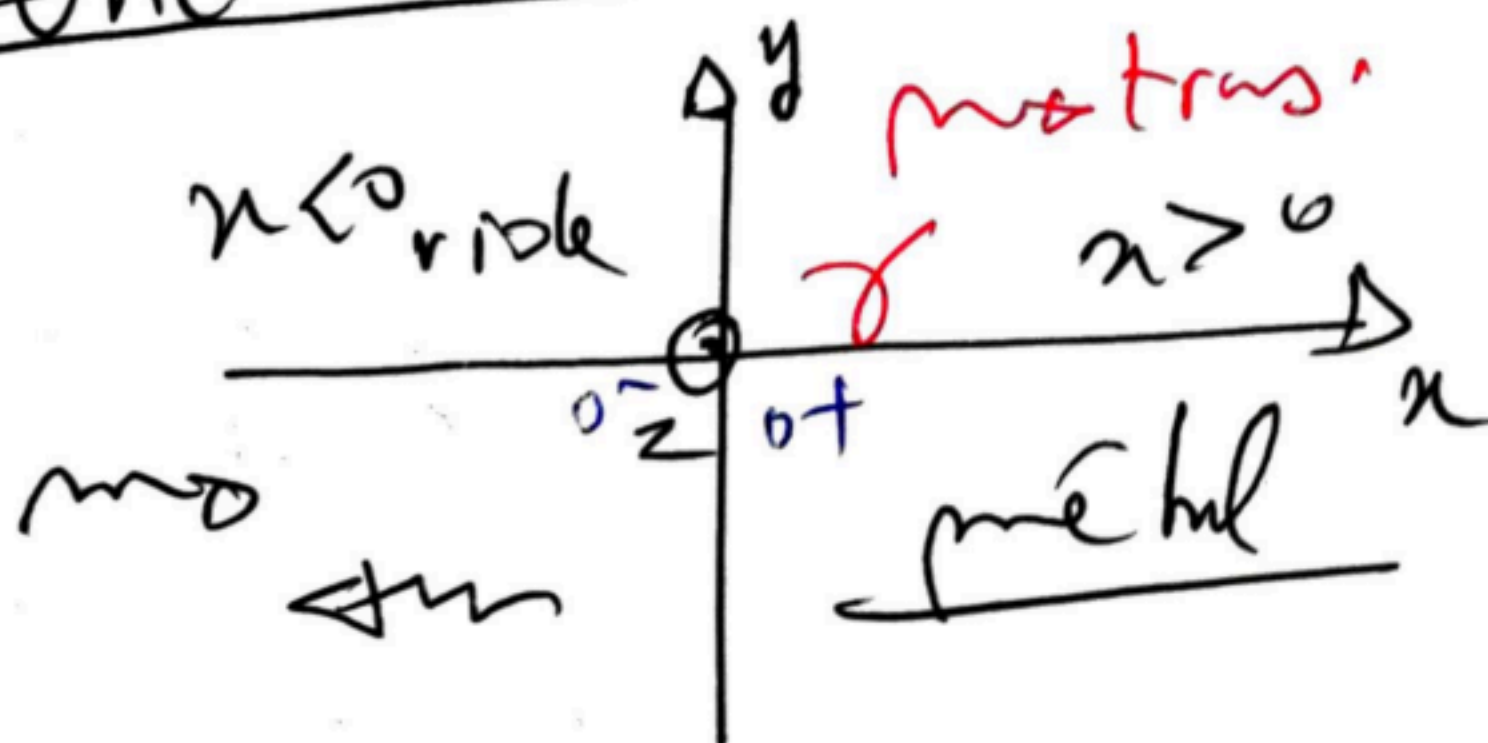


Serie N°2

$$\vec{E}(0,t) = E_0 e^{i\omega t} \vec{e}_y$$



$$\vec{n}_{12} \wedge (\vec{E}_2 - \vec{E}_1) = \vec{0} \Rightarrow E_{2T} = E_{1T}$$

$$\vec{E}_2 - \vec{E}_1 = \frac{\nabla}{\delta} \vec{n}_{12}$$

$$1) \frac{\|\vec{J}\|}{\|\vec{j}\|} \leq 10^{-3} \Rightarrow \left| \frac{\epsilon \frac{\partial \vec{E}}{\partial t}}{\gamma \vec{E}} \right| = \left| \frac{i\omega\epsilon}{\gamma} \right| \leq 10^{-3}$$

$$f \leq f_0 = \frac{\gamma 10^{-3}}{2\pi\epsilon} = 10^{18} \text{ Hz}$$

$$f_0 = 10^{18} \text{ Hz};$$

Hertzienno; IR, visibles, UV proches;
 10^{14} Hz

$$\vec{\omega} \wedge \vec{B} \simeq \mu_0 \gamma \vec{E}$$

UV lointains, RX; $R\gamma$:

$$\vec{\omega} \wedge \vec{B} \simeq \mu_0 \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$2) \operatorname{div} \vec{E} = \frac{\rho}{\epsilon_0} \quad , \quad \left. \begin{array}{l} \operatorname{div} \vec{J}_c + \frac{\partial \rho}{\partial t} = 0 \\ \longrightarrow \rho = \rho_0 e^{+t/\tau} \end{array} \right\}$$

$\operatorname{div} \vec{E} = 0$ le τ localité neutre

$$\boxed{\Delta \vec{E} - \mu_0 \gamma \frac{\partial \vec{E}}{\partial t} = \vec{0}}$$

$$\Delta \vec{B} - \mu_0 \gamma \frac{\partial \vec{B}}{\partial t} = \vec{0}$$

$$3) \vec{E} = \underline{E}_1(x) e^{i\omega t} \vec{u}_y$$

$$\left[\frac{d^2 \underline{E}_1(x)}{dx^2} - i \mu_0 \gamma \omega \underline{E}_1(x) \right] = 0$$

$$x^2 - i \mu_0 \gamma \omega = 0$$

$$x^2 = i \mu_0 \gamma \omega = \frac{(1+i)^2}{\left(\frac{\omega}{\mu_0 \gamma \omega} \right)} = \frac{(1+i)^2}{8}$$

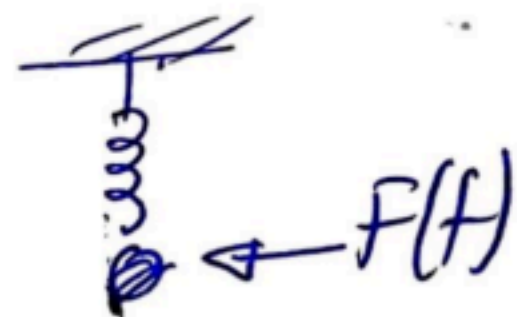
$$\delta = \frac{2}{\sqrt{\mu_0 \gamma \omega}}$$

$$x = \pm \left(\frac{1+i}{8} \right)$$

$$\underline{E}_1(n) = \alpha \exp\left(-\frac{(1+i)n}{\delta}\right) + \beta \exp\left(+\frac{(1+i)n}{\delta}\right)$$

$$\underline{E}_1(n) = \alpha \exp\left(-\frac{(1+i)n}{\delta}\right)$$

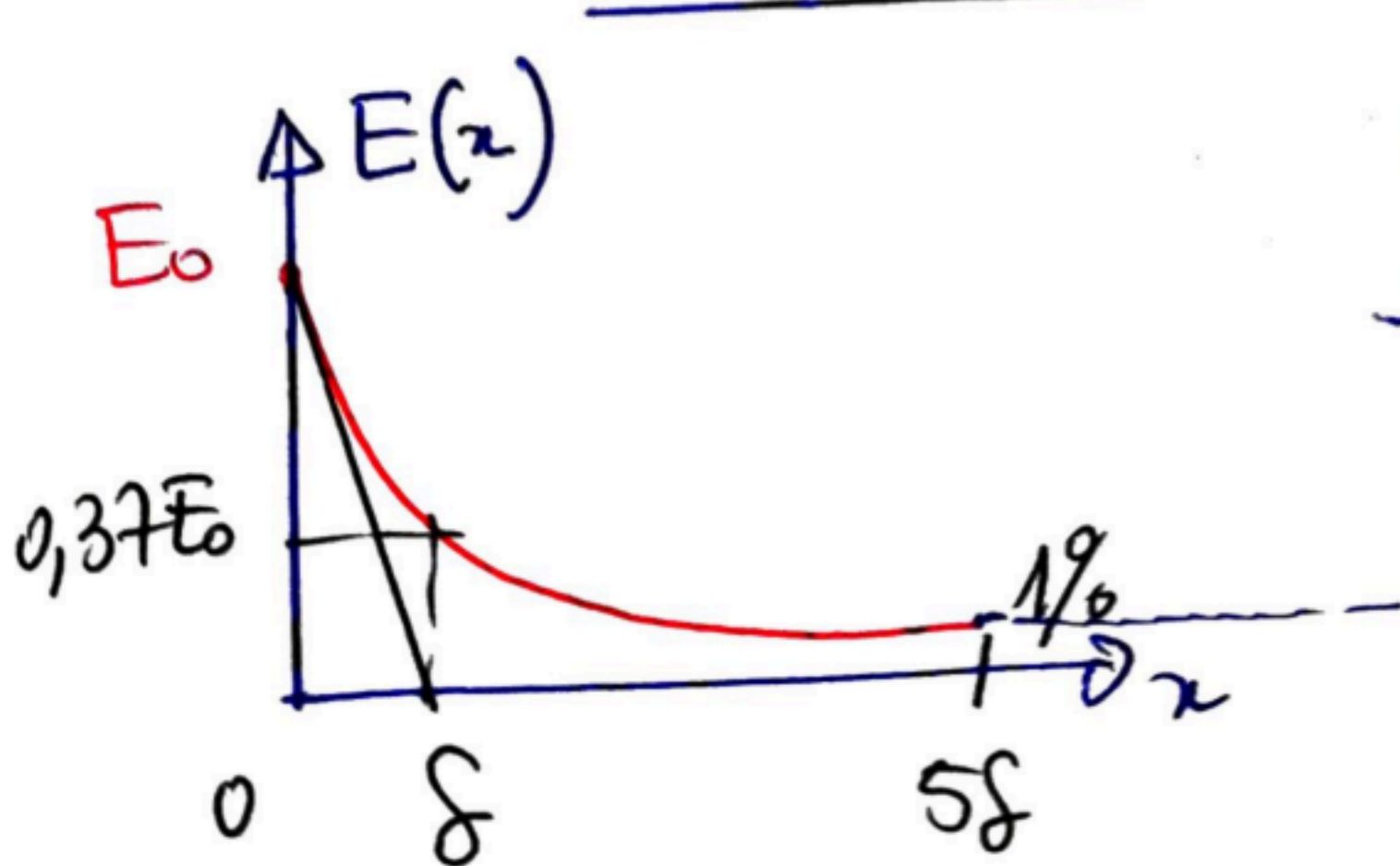
$$\underline{\beta=0} ; \quad \lim_{n \rightarrow \infty} \underline{E}_1(n) = 0$$



$$\begin{aligned} \underline{E}(n,t) &= E_0 \exp\left(-\frac{(1+i)n}{\delta}\right) e^{i(\omega t)} \vec{u}_y \\ &= E_0 e^{-\frac{n}{\delta}} e^{i(\omega t - \frac{n}{\delta})} \vec{u}_y \end{aligned}$$

$$\underline{E}(n,t) = E_0 e^{-k''n} e^{i(\omega t - k'n)} \vec{u}_y$$

$$\underline{k} = k(\omega) ; \quad k'' = \frac{1}{\delta} - i \frac{1}{\delta}$$

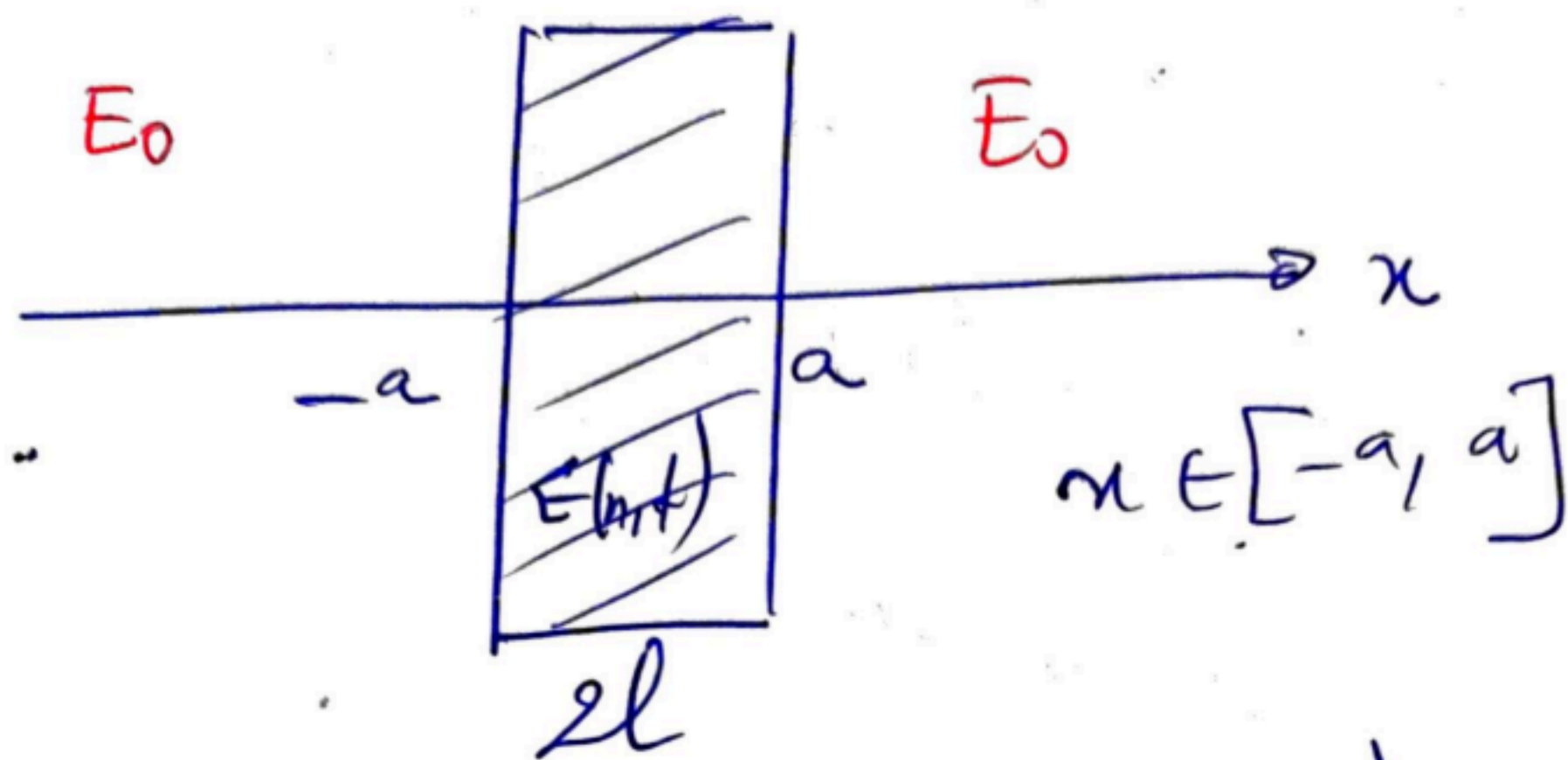


$$\int_0^{5\delta} \rightarrow \int_0^{\infty}$$

$$\delta = \sqrt{\frac{2}{\mu_0 \gamma \omega}}$$

↳ profondeur de pénétration

$\delta \propto \omega^{-1/2}$
 effet de peau ou
 effet pellucide



$$\underline{E}_1(x) = \alpha e^{-\frac{(1+i)x}{\delta}} + \beta e^{+\frac{(1+i)x}{\delta}}$$

$$\Rightarrow \underline{E}_1(-x) = \underline{E}_1(x) \Rightarrow \boxed{\alpha = \beta}$$

$$\underline{E}_1(x) = \alpha \left[2 \cosh\left(\frac{(1+i)x}{\delta}\right) \right]$$

$$E_1(a) = 2\alpha \cosh\left(\frac{(1+i)a}{\delta}\right) = E_0$$

$$\underline{E}_1(x) = \frac{E_0 \cosh\left(\frac{(1+i)x}{\delta}\right)}{\cosh\left(\frac{(1+i)a}{\delta}\right)}$$

M22

2021

$\frac{p_y}{\cos(\omega t)} \rightarrow e^{i\omega t}$
 $\cos^2(\omega t) \rightarrow ??$



$$\underline{k} = \frac{1}{\delta} - i \frac{1}{\delta}$$

$$\underline{k} = \frac{\sqrt{2} e^{-i\frac{\pi}{4}}}{\delta}$$

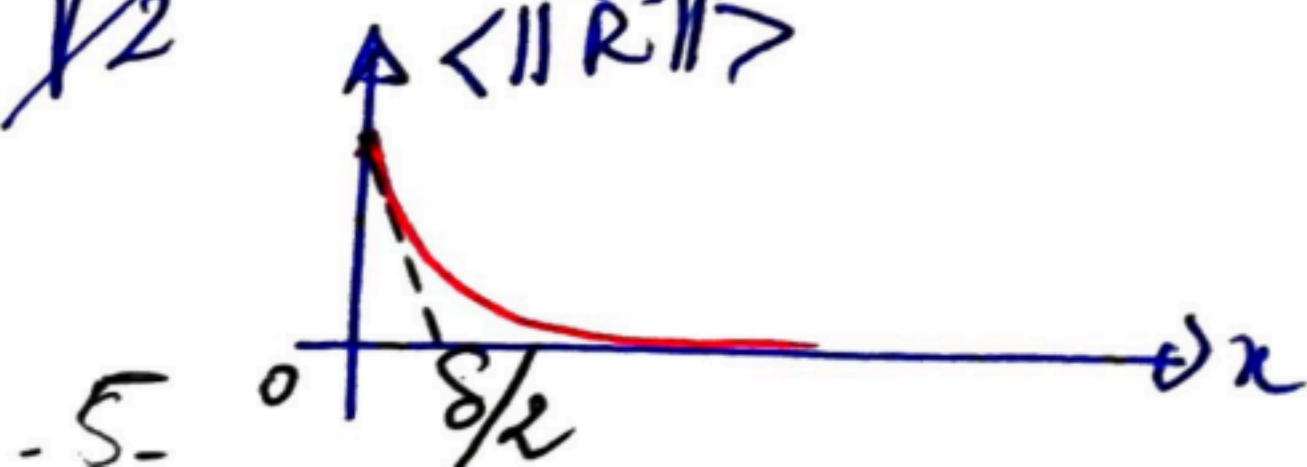
$$b) \underline{B} = \frac{\underline{k} \wedge \underline{E}}{\omega} = \frac{\mu_0 \sqrt{2}}{\delta \omega} E_0 e^{-\frac{x}{\delta}} e^{i(\omega t - \frac{x}{\delta} - \frac{\pi}{4})} \underline{u}_z$$

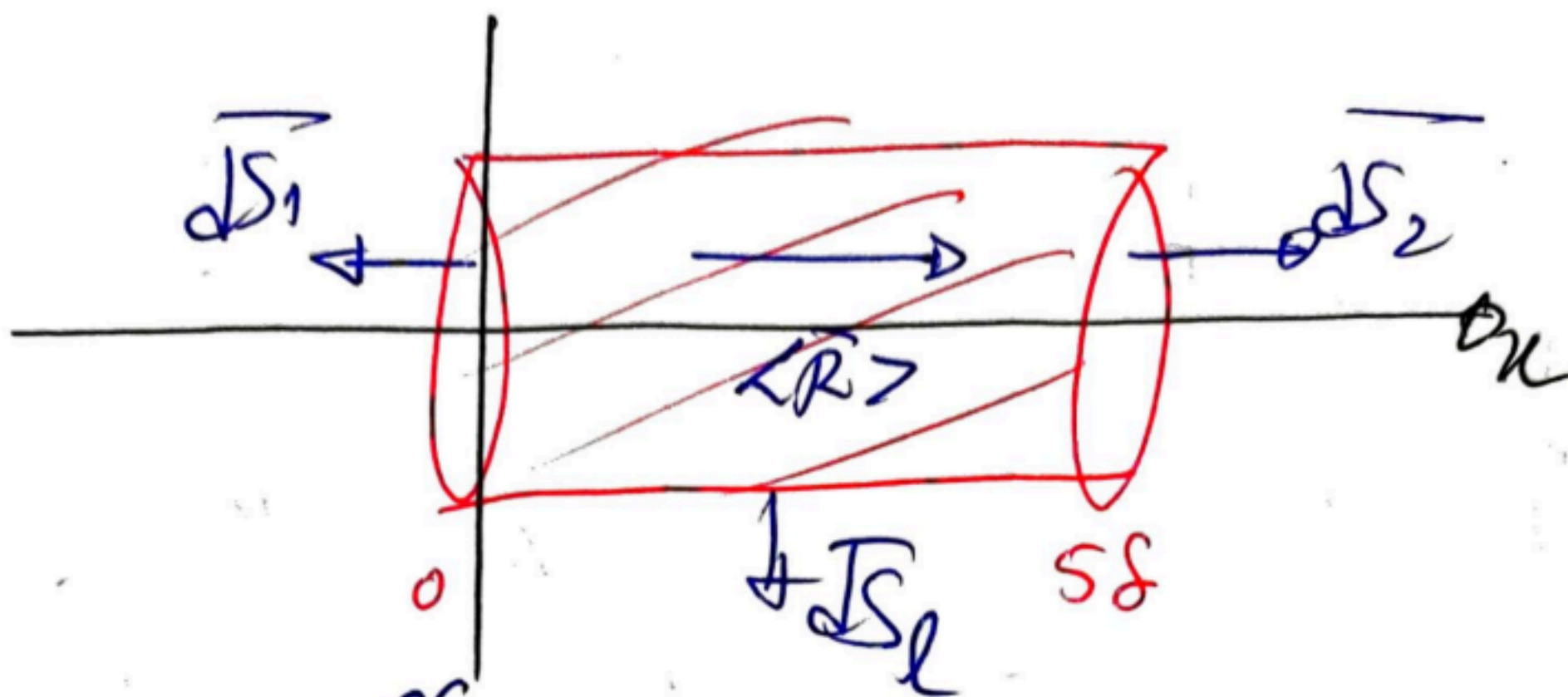
$$c) \underline{R} = \frac{\text{Re}(\underline{E}) \wedge \text{Re}(\underline{B})}{\omega \mu_0}$$

$$= \frac{E_0^2}{\omega \mu_0} \frac{\sqrt{2}}{\delta \omega} e^{-\frac{2x}{\delta}} \cos(\omega t - \frac{x}{\delta} - \frac{\pi}{4}) \underline{u}_x$$

$$\underline{R} = \frac{E_0^2 \sqrt{2}}{\delta \omega \mu_0} \frac{1}{2} \left[\cos(2(\omega t - \frac{x}{\delta} - \frac{\pi}{4})) + \cos(\frac{\pi}{4}) \right] \times \exp(-\frac{2x}{\delta}) \underline{u}_x$$

$$\langle \underline{R} \rangle = \frac{E_0^2}{\delta \omega \mu_0} \frac{1}{2} \frac{\sqrt{2}}{\sqrt{2}} e^{-\frac{2x}{\delta}} \underline{u}_x$$





$$\langle P_{\text{form}} \rangle = \oint \langle \vec{R} \rangle d\vec{S}$$

$$= - \int \langle \|\vec{R}(0, t)\| \rangle dS + \int \langle R(x=5\delta, t) \rangle dS$$

$$= \frac{-E_0^2 S}{2\mu_0 \omega} + \frac{E_0^2 S}{2\mu_0 \omega} e^{-10}$$

$$= - \frac{E_0^2 S}{2\mu_0 \omega} = - \langle P_{\text{oule}} \rangle$$

$\frac{P_g}{\hbar} \langle U_{\text{em}} \rangle =$ ne dépend pas du temps

$$\frac{d\langle U_{\text{em}} \rangle}{dt} = - \oint \langle \vec{R} \rangle d\vec{S} - \iint (\vec{J} \cdot \vec{E}) d\tau$$

0

4) $f = 36 \text{ Hz} = 3 \cdot 10^9 \text{ Hz}$

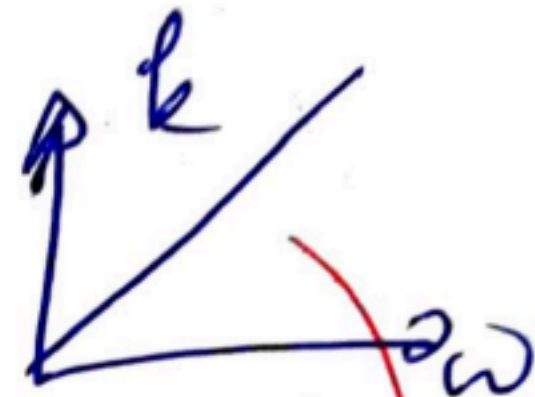
K	η	G	T
10^3	10^6	10^3	10^{12}

viola : $\|\vec{B}\| \leq \frac{\|\vec{E}\|}{c}$

$v_g = c = v_p$;

$k = \frac{\omega}{c}$

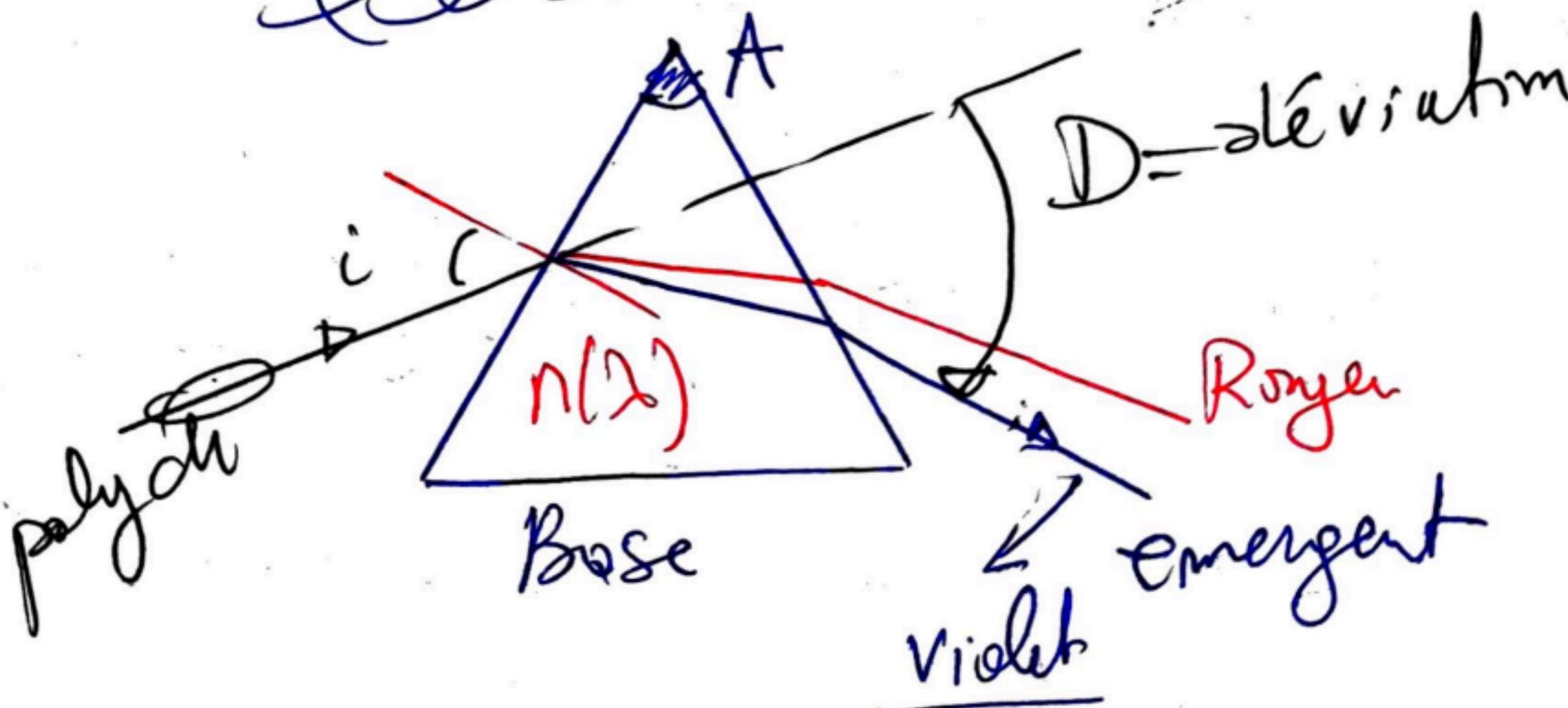
Non dispersif



v_g : ne dépend pas de ω

Milieu Dispersif : exple : Prisme

$n(\lambda) = a + \frac{b}{\lambda^2}$: Cauchy

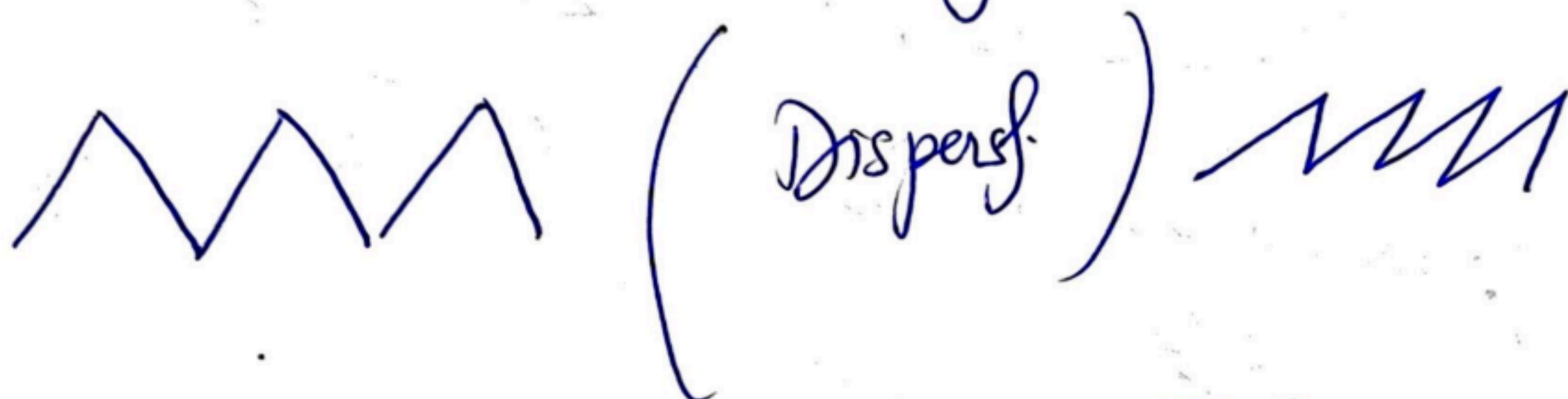


$$v_g = \frac{c}{n(\omega)}, \quad k = \frac{\omega}{c} n(\omega)$$

dépend de ω

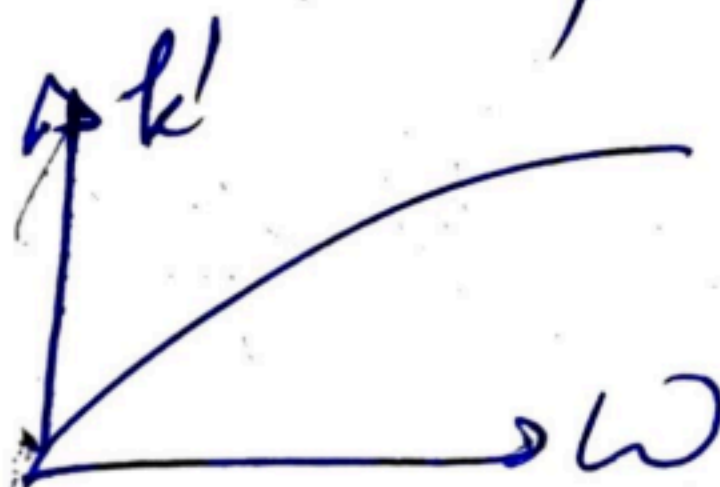
Non linéaire

Mélange et dispersion : distortion de forme du signal



métal $\|\vec{B}\| = \frac{\sqrt{2}}{\delta\omega} \|\vec{E}\|, \quad \epsilon_f = \frac{\pi}{4}$

$$k' = \frac{1}{\delta} = \sqrt{\frac{\mu_0 \gamma \omega}{2}}$$



$$v_g = \frac{\omega}{k'} = \omega \delta = \sqrt{\frac{2\omega}{\mu_0 \gamma}}$$

$$v_g = \frac{d\omega}{dk'}; \quad \frac{dk'}{d\omega} = \frac{1}{v_g} = \sqrt{\frac{\mu_0 \gamma}{2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\omega}}$$

$$\frac{1}{v_g} = \frac{1}{2} \frac{1}{v_g}; \quad \boxed{v_g = 2v_g}$$

$$v_p \cdot v_g = c^2 \quad \text{vide milieu transparent}$$

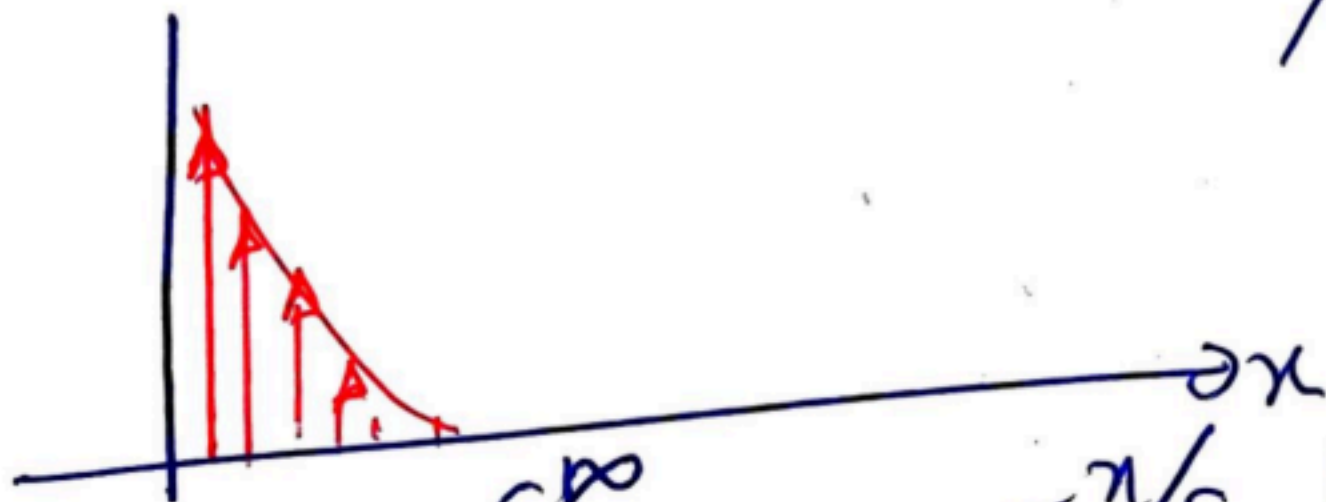
$$v_p \cdot v_g \neq c^2 \quad \text{m\u00e9tal " opaque}$$

$$k'' = 0 \quad \text{: non att\u00e9nu\u00e9e}$$

$$k'' \neq 0 \quad \text{att\u00e9nu\u00e9e}$$

$$5) a) \vec{J} = \gamma \vec{E} = (\gamma E_0) e^{-\gamma x} \cos(\omega t - \gamma x) \vec{u}_y$$

$\gamma \rightarrow A/m^2$



$$J_s = \int_0^{\infty} \gamma E_0 e^{-\gamma x} dx = (\gamma E_0) \delta \quad A/m$$

$$* \quad g(z) = \int_0^{\infty} e^{-z/\delta} \rightarrow \sigma = \int_0^{\infty} \rho_0 e^{-z/\delta} dz$$

$$\boxed{\sigma = \rho_0 \delta}$$

$$\vec{J}_s(t) = (\gamma E_0 \delta) \cos(\omega t)$$

5) b)

$$I(t) = \iint \gamma E_0 e^{-\gamma \delta} \cos(\omega t - \gamma \delta) dx dz$$

$$= \gamma E_0 z_0 \int_0^{\infty} e^{-\gamma \delta} \cos(\omega t - \gamma \delta) d\gamma \quad p(3)$$

$$= \gamma E_0 z_0 \text{ Réel} \left(\int_0^{\infty} e^{i(\omega t)} \times e^{-\left(1+i\right)\frac{\gamma}{\delta}} d\gamma \right)$$

$$= \gamma E_0 z_0 \text{ Réel} \left(e^{i\omega t} \int_0^{\infty} e^{-(1+i)\frac{\gamma}{\delta}} d\gamma \right)$$

$$= \gamma E_0 z_0 \text{ Réel} \left(e^{i\omega t} \frac{\delta}{(1+i)} \right)$$

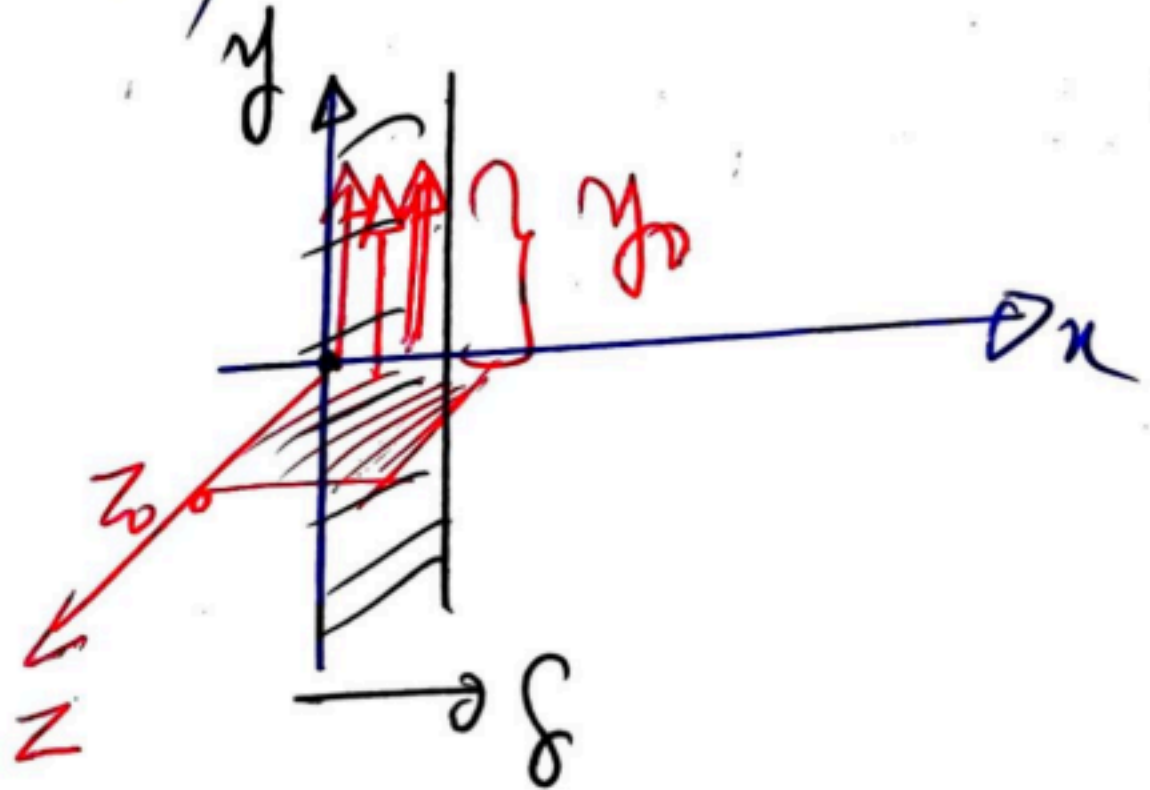
$$I(t) = \frac{\gamma E_0 z_0 \delta}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$I_{\text{eff}} = \frac{\gamma E_0 z_0 \delta}{2} = \sqrt{\langle I^2(t) \rangle}$$

$$= \sqrt{\frac{1}{T} \int I^2(t) dt}$$

$$\begin{aligned}
 c) \langle P_{\text{mle}} \rangle &= \iiint \langle \vec{j} \cdot \vec{E} \rangle \cdot dx dy dz \\
 &= \gamma_0 z_0 \int_0^\infty \langle \gamma E_0^2 e^{-2\gamma/\delta} \cos^2(\omega t - \gamma/\delta) \rangle dx \\
 &= \frac{\gamma_0 z_0 \gamma E_0^2}{2} \int_0^\infty e^{-2\gamma/\delta} dx \\
 &= \frac{\gamma_0 z_0 \gamma E_0^2}{2} \frac{\delta}{2} = \frac{1}{4} \langle P_{\text{mle}} \rangle
 \end{aligned}$$

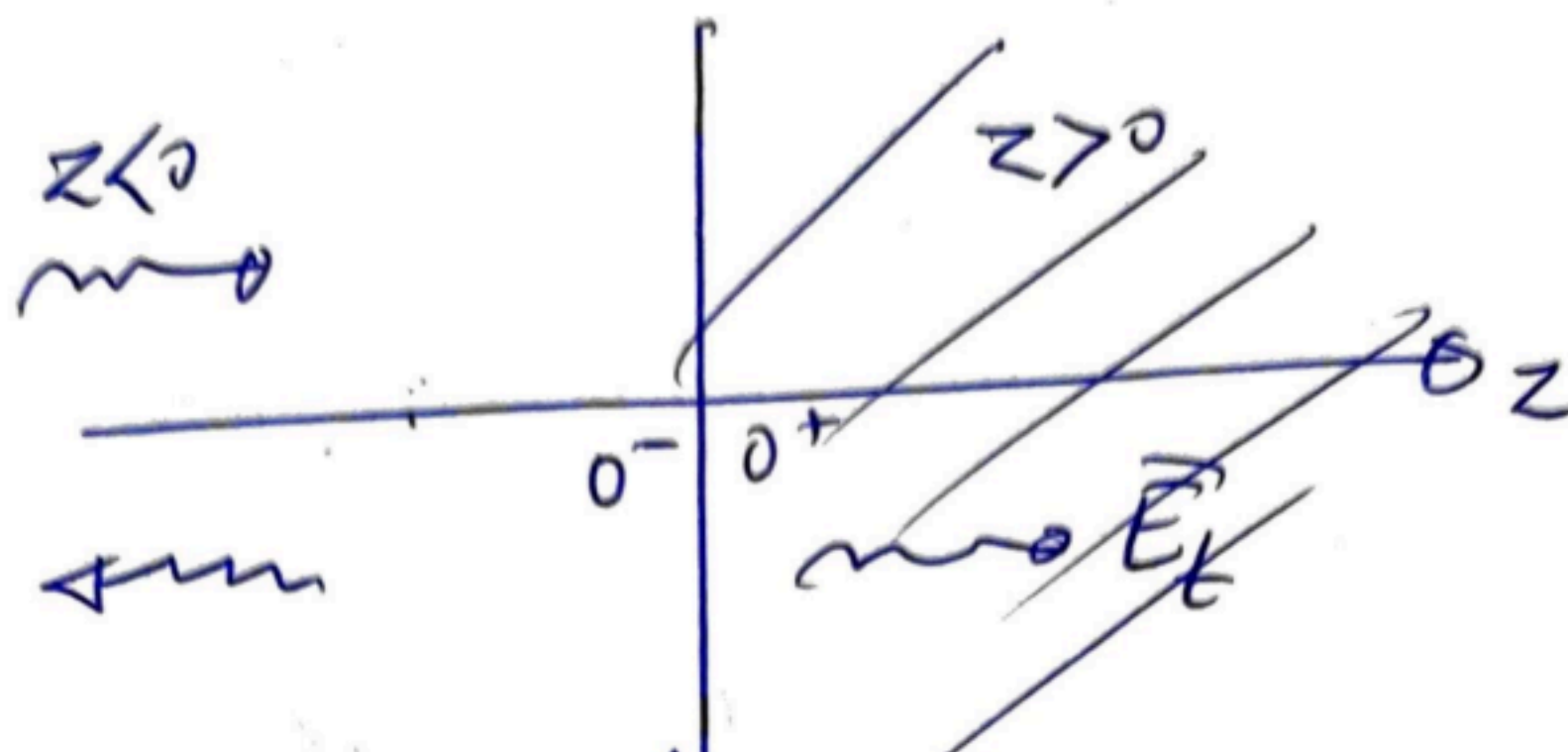
$$\begin{aligned}
 d) \eta &= \frac{\langle P_{\text{mle}} \rangle}{\langle I^2 \rangle} = \frac{\gamma_0 z_0 \gamma \delta}{4} \frac{4}{\gamma^2 z_0^2 \delta^2} \\
 &= \frac{1}{\gamma} \left(\frac{\gamma_0}{\delta z_0} \right)
 \end{aligned}$$



$$\begin{aligned}
 e) \langle U_m \rangle &= \\
 \frac{\langle U_m \rangle}{\langle I^2(t) \rangle} &= \frac{1}{2} L
 \end{aligned}$$

$$L = \frac{\gamma}{\omega}$$

Ex2



$$\vec{E}_t = E_0 e^{i(\omega t - k_z z)} \vec{u}_x$$

$$\textcircled{\text{Div}} \vec{E}_t = (-i k_z) \vec{E}_t = 0 \Rightarrow (T.E)$$

$\int_{-\infty}^{\infty}$

$$\nabla \times \vec{B} = \mu_0 \left(\gamma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\Delta \vec{E} - \mu_0 \gamma \frac{\partial \vec{E}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$$

$$\left(-k_z^2 - \mu_0 \gamma i \omega + \frac{\omega^2}{c^2} \right) \vec{E}_t = \vec{0}$$

$$k_z^2 = \frac{\omega^2}{c^2} - i \mu_0 \gamma \omega$$

$$k_z^2 = \frac{\omega^2}{c^2} \left[1 - \frac{i \gamma}{\epsilon_0 \omega} \right]$$

$$b) \underline{k}_2 = k' - i k''$$

$$k_2^2 = (k'^2 - k''^2) - i 2k'k''$$

$$\begin{cases} k'^2 - k''^2 = \omega^2/c^2 \\ 2k'k'' = \mu_0 \gamma \omega > 0 \end{cases}$$

$$2k'k'' = \mu_0 \gamma \omega > 0$$

$$|\underline{k}_2|^2 = |\underline{k}_2|^2$$

$$\sqrt{\frac{\omega^4}{c^4} + (\mu_0 \gamma \omega)^2} = k'^2 + k''^2$$

$$2k'^2 = \frac{\omega^2}{c^2} + \sqrt{\frac{\omega^4}{c^4} + (\mu_0 \gamma \omega)^2}$$

$$2k''^2 = -\frac{\omega^2}{c^2} + \sqrt{\frac{\omega^4}{c^4} + (\mu_0 \gamma \omega)^2}$$

$$2k'^2 = \frac{\omega^2}{c^2} \left[1 + \sqrt{1 + \left(\frac{\gamma}{\epsilon_0 \omega} \right)^2} \right]$$

$$\underline{\underline{\frac{\gamma}{\epsilon_0 \omega} \gg 1}} ; 2k'^2 = \frac{\omega^2}{c^2} \left(\frac{\gamma}{\epsilon_0 \omega} \right) = \omega \gamma \mu_0$$

$$k' = \frac{1}{\delta}$$

$$2) b) \frac{\gamma}{\epsilon_0 \omega} = \frac{\gamma}{\epsilon_0 2\pi \nu} > \frac{\gamma}{2\pi \epsilon_0 \nu_0} = 108$$

$$\frac{\gamma}{\epsilon_0 \omega} \gg 1 \Rightarrow \left| \frac{\gamma E}{i \epsilon_0 \omega E} \right| = \left| \frac{\gamma_c}{jD} \right| \gg 1$$

$$\sqrt{\rho} \bar{B} \approx \rho (\gamma \bar{E})$$

$$\underline{k}_2^2 = \frac{\omega^2}{c^2} \left(-i \frac{\gamma}{\epsilon_0 \omega} \right) = \frac{(1-i)^2}{2 \epsilon_0 c^2} \omega \gamma$$

$$= \frac{(1-i)^2}{\left(\frac{2}{\gamma \omega \rho} \right)} = \left(\frac{1-i}{\delta} \right)^2$$

$$\underline{k}_2 = \left(\frac{1}{\delta} \right) - i \frac{1}{\delta}$$

$$\underline{E}_t = \underline{E}_{0t} e^{-z/\delta} e^{i(\omega t - z/\delta)} \underline{D}_x$$

$$3) \alpha = \frac{v \cdot y}{c} = \frac{\omega \delta}{c} = \frac{k_2}{k_2'} = \frac{\omega}{c} \sqrt{\frac{2}{\epsilon_0 \gamma \omega}}$$

$$\alpha = \sqrt{\frac{2 \omega \epsilon_0}{\gamma}} \ll 1$$

$$4) \underline{B}_t = \frac{\sqrt{2}}{\delta\omega} \underline{E}_{ot} e^{-\frac{z}{\delta}} e^{i(\omega t - \frac{z}{\delta} - \frac{\pi}{4})} \underline{u}_y$$

$$5) \underline{R} = \frac{\underline{E}_{or}}{\underline{E}_{oi}} \quad \underline{n}_{12} \wedge (\underline{E}_2 - \underline{E}_1) = \underline{0}$$

$$\underline{t} = \frac{\underline{E}_{ot}}{\underline{E}_{oi}} \quad \underline{n}_{12} \wedge (\underline{B}_2 - \underline{B}_1) = \mu_0 \underline{j}_s = \underline{0}$$

$$\begin{cases} \underline{E}_{2T} = \underline{E}_{1T} \\ \underline{B}_{2T} = \underline{B}_{1T} \end{cases}$$

$$\begin{cases} \underline{E}_i + \underline{E}_r = \underline{E}_t \\ \underline{B}_i + \underline{B}_r = \underline{B}_t \end{cases} \quad z=0.$$

$$\begin{cases} \underline{E}_{oi} + \underline{E}_{or} = \underline{E}_{ot} \\ \frac{\underline{E}_{oi}}{c} - \frac{\underline{E}_{or}}{c} = \frac{\sqrt{2}}{\delta\omega} \underline{E}_{ot} e^{-i\frac{R}{\delta}} \end{cases}$$

$$\begin{cases} 1 + \underline{r} = \underline{t} \\ 1 - \underline{r} = \frac{\sqrt{2}c}{\delta\omega} \underline{t} e^{-i\frac{\pi}{4}} \end{cases}$$

$$1 - \underline{r} = \underline{t} \frac{(1-i)}{\alpha}$$

$$1 + \underline{r} = \underline{t}$$

$$r_E = \frac{E_{or}}{E_{oi}}$$

$$1 - \underline{r} = \underline{t} \frac{(1-i)}{\alpha}$$

$$r_B = \frac{B_{or}}{B_{oi}}$$

$$\underline{t} = \underline{t} \left(1 + \frac{(1-i)}{\alpha} \right)$$

$$\underline{t} = \frac{2\alpha}{(\alpha+1) - i}$$

$$\underline{r} = \frac{(\alpha-1) + i}{(\alpha+1) - i} = \underline{t} - 1$$

b) $R = |\underline{r}|^2$ دواج

$$R = \frac{\langle \|\vec{\pi}_n\| \rangle}{\langle \|\vec{\pi}_i\| \rangle} = \left| \frac{E_{or}}{E_{oi}} \right|^2 = |\underline{r}|^2$$

$$\langle \vec{\pi}_i \rangle = \frac{E_{oi}^2}{2\mu c} \vec{e}_n$$

$$\langle \vec{\pi}_n \rangle = \frac{-|E_{or}|^2}{2\mu c} \vec{e}_n$$

$$R = \frac{(\alpha - 1)^2 + 1}{(\alpha + 1)^2 + 1}$$

$$T = 1 - R = \frac{4\alpha}{(\alpha + 1)^2 + 1}$$

$$\langle \|\vec{T}_i\| \rangle = \langle \|\vec{T}_n\| \rangle + \langle \|\vec{T}_t\| \rangle$$

$$\boxed{1 = R + T}$$

$$T = \frac{\langle \|\vec{T}_t\| \rangle}{\langle \|\vec{T}_i\| \rangle};$$

$$\langle \vec{T}_t \rangle = \frac{1}{2\mu} \text{Re}[\vec{E}_t \wedge \vec{B}_t^*]$$

$$= \frac{1}{2\mu} \text{Re} \left[\underline{E}_0 e^{-\frac{z}{\delta}} e^{i(\omega t - \frac{z}{\delta})} \right]$$

$$\frac{\sqrt{2}}{\delta \omega} \underline{E}_0^* e^{-\frac{z}{\delta}} e^{-i(\omega t - \frac{z}{\delta} - \frac{\pi}{4})}$$

$$= \frac{1}{2\mu \delta \omega} |\underline{E}_0|^2 e^{-\frac{2z}{\delta}} \quad \vec{e}_z = \langle \vec{T}_t(z) \rangle$$

$$T = \frac{1}{\delta\omega} \frac{|E_{0t}|^2}{(E_{0i})^2} C = |t|^2 \frac{C}{\delta\omega} = \frac{|t|^2}{\alpha}$$

$$T = |t|^2 \frac{k_2'}{k_1}$$

$$r_{12} = \frac{k_1 - k_2}{k_1 + k_2}$$

$$t_{12} = \frac{2k_1}{k_1 + k_2}$$

$$R = |r_{12}|^2, \quad T = |t_{12}|^2 \frac{k_2'}{k_1}$$

$$r = \frac{|E_{0t}|}{E_{0i}}$$

k_1	k_2
m_0	m_0
(1)	(2)

on $\alpha \ll 1$

$$R = \frac{(1-\alpha)^2 + 1}{(1+\alpha)^2 + 1} \approx \frac{1-2\alpha+1}{1+2\alpha+1}$$

$$= (1-\alpha)(1-\alpha) \approx (1-2\alpha)$$

$$T = \frac{4\alpha}{(1+\alpha)^2 + 1} = \frac{4\alpha}{1+2\alpha+1} = 2\alpha - 2\alpha^2 \approx 2\alpha$$

$$7) \langle I_{\text{rule}} \rangle = \langle \|\vec{\pi}_i\| \rangle S - \langle \|\vec{\pi}_{\text{all}}\| \rangle S$$

$$= S \langle \|\vec{\pi}_i\| \rangle \left(\underbrace{1 - R}_{\rightarrow T \approx 2\alpha} \right)$$

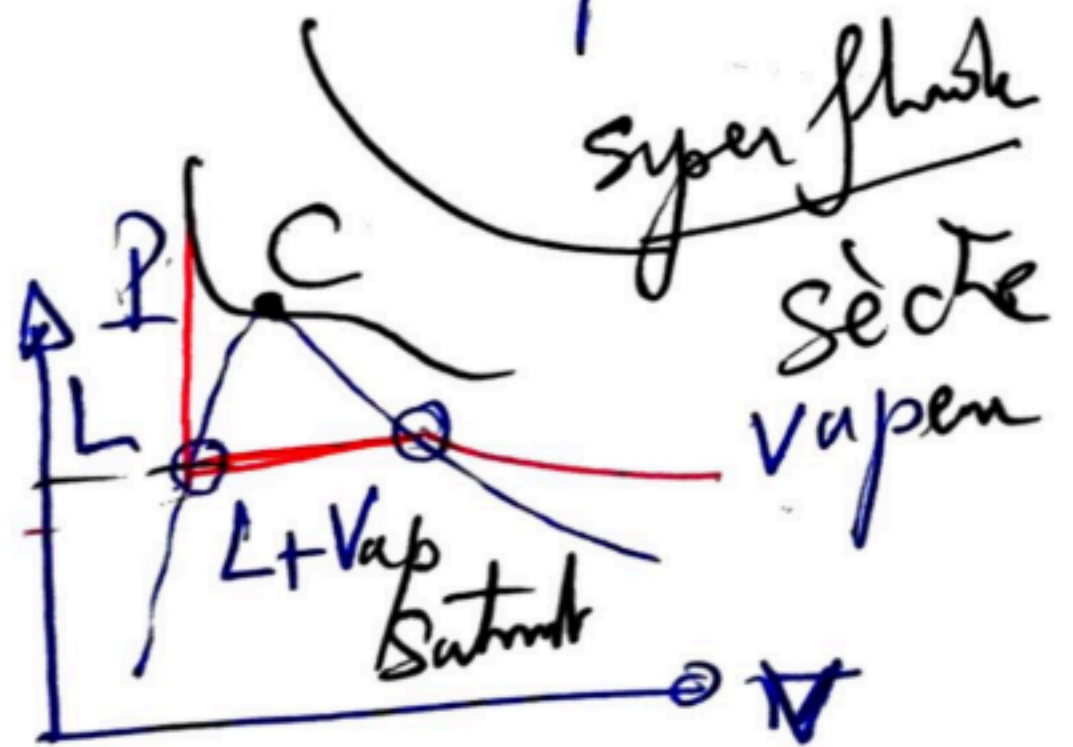
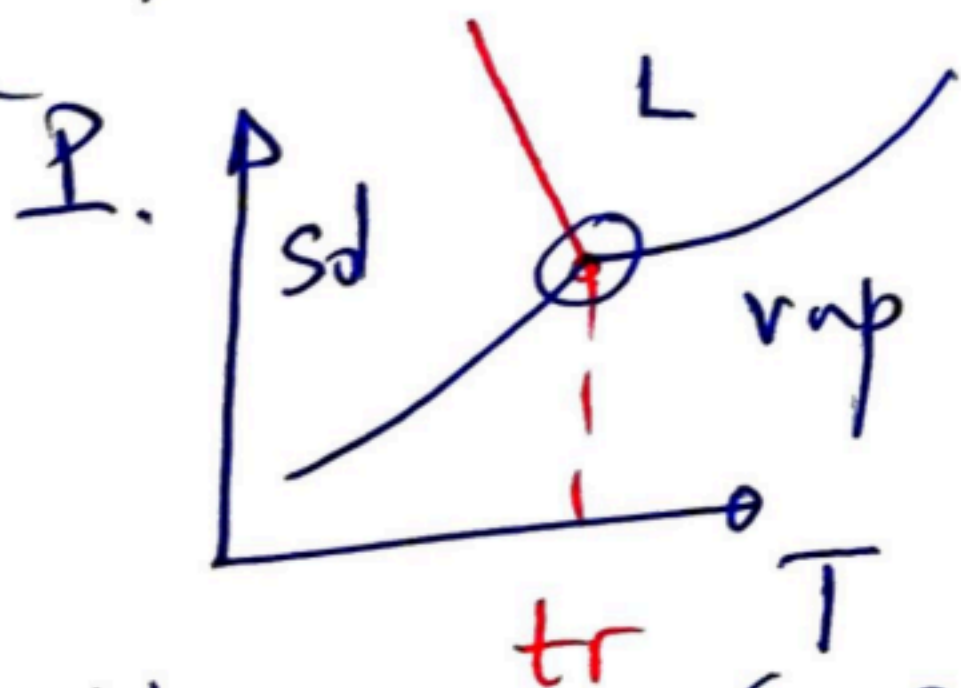
$$= S \frac{\epsilon_0 E_0^2 c}{2} 2\alpha$$

$$8) \vec{E} = \vec{0}; \vec{B} = \vec{0}, \gamma \rightarrow \infty$$

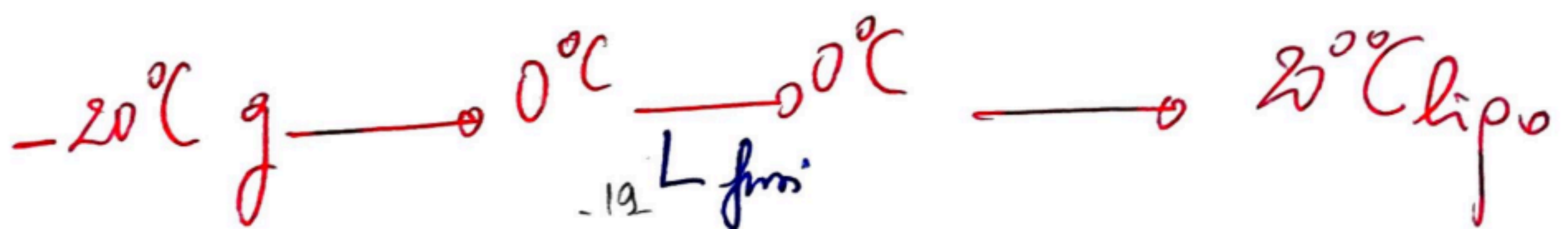
$$\vec{J} = \vec{0} \quad \vec{S} = \vec{0} \quad p_J = 0.$$

9) a) $L_{\text{fusion}} =$ chaleur nécessaire pour
 le transition physique solide $\rightarrow$ liquide.

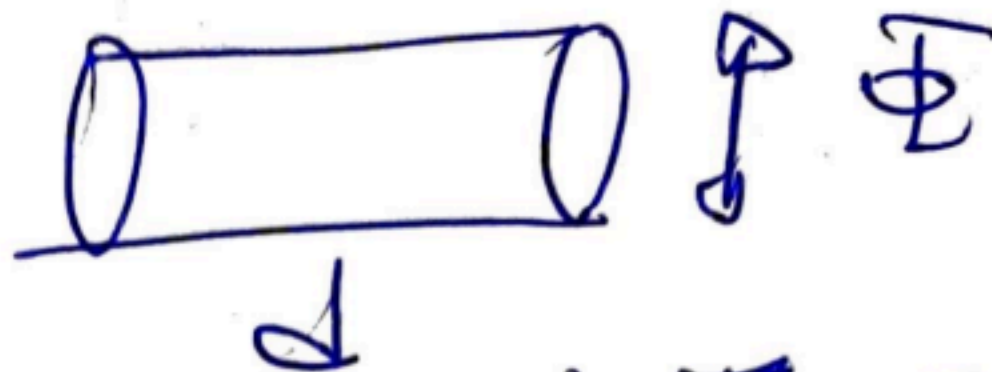
(Isobare; isotherme)



$$b) \Delta H = m \left(c(T_f - T_0) + L_{\text{fusion}} \right)$$



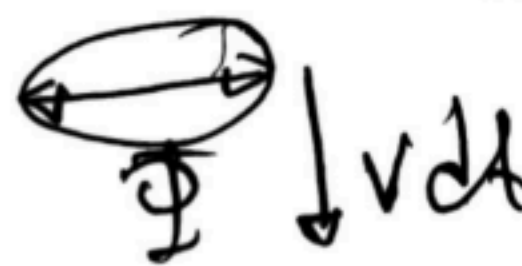
$$m = \rho \cdot \left(\frac{\pi \Phi^2}{4} \right) d$$



$$b) \underline{P}_t = (1 - R) \underline{P}_i = (1 - 0,72) \times 1700$$

$$c) \underline{P}_R = (1 - R) \underline{P}_i = \underline{P}_t$$

$$(1 - R) \underline{P}_i dt = \int \underbrace{\Phi d}_{\text{red}} (v dt) [c(T_f - T_0) + L_f]$$



$$V = (\quad)$$