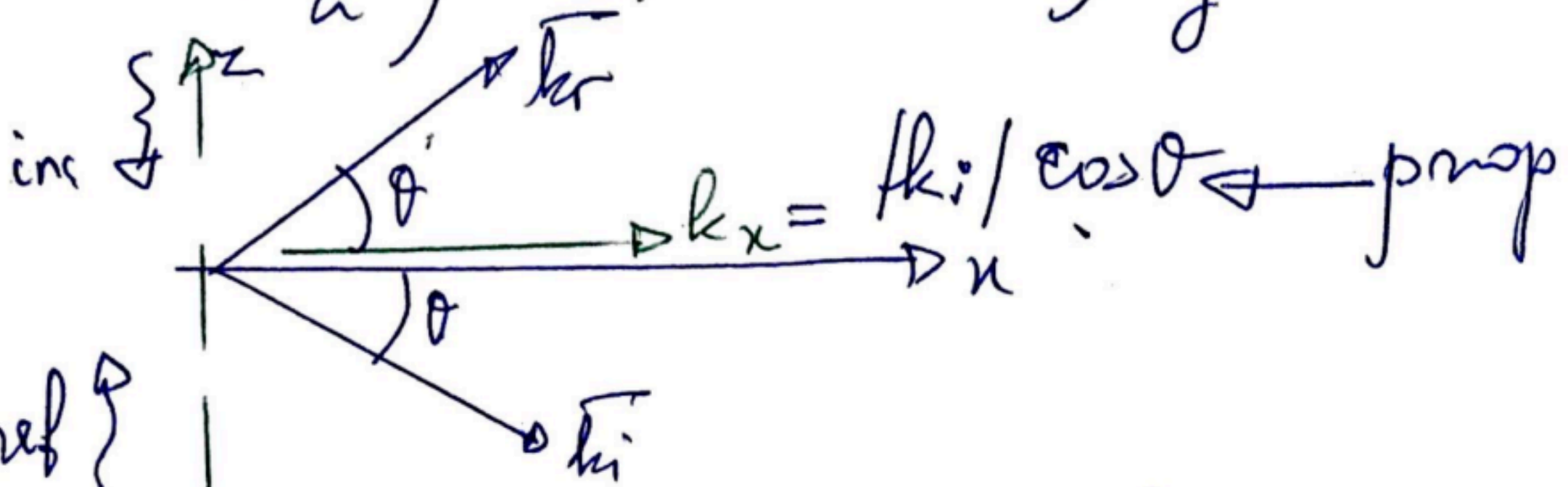
$z=a$; $\sin(\gamma a) = 0$

$\gamma a = n\pi \Rightarrow \gamma = \frac{n\pi}{a}$)

$$E_0(z) = B_1 \sin\left(\frac{n\pi z}{a}\right)$$

on pose $B_1 = E_0$

$$\vec{E}_n = E_0 \sin\left(\frac{n\pi z}{a}\right) \cos(\omega t - kx)\, \vec{u}_y$$



$$k_z = |k_i| \sin\theta \longleftarrow \text{stationnaire}$$

$$\text{or} \left(\frac{\omega^2}{c^2} - k^2\right) = \Omega^2 = \left(\frac{n\pi}{a}\right)^2$$

$$k^2 = \left(\frac{\omega^2}{c^2} - \frac{n^2\pi^2}{a^2}\right)$$

onde se propage : $\underline{k^2 > 0}$ $\quad \omega > \frac{n\pi}{a} c$

$$\Rightarrow f > c\frac{\pi n}{a 2\pi} = n\left(\frac{c}{2a}\right) = n f_1$$

$$f > f_{n\,\text{coupure}} = n f_1$$

$\vec{E}_1$, $\exists$ et $\vec{E}_2 \nexists$

$$f_1 < f_{GBF} < f_2 < f_3.$$

3) ~~$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega}$~~ حل آخر

في guide / mode

$$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{vmatrix} \frac{\partial}{\partial x} \\ 0 \\ \frac{\partial}{\partial z} \end{vmatrix} \wedge \begin{vmatrix} E_y(x,z,t) \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} -\frac{\partial E}{\partial z} \\ 0 \\ \frac{\partial E}{\partial x} \end{vmatrix} = \begin{vmatrix} -\frac{\partial B_x}{\partial t} \\ -\frac{\partial B_y}{\partial t} \\ -\frac{\partial B_z}{\partial t} \end{vmatrix}$$

* $\frac{\partial B_x}{\partial t} = + E_0 \frac{n\pi}{a} \cos\left(\frac{n\pi z}{a}\right) \cos(\omega t - kx)$

$B_x = \frac{E_0 \, n\pi}{\omega a} \cos\left(\frac{n\pi z}{a}\right) \cdot \sin(\omega t - kx)$ $\quad \vec{B_x} \parallel \vec{k}$

$B_y = 0$. pas de sources statiques longit

* $\frac{\partial B_z}{\partial t} = - E_0 \, k \sin\left(\frac{n\pi z}{a}\right) \sin(\omega t - kx)$

$B_z = + \frac{E_0 k}{\omega} \sin\left(\frac{n\pi z}{a}\right) \cos(\omega t - kx)$ $\quad \vec{B_z} \perp \vec{k}$: tranvers

guide d'onde T.E

ou T.M

$\vec{E} \perp \vec{k}$

$\Rightarrow \theta (T.E)$

$\vec{B} = \begin{cases} \vec{B_x} \parallel \vec{k} \\ \vec{B_z} \perp \vec{k} \end{cases}$

$(T.EM) \Rightarrow B_x = 0 \; ; \; \cos\left(\frac{n\pi z}{a}\right) = 0$

$$\frac{n\pi z}{a} = m\pi + \frac{\pi}{2}$$

$$4)\ \vec{R} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{vmatrix} 0 & 0 & B_x \\ E_y & 0 & 0 \\ 0 & 0 & B_z \end{vmatrix}$$

$$= \frac{1}{\mu_0} \begin{vmatrix} E_y B_z \\ 0 \\ -E_y B_x \end{vmatrix}$$

$$\langle \vec{R} \rangle = \langle R_x \rangle \vec{u}_x$$

$$\langle R_z \rangle = 0.$$

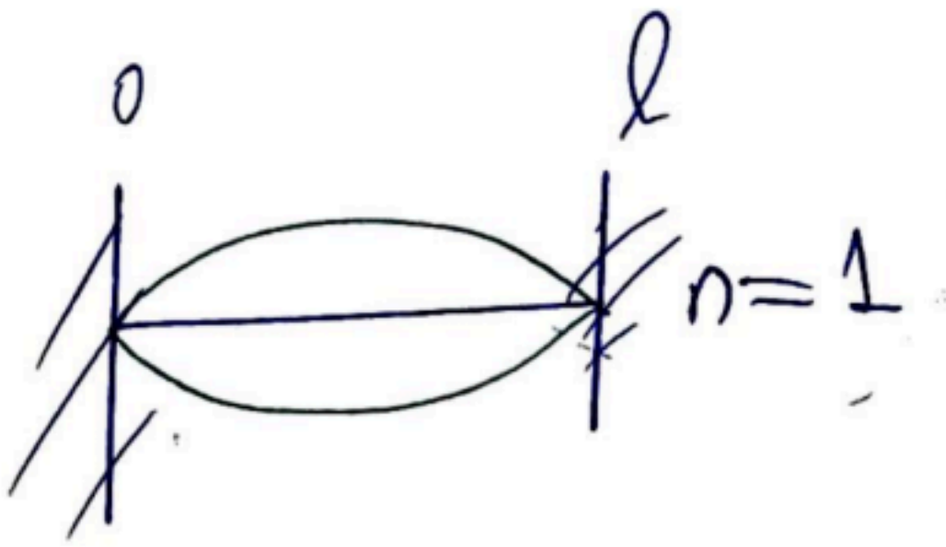
$$k^2 = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_c^2}{\omega^2}\right]$$

$$v_g v_\varphi = c^2$$

PT2 2011

Corde vibrante

$$\left.\begin{array}{l} y(0,t)=0 \\ y(l,t)=0 \end{array}\right\} \quad l=\frac{\lambda_1}{2}$$

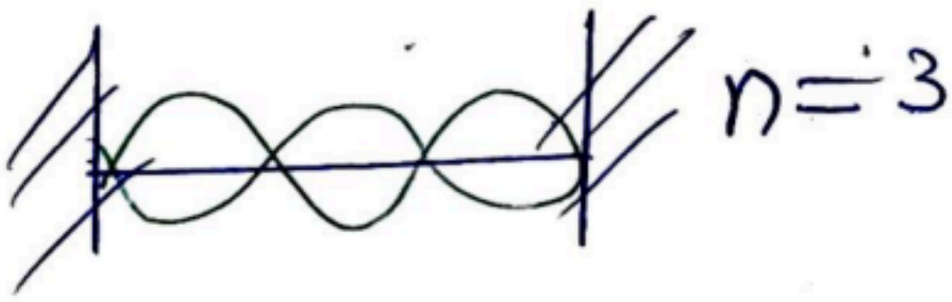


$$l=\lambda_2$$



$$\boxed{l=\frac{n\lambda_n}{2}}$$

$$l=\frac{3\lambda_3}{2}$$



$$\frac{\partial^2 y}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}=0 \quad ; \quad v=\sqrt{\frac{T}{\mu}}$$

$\mu =$ la masse linéïque.

$$y(x,t)=y_1(x-ct)+y_2(x+ct)$$

$$=f(x)\,g(t)$$

$$f''(x)\,g(t)=\frac{1}{v^2}\,g''(t)\,f(x)$$

$$\frac{f''(x)}{f(x)}=\frac{1}{v^2}\frac{g''(t)}{g(t)}=K<0$$

$$K = -k^2 < 0$$

$$\frac{f''(x)}{f(x)} = -k^2 \; ; \quad f''(x) + k^2 f(x) = 0$$

$$f(x) = f_0 \sin(kx + \varphi)$$

$$x = 0 \Rightarrow \sin\varphi = 0 \qquad \boxed{\varphi = 0}$$

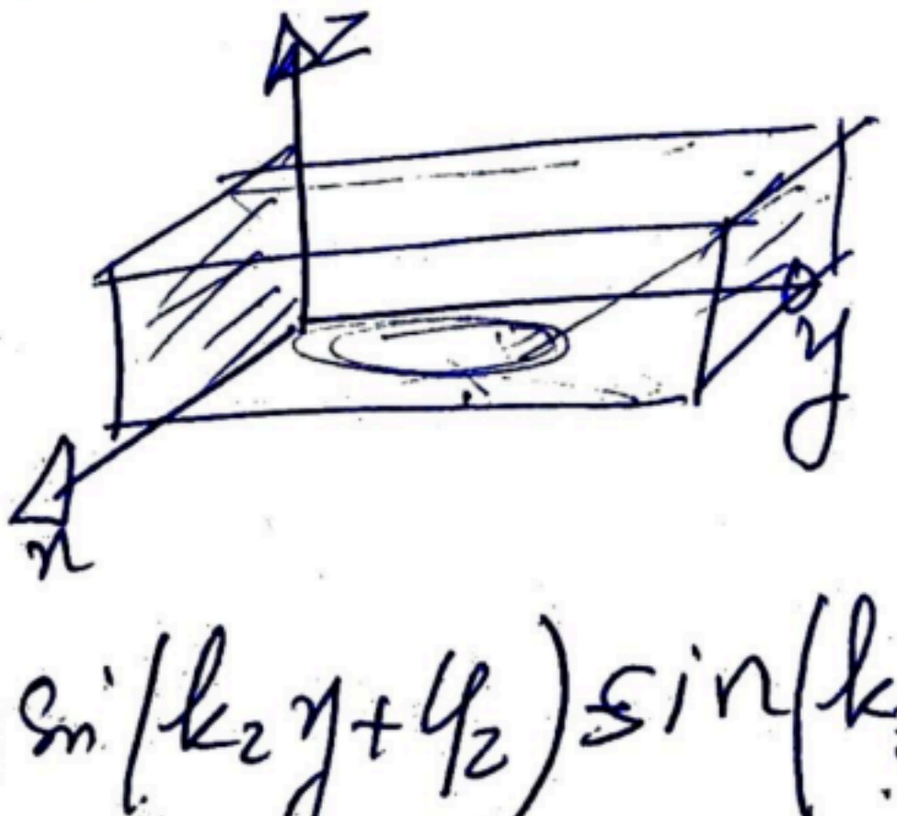
$$x = l \Rightarrow \sin(kl) = 0 \qquad kl = n\pi$$

$$k_n = \boxed{\frac{n\pi}{l} = \frac{2\pi}{\lambda_n}} = \frac{\omega_n}{v} = \frac{2\pi f_n}{v}$$

$$\boxed{l = \frac{n\lambda_n}{2}}$$

Cavité

$a < b < c$



$$E_x = E_1 \cos(k_1 x + \varphi_1) \sin(k_2 y + \varphi_2) \sin(k_3 z + \varphi_3) e^{j\omega t}$$

1) $\vec{\Delta}\vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$

$$k_1 = \frac{m\pi}{a} \qquad \varphi_1 = 0$$

$$k_2 = \frac{n\pi}{b} \qquad \varphi_2 = 0$$

$$k_3 = \frac{p\pi}{c} \qquad \varphi_3 = 0$$

$$\left(\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{c^2}\right)\pi^2 = \frac{\omega^2}{c^2}$$

$$f = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{c^2}}$$

$$f_{m,n,p}$$

Sachant $a < b < c$

$$\frac{1}{a} > \frac{1}{b} > \frac{1}{c}$$

$$f_{min} = f_{0,1,1}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_x(x,y,z,t)=0$$

$$\left[-k_1^2-k_2^2-k_3^2-\frac{1}{c^2}(-\omega^2)\right]E_x=0$$

$$\boxed{k_1^2+k_2^2+k_3^2=\frac{\omega^2}{c^2}} \quad (1)$$

$$\vec{\nabla}\left|\begin{matrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{matrix}\right.$$

$$div\vec{E}=0 \Rightarrow \vec{\nabla}\cdot\vec{E}=0$$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0.$$

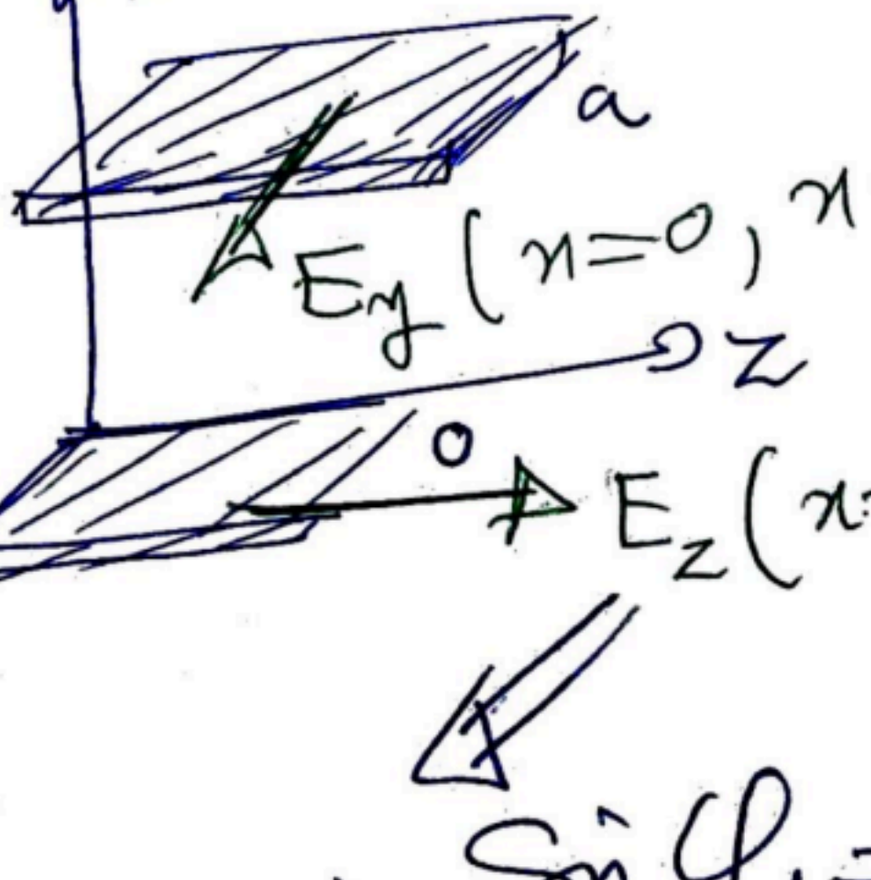
$$\boxed{k_1E_1+k_2E_2+k_3E_3=0} \quad (2)$$

$$E_{2T}=E_{1T}=0$$

métal parfait

$x=a;$

2) $x=0$



$$E_y(x=0,x=a,y,z,t)=0$$

$$E_z(x=0,x=a,y,z,t)=0$$

$$x=0 \Rightarrow \sin\varphi_1=0 \qquad \varphi_1=0$$

$$x=a \Rightarrow \sin(k_1a)=0 \qquad k_1a=m\pi$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \begin{vmatrix} \frac{\partial}{\partial x} & & E_x \\ \frac{\partial}{\partial y} & \wedge & E_y \\ \frac{\partial}{\partial z} & & E_z \end{vmatrix}$$

$$W_e = \iiint \frac{\varepsilon_0}{2} \left(E_x^2 + E_y^2 + E_z^2 \right) dx\,dy\,dz$$

$$\iiint \frac{\varepsilon_0 E_x^2}{2} dx\,dy\,dz = \frac{\varepsilon_0}{2} E_1^2 \int_0^a \cos^2\left(\frac{n\pi x}{a}\right) dx \times \cos^2(\omega t)$$

$$\int_0^b \sin^2\left(\frac{n\pi y}{b}\right) dy \times \int_0^c \sin^2\left(\frac{n\pi z}{c}\right) dz$$

$$= \frac{\varepsilon_0 E_1^2}{2} \cdot \frac{a}{2} \cdot \frac{b}{2} \cdot \frac{c}{2} \cos^2(\omega t)$$

$$\frac{1}{a} \int_0^a \cos^2\left(\frac{n\pi x}{a}\right) dx = \frac{1}{2}$$

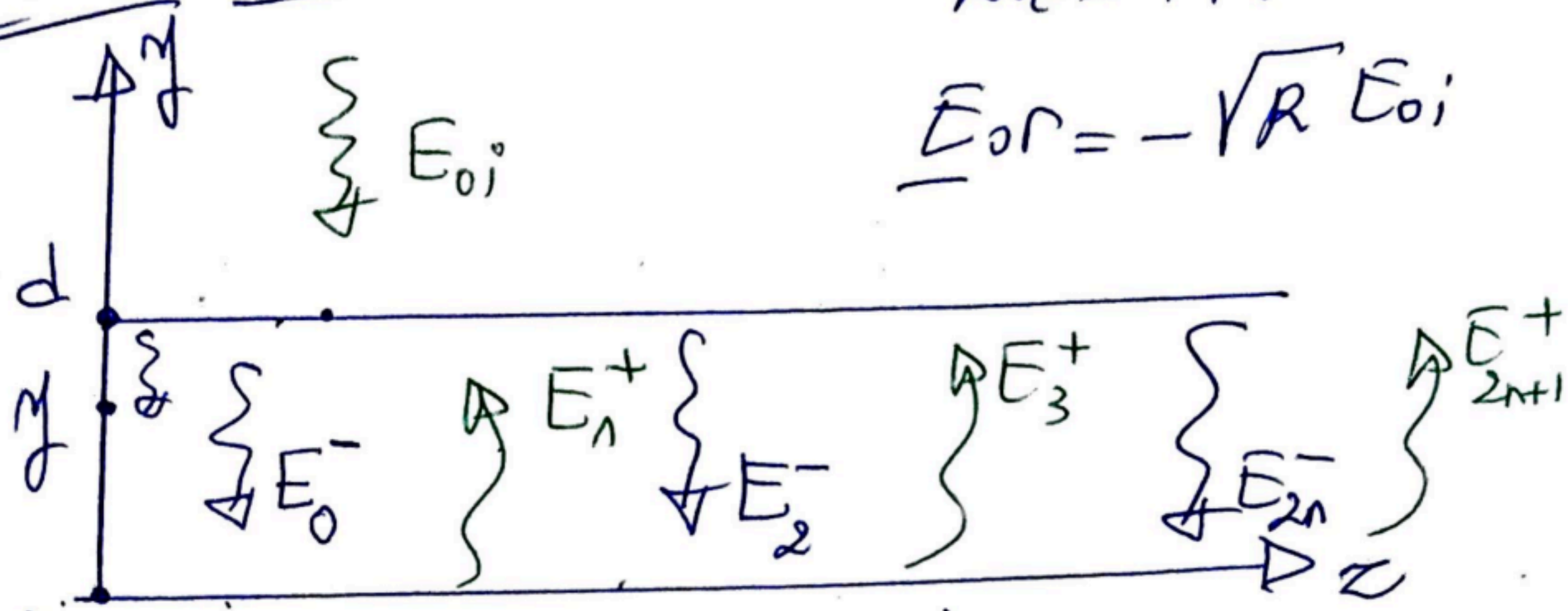
$$\langle W_e \rangle = \frac{\varepsilon_0}{2} \left(E_1^2 + E_2^2 + E_3^2 \right) \frac{abc}{8} \times \frac{1}{2}$$

Cavité résonante

MP2 2004

$$E_{ot} = +\sqrt{T}\, E_{oi}$$

$$E_{or} = -\sqrt{R}\, E_{oi}$$



$$\vec{E}_{incidente} = E_{oi}\, e^{i(\omega t)}\, \vec{u}_x$$

$$\vec{E}^- = E_{oi}\sqrt{T}\, e^{i(\omega t - \vec{k}\vec{r})}\, \vec{u}_x$$

$$\vec{k} = -\frac{2\pi}{\lambda}\vec{u}_y \qquad \vec{r} = \begin{vmatrix} x \\ -(d-y) \\ z \end{vmatrix}$$

$$\vec{E}_0^- = E_{oi}\sqrt{T}\, e^{i(\omega t - k(d-y))}\, \vec{u}_x$$

$$\vec{E}_0^- = E_{oi}\sqrt{T}\left(e^{-ikd}\right) e^{i(\omega t + ky)}\, \vec{u}_x \quad (oy)^-$$

$$\vec{E}_0^-(y=0,t) = E_{oi}\sqrt{T}\, e^{(-ikd)}\, e^{i(\omega t + ky)}\, \vec{u}_x$$

$$\vec{E}_1^{+} = \left(E_{0i}\sqrt{T}\right)\left(-\sqrt{R}\right)e^{-ikd}\, e^{i(\omega t - ky)}\,\vec{u}_n$$

2) $$\vec{E}_2^{-} = \left(E_{0i}\sqrt{T}\right)\left(-\sqrt{R}\right)^2 e^{-i2kd}\, e^{i(\omega t - k(d-y))}$$

$$\vec{E}_2^{-} = \left(E_{0i}\sqrt{T}\right)\left(-\sqrt{R}\right)^2 e^{-i3kd}\, e^{i(\omega t + ky)}\,\vec{2}\vec{u}_n$$

3) $$\vec{E}_3^{+} = \left(E_{0i}\sqrt{T}\right)\left(-\sqrt{R}\right)^3 e^{-i3kd}\, e^{i(\omega t - ky)}\,\vec{4}\vec{u}$$

$$\vec{E}_{2n}^{-} = \left(E_{0i}\sqrt{T}\right)\left(-\sqrt{R}\right)^{2n} e^{-i(2n+1)kd}\, e^{i(\omega t + ky)}$$

$$\vec{E}_{2n+1}^{+} = \left(E_{0i}\sqrt{T}\right)\left(-\sqrt{R}\right)^{(2n+1)} e^{-i(2n+1)kd}\, e^{i(\omega t - ky)}$$

$$\vec{E}^{-} = \sum \vec{E}_{2n}$$

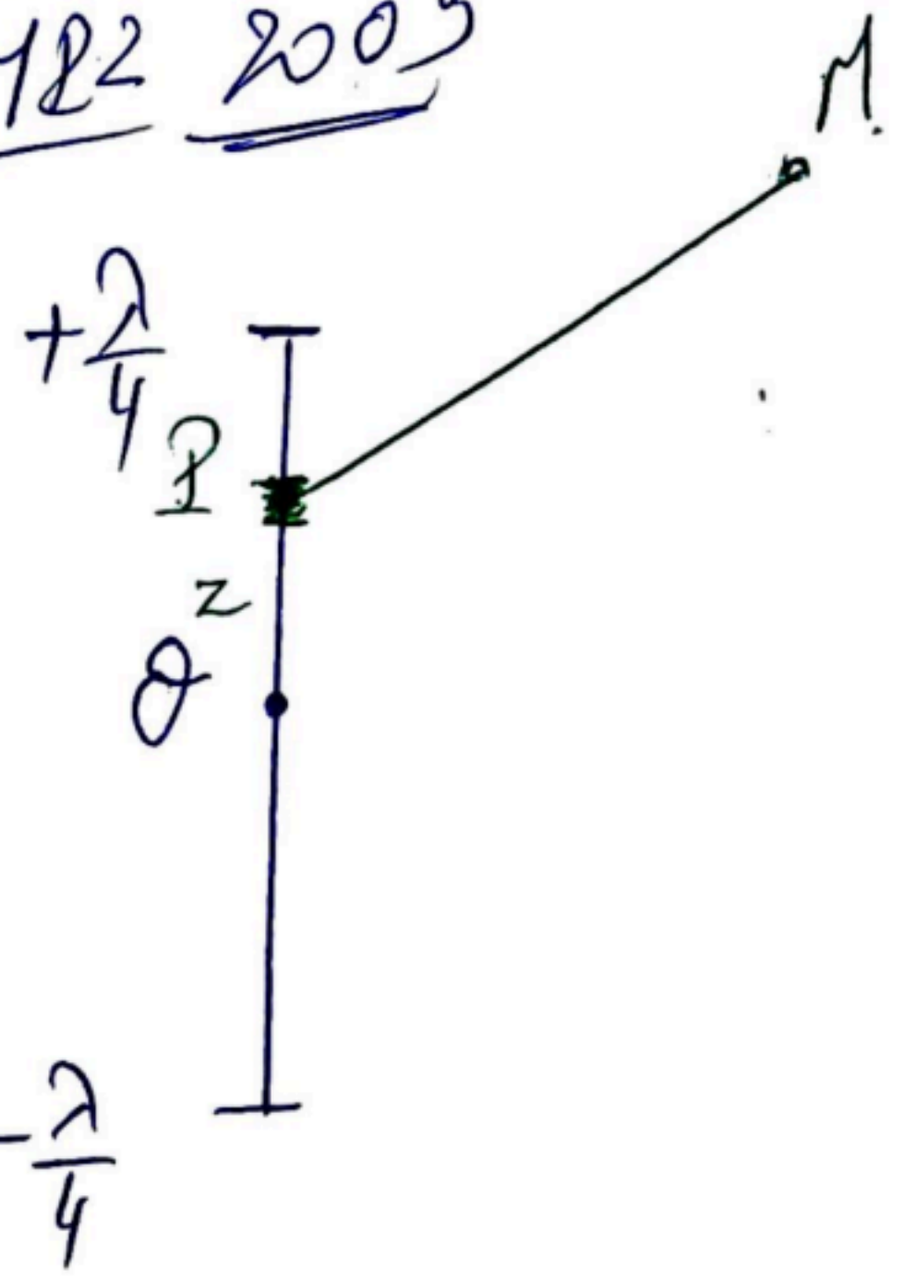
$$\sum_{0}^{\infty} R^n \left(e^{-i2kd}\right)^n = \frac{1}{1 - Re^{-i2kd}}$$

$R < 1$ $\qquad R + T = 1$

Antennes : RT2 2013
M22 2003

M22 2016

$$i(z,t) = I_0 \cos(\omega t) \cos\left(\frac{2\pi z}{\lambda}\right)$$



$$l = \frac{\lambda}{2}$$

$$OP = z$$

$$\delta p = q(z,t)\, dz$$

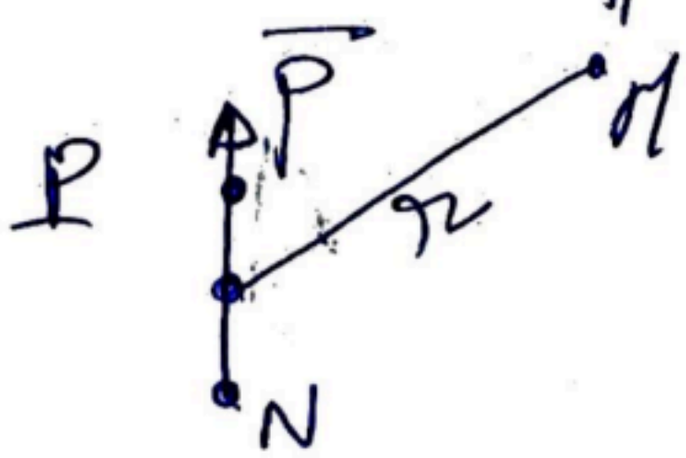
$$\delta p = \int i(z,t)\, dt \; dz$$

$$\delta p = \frac{I_0}{\omega} \sin(\omega t) \cos\left(\frac{2\pi z}{\lambda}\right) dz$$

$$\underline{\delta p} = \frac{I_0}{\omega} \cos\left(\frac{2\pi z}{\lambda}\right) e^{j\left(\omega t - \frac{\pi}{2}\right)} dz$$

$$\delta p = -j \frac{I_0}{\omega} \cos\left(\frac{2\pi z}{\lambda}\right) e^{j\omega t}\, dz$$

$$\vec{B}_{dipôle} = \frac{\mu_0 \ddot{\vec{p}}\left(t - \frac{r}{c}\right)}{4\pi c r} \sin\theta\, \vec{e}_{\theta\varphi}$$



$$\overrightarrow{dB} = \frac{\mu_0 \sin\theta}{4\pi c(PM)} \left[\frac{\partial^2(\delta p)}{\partial t^2}\right]_{\left(t-\frac{PM}{c}\right)} \overrightarrow{u_\varphi}$$



$$(PM)^2 = z^2 + r^2 - 2rz\cos\theta$$

$$(PM)^2 = r^2\left[1 + \frac{z^2}{r^2} - \frac{2z}{r}\cos\theta\right]$$

$$PM = r\left[1 + \frac{z^2}{2r^2} - \frac{z}{r}\cos\theta\right]$$

$$PM \simeq r\left[1 - \frac{z}{r}\cos\theta\right] = r - z\cos\theta$$

$$\frac{1}{PM} = \frac{1}{r}\left(1 + \frac{z}{r}\cos\theta\right) = \frac{1}{r} + \frac{z}{r^2}\cancel{\cos\theta}$$

$$\overrightarrow{dB} = \frac{\mu_0 j \sin\theta}{4\pi c r} I_0 \omega \cos\left(\frac{2\pi z}{\lambda}\right) e^{j(\omega t - \frac{\omega}{c}r + \frac{\omega z}{c}\cos\theta)} dz$$

$$\overrightarrow{B} = \frac{\mu_0 j \sin\theta}{4\pi c r} I_0 \omega \, e^{j(\omega t - kr)} \int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \cos\left(\frac{2\pi z}{\lambda}\right) e^{jkz\cos\theta} dz$$

$$\overrightarrow{B} = \frac{\vec{k} \wedge \vec{E}}{\omega}$$