

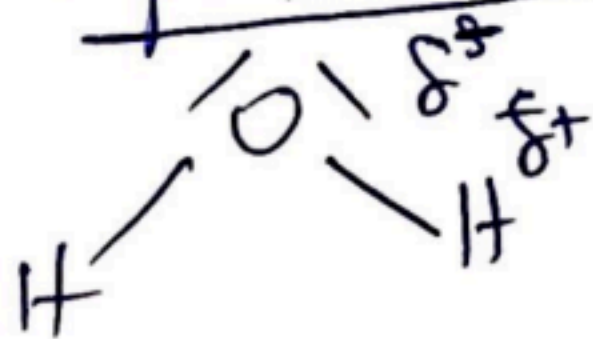
PT2 2023

Semester 3

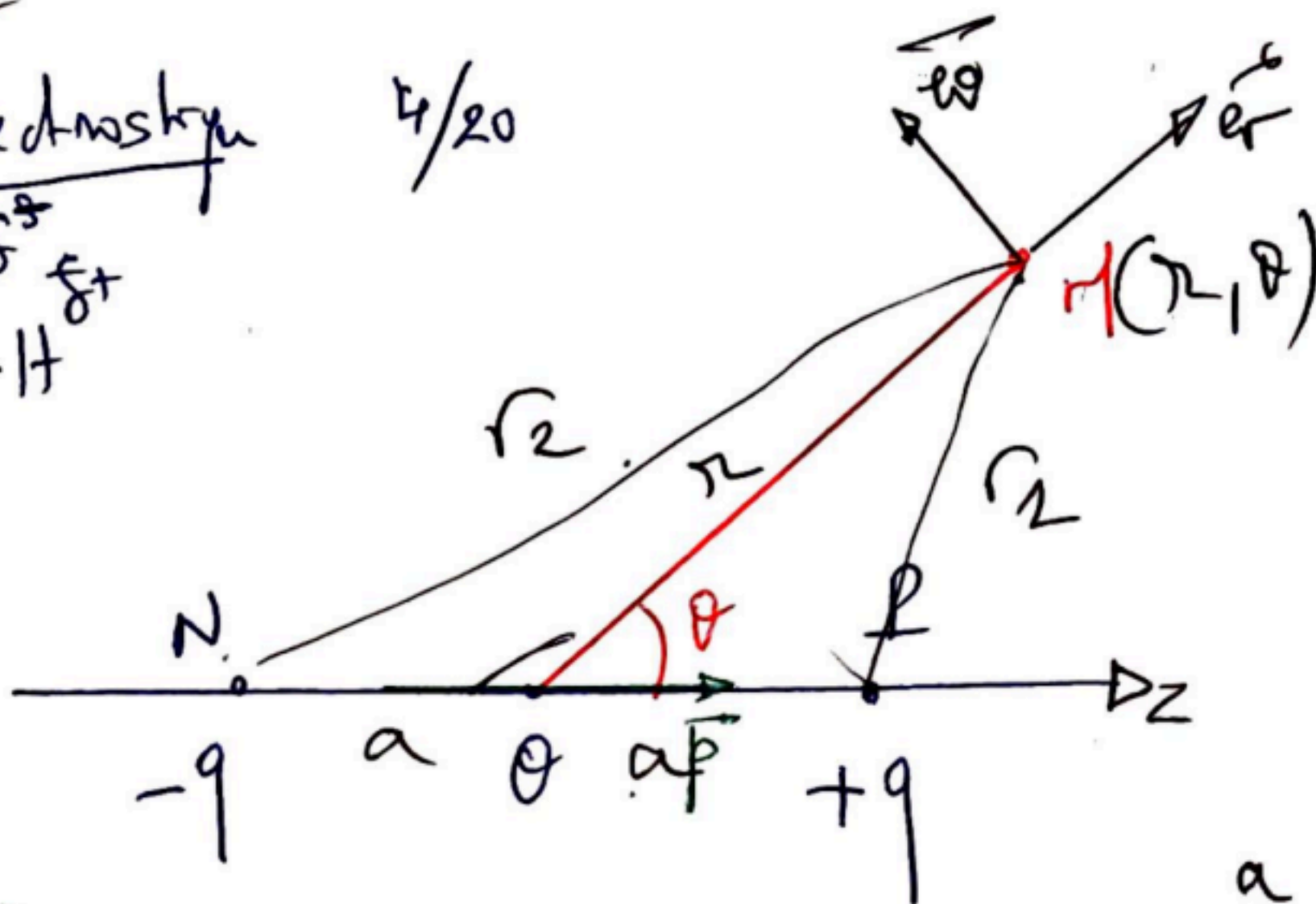
MP2 = 2016

Dipole électrique

4/20



$2a \ll r$



$\frac{a}{r} \sim 10^{-6}$

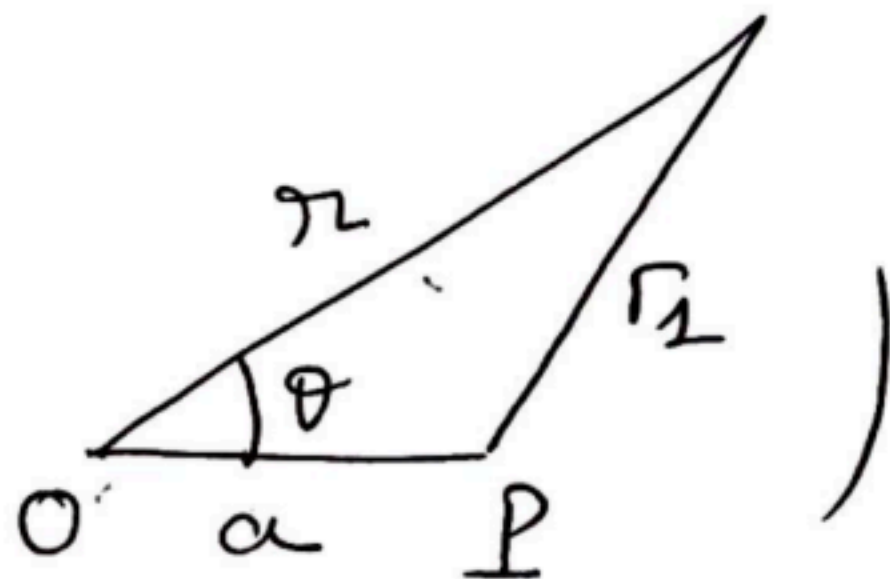
$$\vec{p} = q \cdot \overrightarrow{NP}$$

$$\|\vec{p}\| = \text{C.m.}; \quad \frac{1}{3} 10^{-23} \text{ C.m.} = 1 \text{ Debye}$$

$$V(M) = \frac{q}{4\pi\epsilon} \left(\frac{1}{PM} - \frac{1}{NM} \right); \quad \begin{aligned} PM &= r_1 \gg a \\ NM &= r_2 \gg a \end{aligned}$$

$$= \frac{q}{4\pi\epsilon} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$r_1^2 = a^2 + r^2 - 2ar \cos \theta$$



$$r_1^2 = r^2 \left[1 + \frac{a^2}{r^2} - \frac{2a}{r} \cos \theta \right]$$

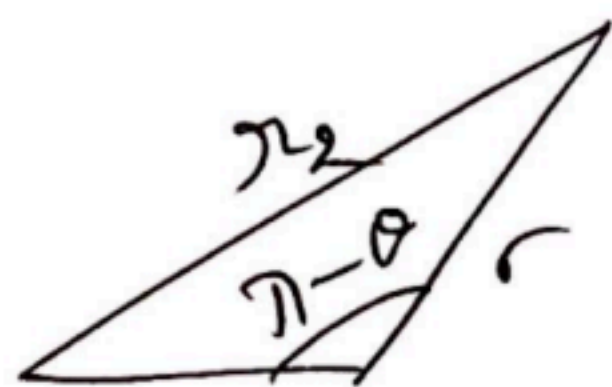
$$r_1 = r \left[1 + \underbrace{\left(\frac{a^2}{r^2} \right)}_{10^{-12}} - \underbrace{\frac{2a}{r} \cos \theta}_{10^{-6}} \right]^{1/2}$$

$$r_1 \approx r \left[1 + \frac{a^2}{2r^2} - \frac{a}{r} \cos \theta \right]$$

$$r_1 = r \left[1 - \frac{a}{r} \cos \theta \right]; \quad \frac{1}{r_1} = \frac{1}{r} \left[1 - \frac{a}{r} \cos \theta \right]^{-1}$$

$$\frac{1}{r_1} \approx \frac{1}{r} \left(1 + \frac{a}{r} \cos \theta \right) \quad 2/100$$

$$\text{de m } \frac{1}{r_2} \approx \frac{1}{r} \left(1 - \frac{a}{r} \cos \theta \right)$$



$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{q}{4\pi\epsilon_0} \left(\frac{2a \cos \theta}{r^2} \right)$$

$$2aq = N.P. \cdot q = p_0 \Rightarrow$$

$$V = \frac{p_0 \cos \theta}{4\pi\epsilon_0 r^2}$$

d'écrit

$$\vec{E} = - \frac{\partial V}{\partial r} \vec{e}_r - \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{e}_\theta - \frac{1}{r \sin \theta} \frac{\partial V}{\partial \varphi} \vec{e}_\varphi$$

$$E_\varphi = 0, \quad E_r = \frac{2p_0 \cos \theta}{4\pi\epsilon_0 r^3}$$

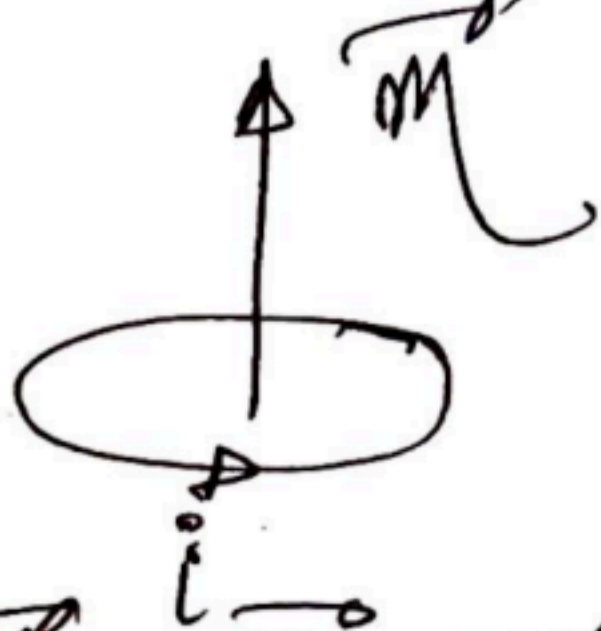
$$E_\theta = \frac{p_0 \sin \theta}{4\pi\epsilon_0 r^3}$$

$$\vec{E}_{\text{dipole}} = \frac{p_0 \cos}{4\pi \epsilon r^3} (2\cos\theta \vec{e}_r + \sin\theta \vec{e}_\theta)$$

$$\vec{B}_{\text{dipole}} = \frac{\mu_0 m_0}{4\pi r^3} (2\cos\theta \vec{e}_r + \sin\theta \vec{e}_\theta)$$

$$\vec{m} = \text{moment magnétique} = \underline{\underline{iS}}$$

$$\vec{p} = qN\vec{L}$$



les lignes de champs

$$\vec{E}: \vec{L} \wedge \vec{E} = \vec{0}$$

$$\vec{L} = \frac{dr}{r d\theta} \quad \left| \begin{array}{c} E_r \\ E_\theta \\ 0 \end{array} \right.$$

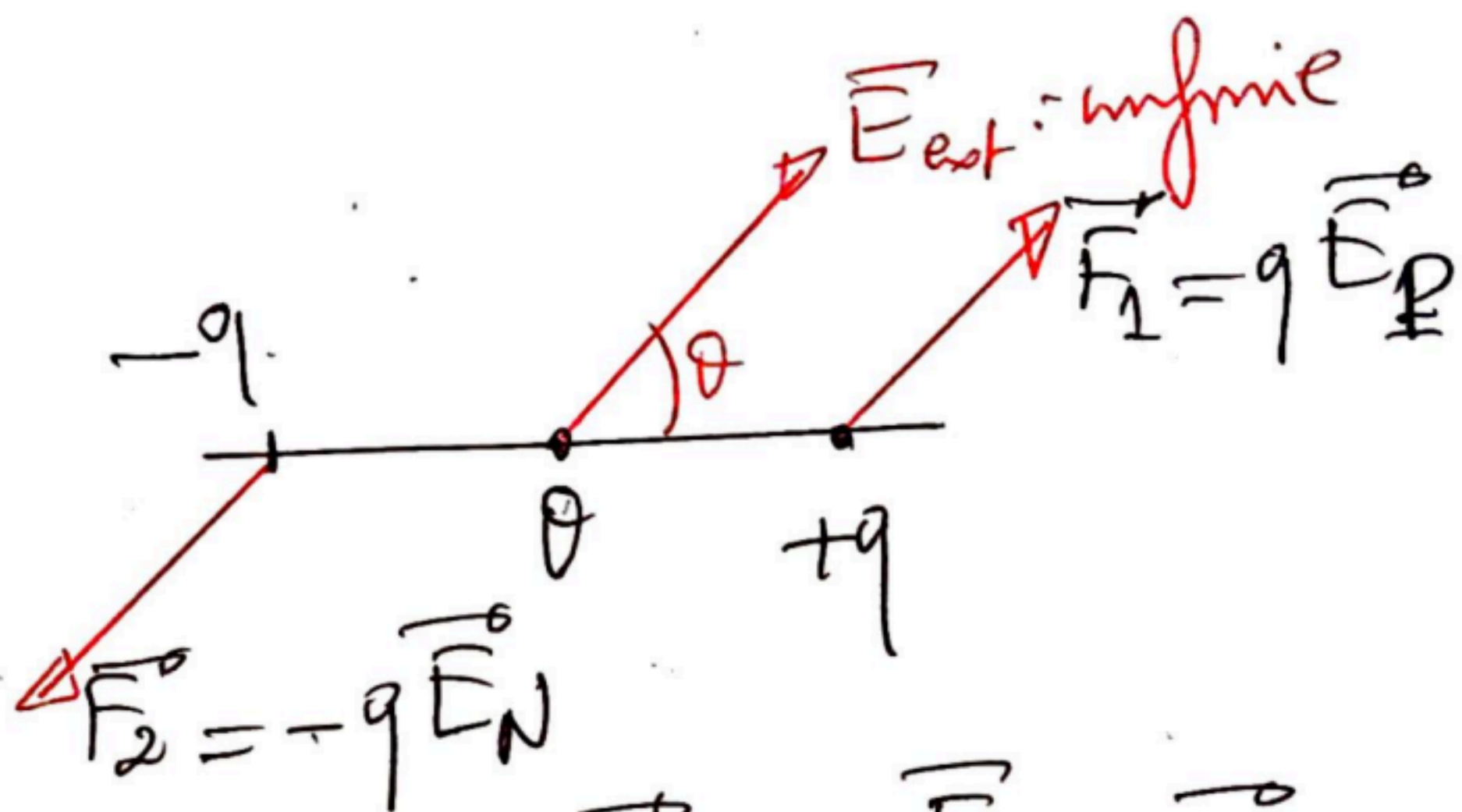


$$\frac{dr}{E_r} = \frac{r d\theta}{E_\theta} ; \quad \frac{dx}{E_x} = \frac{dy}{E_y}$$

$$\frac{dr}{r} = \frac{E_r}{E_\theta} d\theta = \frac{2\cos\theta d\theta}{\sin\theta}$$

$$\ln r = 2 \ln(\sin\theta) + \text{cte}$$

$$\boxed{r = r_0 \sin^2\theta}$$



$$\text{or } \vec{E}_{ext} = \vec{E}_P = \vec{E}_N \Rightarrow \vec{F}_{tot} = \vec{0}$$

$$\vec{M}(\vec{F}_P) = \vec{OP} \wedge (q \vec{E}_{ext})$$

$$\vec{M}(\vec{F}_N) = \vec{ON} \wedge (-q \vec{E}_{ext})$$

$$\vec{\Gamma} = (\vec{OP} - \vec{ON}) \wedge q \vec{E}_{ext} = q \vec{NP} \wedge \vec{E}_{ext}$$

$$\boxed{\vec{\Gamma} = \vec{p} \wedge \vec{E}_{ext}}$$

$$\hookrightarrow \text{N.m}$$

$$\vec{\Gamma} = \vec{m} \wedge \vec{B}_{ext}$$

$\hookrightarrow \text{N.m} \quad \text{A.m}^2$

Ruyant

$$\theta = (\vec{P}, \vec{OM})$$

$$1) \underline{NP} = za e^{j\omega t}$$

$$NP = za \cos(\omega t)$$

$$\vec{P} = q \vec{NP} = (q za) \cos(\omega t) \vec{u}_z$$

$za \ll OM$
approx dipole

$\vec{P}(t) : \text{oscillant}$

$$(zaq) = p_0 : \text{C.m.}$$

$$\vec{P}(t) = q(t) \vec{NP} : \text{Antenne}$$

$$2) \underline{v \ll c} \Rightarrow \frac{dOP}{dt} = \frac{d[a \cos(\omega t)]}{dt}$$

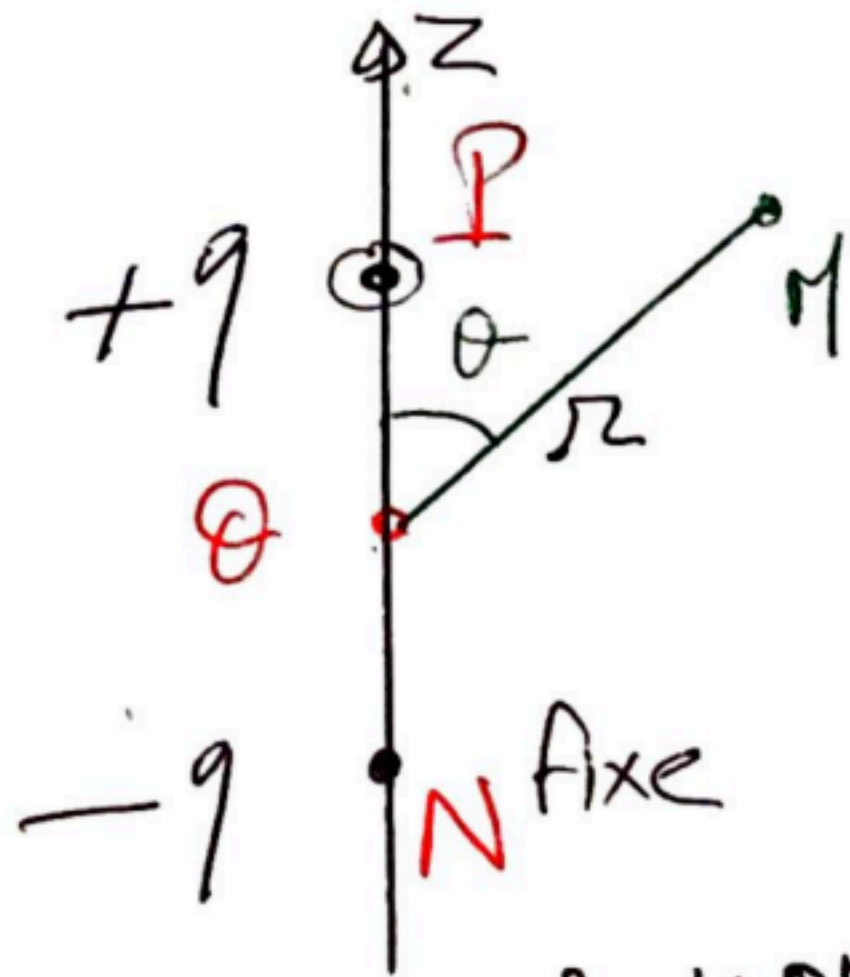
$$v(t) = -a\omega \sin(\omega t) ; v_{\max} = a\omega$$

$$\frac{v_{\max}}{c} = \frac{a\omega}{c} = ak = a \frac{2\pi}{\lambda} \ll 1$$

$$\Rightarrow \boxed{za \ll \lambda} \quad \text{A.R.Q.S.}$$

$$V(r) = \dots \quad t = t - \frac{(r)}{c}$$

$$\Rightarrow t' \simeq t - \frac{OM}{c} \quad (1)$$



$$b) \lambda \in [0,32 \mu\text{m} - 0,75 \mu\text{m}]$$

violet

→ orange

$$\vec{A}, \vec{B} = \nabla \times \vec{A}; \quad \vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

hors programme

$$\underline{V} = \frac{P \cos \theta}{4\pi \epsilon r^2} (1 + j k r); \quad \vec{k} = \frac{\omega}{c} \vec{e}_r$$

$$\vec{r} = r \vec{e}_r$$

3) Cas statique $\underline{\omega = 0}, \underline{k = 0}$

$$\underline{V} = \frac{P_0 \cos \theta}{4\pi \epsilon r^2}$$

$$b) 1^{er} \text{ terme : } \frac{P \cos \theta}{4\pi \epsilon r^2} \sim \frac{1}{r^2}$$

$$2^{eu} \text{ terme : } \frac{j k P \cos \theta}{4\pi \epsilon r} \sim \frac{1}{r}$$

$$r \gg \lambda; \quad (kr) = \left(\frac{2\pi}{\lambda} r \right) \gg 1 \Rightarrow V \simeq \frac{j \omega P \cos \theta}{4\pi \epsilon c r}$$

$$\underline{V} = \frac{\dot{p} \cos \theta}{4\pi \epsilon_0 r^2}$$

$$\frac{dp}{dt} = \dot{p} = j\omega p \quad ; \quad \frac{d^2 p}{dt^2} = \ddot{p} = -\omega^2 p$$

By:
$$\underline{V} = \frac{1}{4\pi \epsilon_0} \left(\frac{p}{r^2} + \frac{\dot{p}}{rc} \right) \cos \theta$$

$r \gg \lambda$:
$$\left| \frac{p}{r^2} \right| \ll \left| \frac{\dot{p}}{rc} \right|$$

$$\frac{\left| \frac{p}{r^2} \right|}{\left| \dot{p}/rc \right|} = \frac{\left| \frac{p}{r^2} \right|}{\left| \frac{rc}{r^2} \right|} = \frac{c}{\omega r} = \frac{1}{rk} \ll 1$$

$kr \gg 1$,

$$4) \quad \underline{B} = j\omega (1 + jkr) \frac{\mu_0}{4\pi} \frac{p \sin \theta}{r^2} \underline{e}_\varphi$$

$r \gg \lambda$:
$$\underline{B} = \frac{-\omega^2 \mu_0 p \sin \theta}{4\pi c r} \underline{e}_\varphi$$

$$-\omega^2 p = \ddot{p}$$

$$k = \frac{\omega}{c}$$

$$\vec{B}'(r \gg \lambda) = \frac{\mu_0 \ddot{p} \sin \theta}{4\pi r c} \vec{e}_\varphi$$

$$5) a) \vec{E}(\omega=0) = \frac{p_0}{4\pi\epsilon_0 r^3} (2\cos\theta \vec{e}_r + \sin\theta \vec{e}_\theta)$$

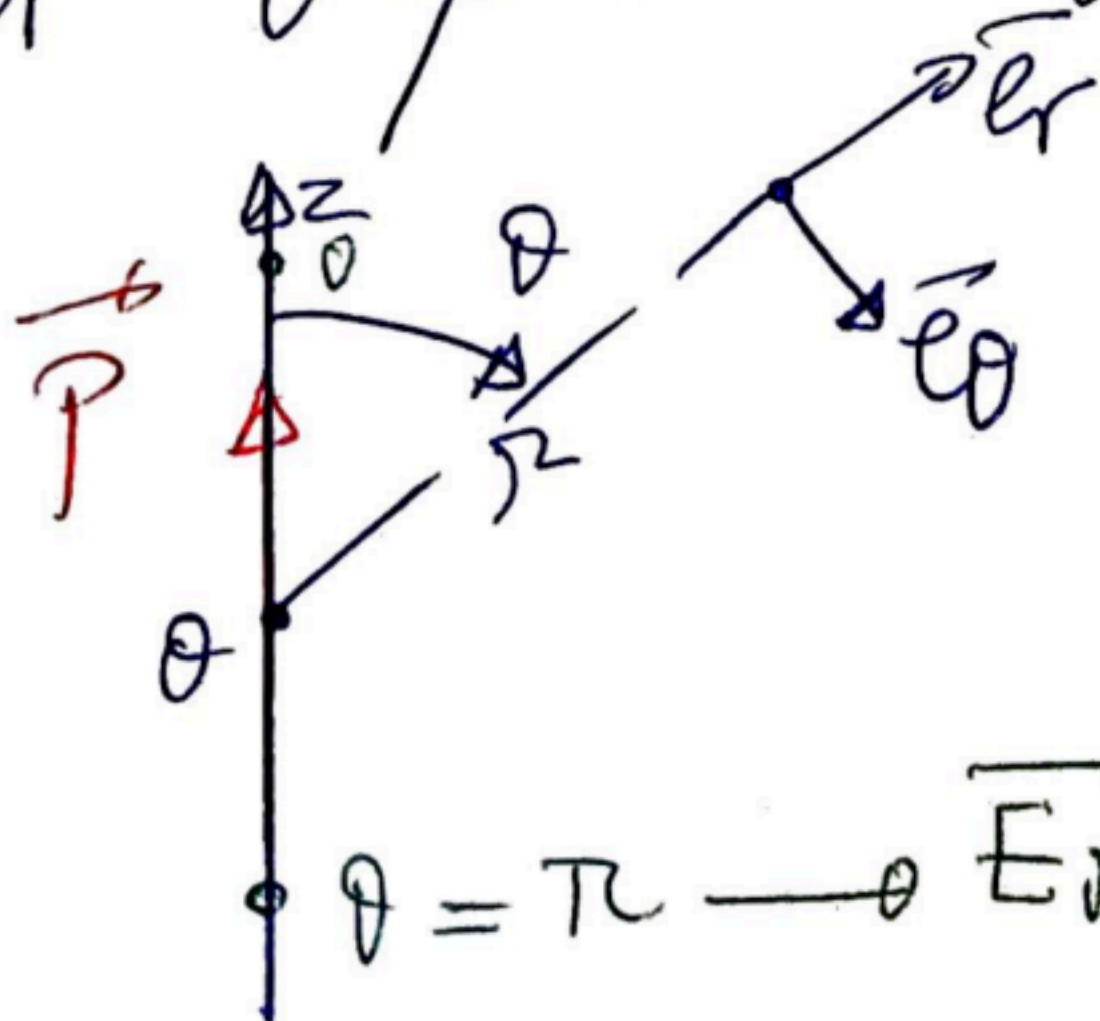
$$b) \vec{E}(r \gg \lambda) \simeq \begin{cases} E_r = \frac{2p \cos\theta}{4\pi\epsilon_0} j \frac{k}{r^2} \sim 1/r^2 \\ E_\theta = \frac{p \sin\theta}{4\pi\epsilon_0} \left(-\frac{k^2}{r} \right) \sim 1/r \end{cases}$$

$(kr) \gg 1$
 $(kr)^2 \gg 1, (kr)$

$$\left| \frac{E_\theta}{E_r} \right| = \frac{k^2}{2r} \frac{r^2}{k} = \frac{kr}{2} \gg 1$$

$$c) \vec{E}(r \gg \lambda) \simeq \frac{\ddot{p} \sin\theta}{4\pi\epsilon_0 c^2 r} \vec{e}_\theta$$

et $\theta \neq 0$ $\theta \neq \pi$



$\theta = \pi \longrightarrow E_{rayonnée} = \vec{0}$

$r \gg \lambda$ onde $\vec{r}$ localent plane

$$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} = \frac{\vec{e}_r \wedge E_0 \vec{e}_\theta}{c} = \frac{E_0}{c} \vec{e}_\varphi$$

$$(\vec{e}_r \wedge \vec{B}) = \frac{\vec{e}_r \wedge (\vec{e}_r \wedge \vec{E})}{c} = -\frac{\vec{E}}{c} (\vec{e}_r \cdot \vec{e}_r)$$

$$\vec{E} = -c (\vec{e}_r \wedge B_\varphi \vec{e}_\varphi) = c B_\varphi \vec{e}_\theta$$

II) $r \gg \lambda$: $\vec{E} \approx \frac{\ddot{p} \sin \theta}{4\pi \epsilon_0 c^2 r} \vec{e}_\theta$

$$\vec{B} = \frac{\mu_0 \dot{p} \sin \theta}{4\pi c r} \vec{e}_\varphi$$

1a)

$$\frac{\|\vec{E}\|}{\|\vec{B}\|} = c, \quad \mu_0 \epsilon_0 c^2 = 1$$

$$\vec{B} \perp \vec{E} \perp \vec{k} = \frac{2\pi}{\lambda} \vec{e}_r$$

1b) l'onde $\vec{r}$ localent plane

$$2) \vec{R} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{\vec{E}_\theta \wedge B_\varphi}{\mu_0} = \frac{E_\theta B_\varphi}{\mu_0} \vec{e}_r$$

$$\vec{R} = \frac{E_\theta^2}{\mu_0 c} \vec{e}_r \quad b) \vec{V}_e = \frac{\langle \vec{R} \rangle}{\langle u_{em} \rangle}$$

radial

c) $\vec{R} \rightarrow$ Rayons lumineux.

$$3) I = \langle \|\vec{R}\| \rangle = \text{Watt/m}^2$$

$$I = \frac{1}{\mu_0 c} \frac{\langle (\ddot{p})^2 \sin^2 \theta \rangle}{16 \pi^2 \epsilon^2 c^4 r^2}, \quad \sin^2 \theta = d\Omega$$

$$I = \frac{\langle (\ddot{p})^2 \rangle \sin^2 \theta}{\mu_0 c 16 \pi^2 \epsilon^2 c^4 r^2}$$

$$p = p_0 \cos(\omega t - kr)$$

$$(\ddot{p})^2 = (-\omega^2 p_0)^2 \cos^2(\omega t - kr)$$

$$\langle (\ddot{p})^2 \rangle = \frac{1}{2} p_0^2 \omega^4$$

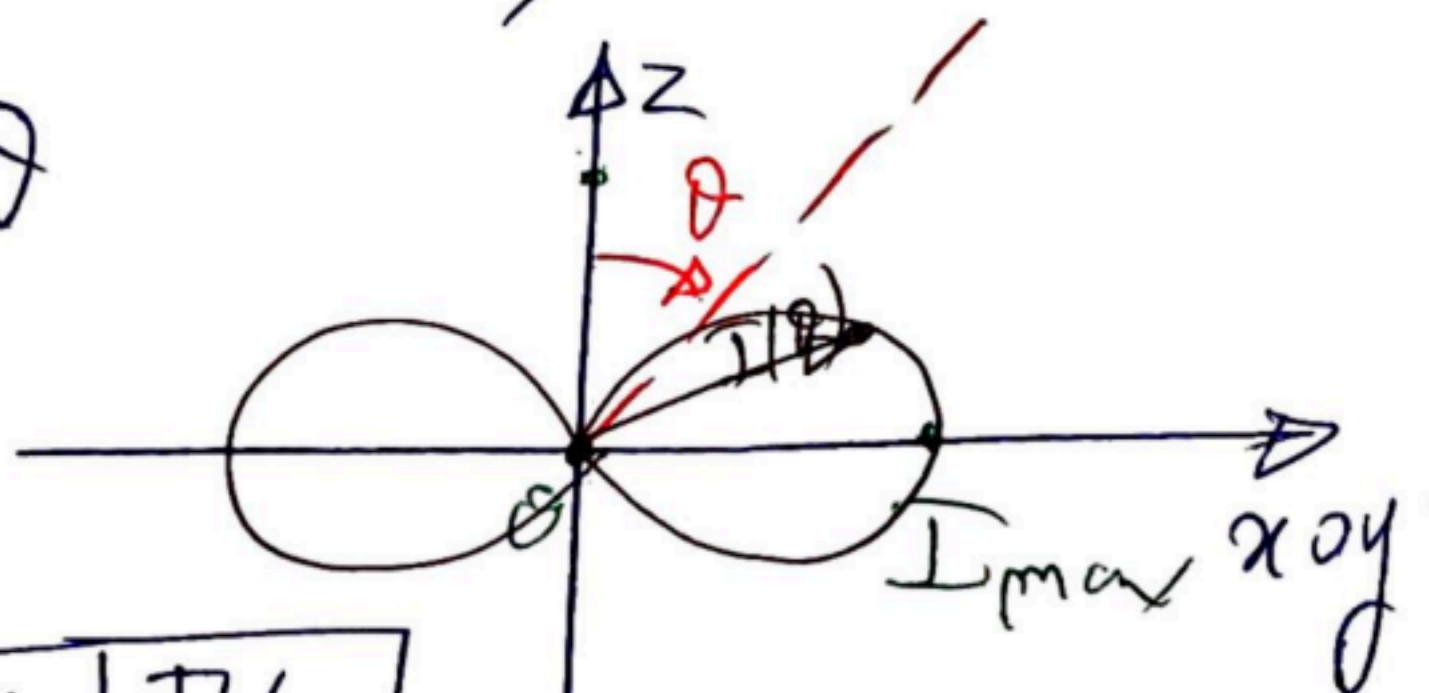
$$I = \frac{p_0^2 \omega^4 \sin^2 \theta}{32 \pi^2 \epsilon_0 c^3 r^2} ; \quad \mu_0 \epsilon_0 c^2 = 1$$

4) a) $I(\theta) = I_{\max} = \frac{p_0^2 \omega^4}{32 \pi^2 \epsilon_0 c^3} \frac{1}{r^2}$

$$\theta = \frac{\pi}{2} ; \quad \theta \in [0, \pi]$$

$$I(\theta) = I_{\min} = 0 \quad \theta = 0, \theta = \pi$$

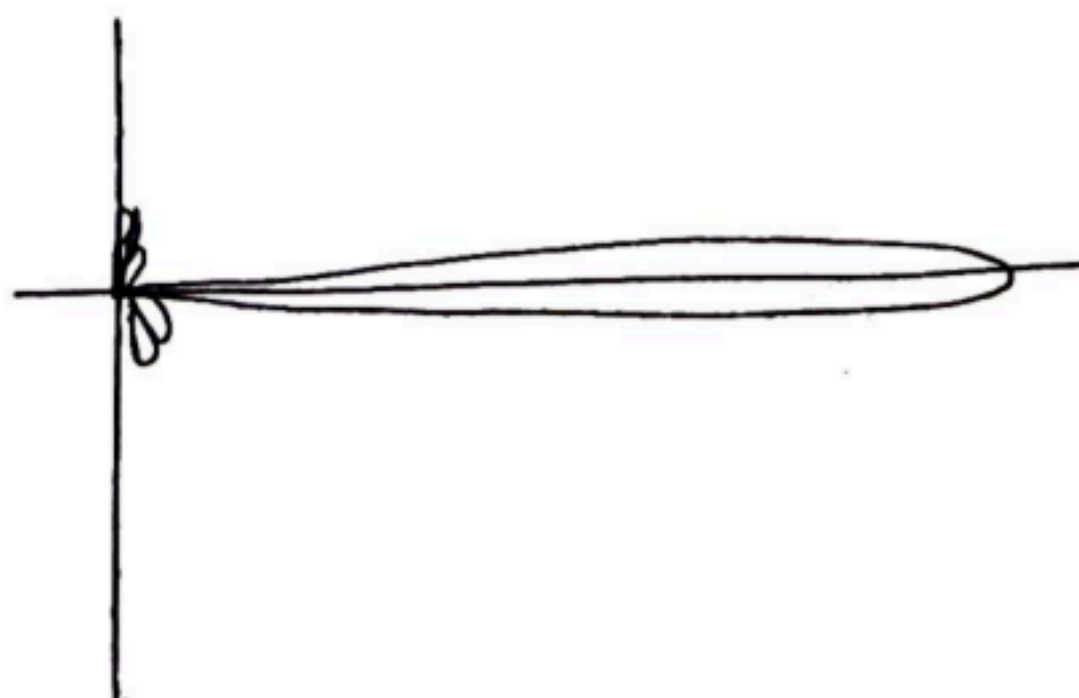
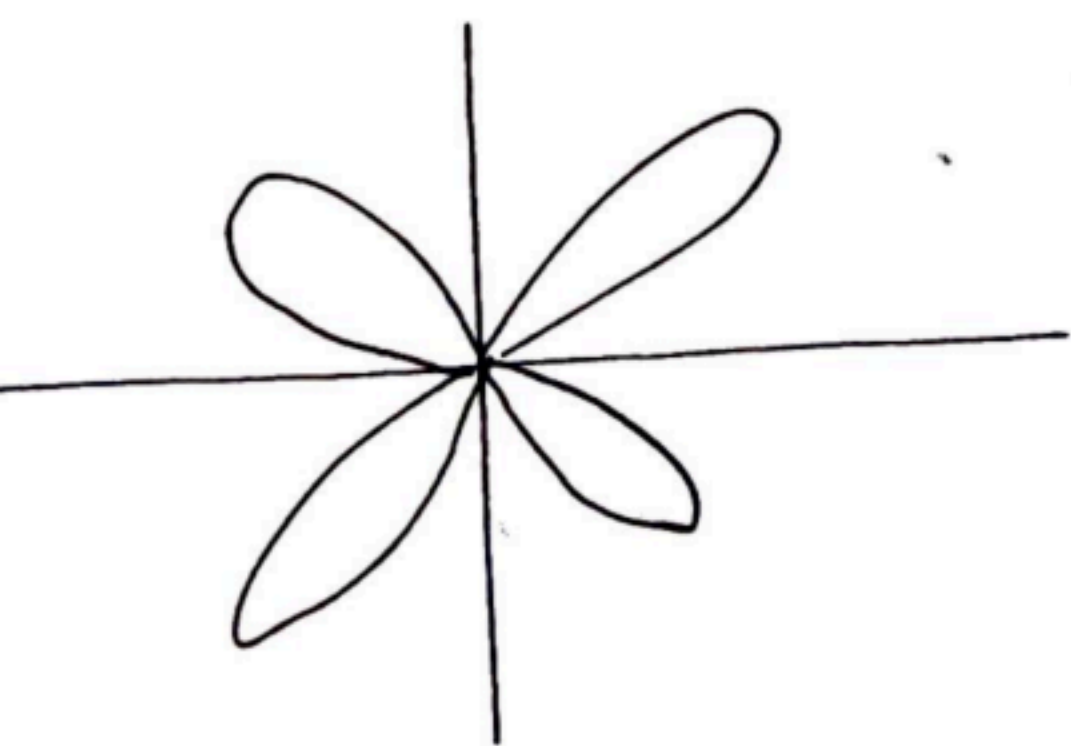
b) $\frac{I(\theta)}{I_{\max}} = \sin^2 \theta$



θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin^2 \theta$	0	$1/4$	$1/2$	$3/4$	1

Antenne

Antenne



$$P = \oint \langle \vec{R} \rangle dS = \oint \frac{p_0^2 \omega^4 \sin^2 \theta}{32 \pi^2 \epsilon_0 c^3 r^2} r^2 \sin \theta d\theta d\varphi$$

$$= \frac{p_0^2 \omega^4}{32 \pi^2 \epsilon_0 c^3} \int_0^\pi \sin^3 \theta d\theta \int_0^{2\pi} d\varphi$$

$\frac{4}{3}$

$$P = \frac{p_0^2 \omega^4}{12 \pi \epsilon_0 c^3} \quad \text{pas de moment}$$

$P = ds$

Rq: onde : on distingue

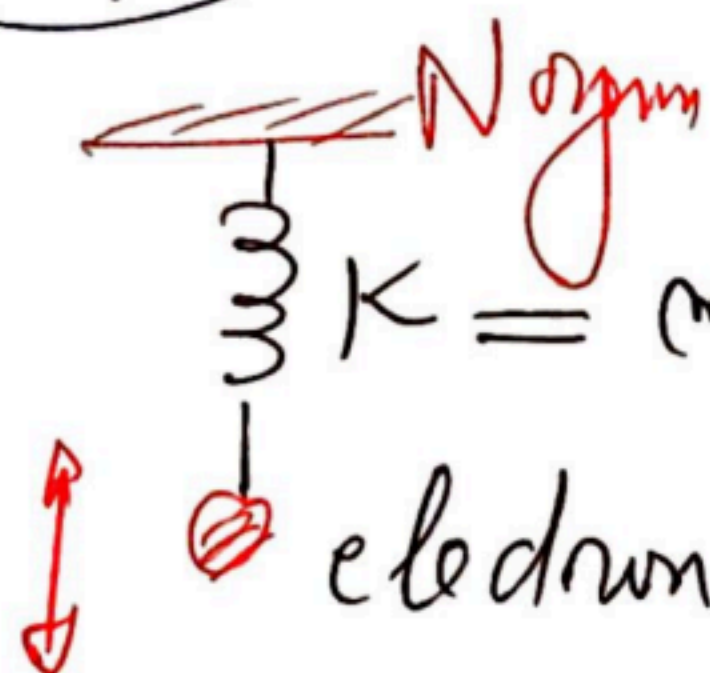
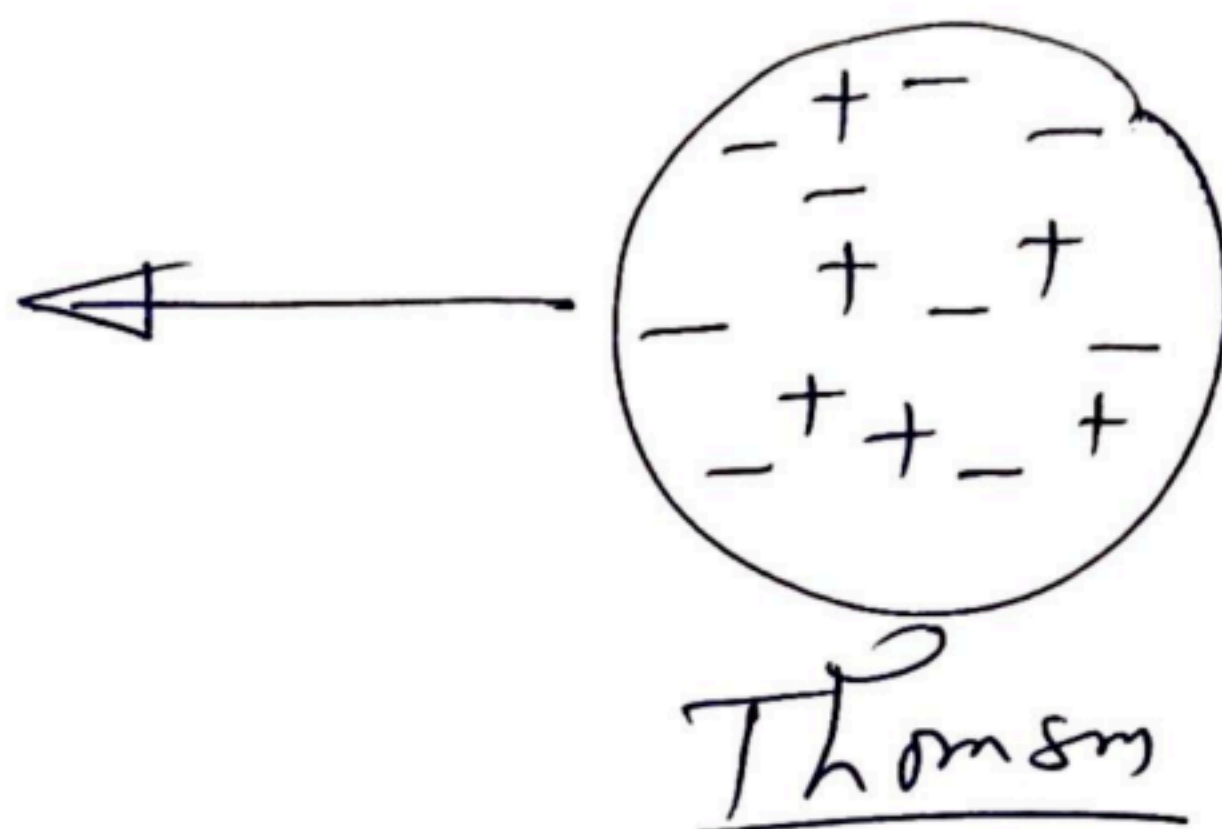
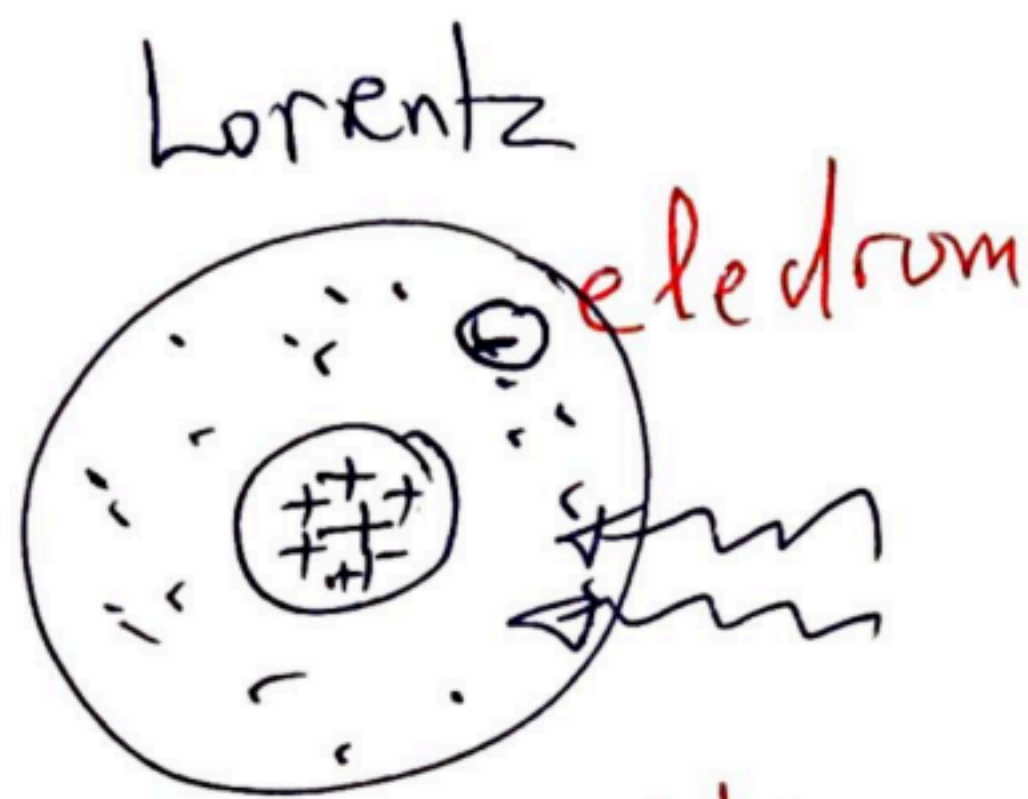
$$\vec{E}_{\text{sphérique}} = \frac{\bar{E}_0 \cos(\omega t - kr)}{r} \vec{e}_\theta$$



onde anisotrope
dépend de θ :

$$\frac{P_{\text{violet}}}{P_{\text{rouge}}} = \left(\frac{\omega_v}{\omega_R} \right)^4 = \left(\frac{\lambda_R}{\lambda_v} \right)^4 = \left(\frac{0,8}{0,4} \right)^4 = 16$$

$\left(\frac{2,75}{0,32} \right)^4 \approx 30$

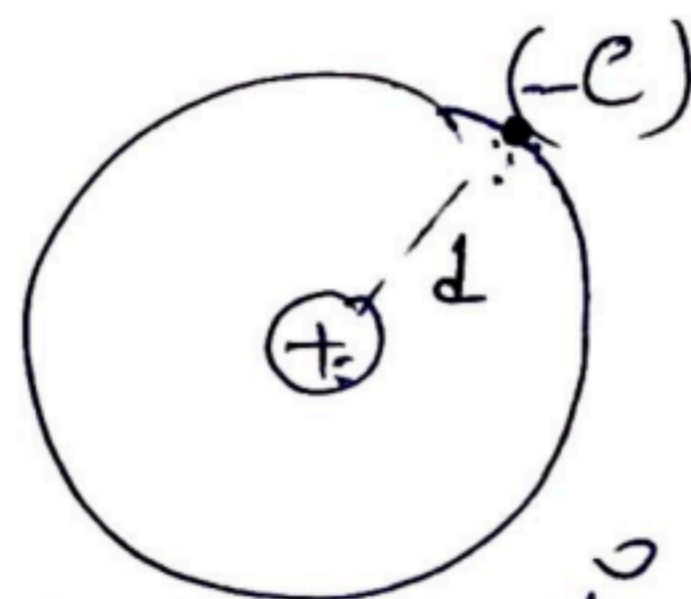


$$K = m \omega_0^2$$

$$\vec{F}_e = -m \omega_0^2 \vec{r}$$

Modèle de Jeans - Rayleigh

$$\vec{E} = \vec{E}_0 \cos(\omega t)$$



$$E_0 \cos\left(\omega\left(t - \frac{x}{c}\right)\right), \quad x_{\max} = d \approx 0,5 \text{ \AA}$$

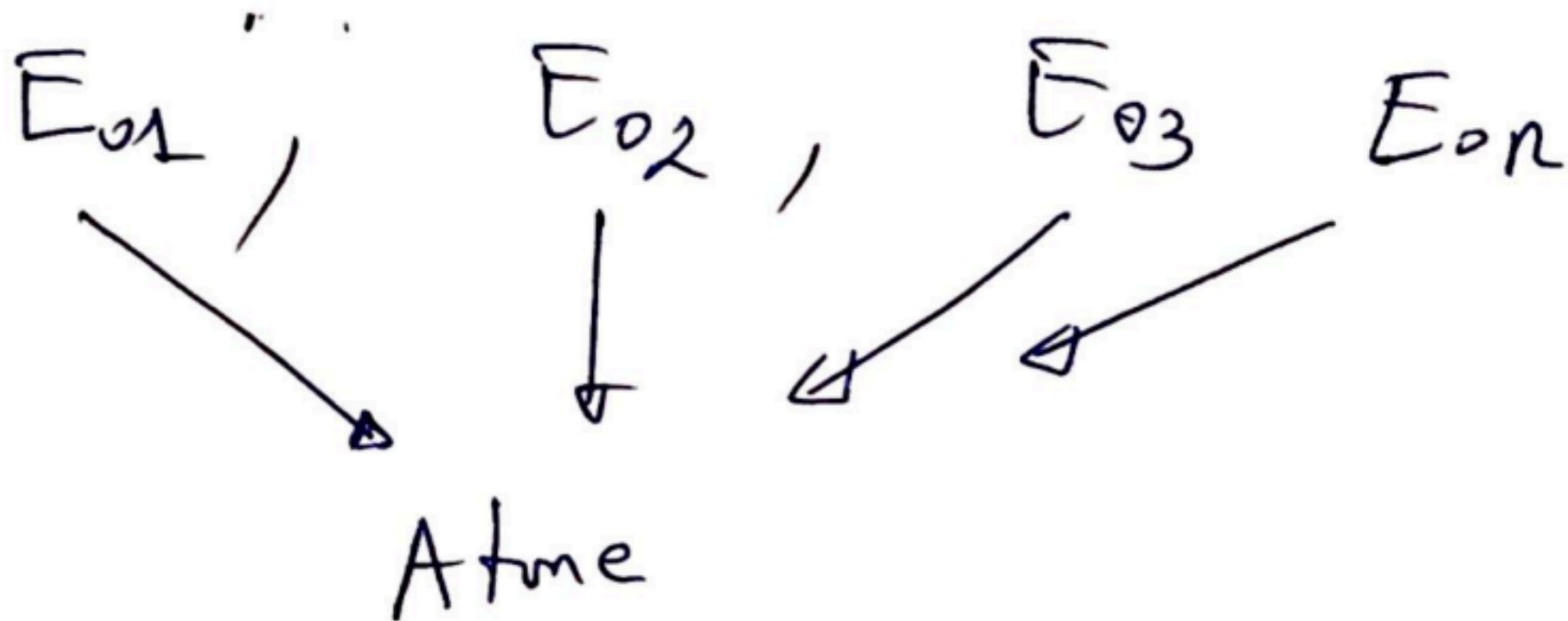
$$\frac{\omega d}{c} = \frac{2\pi d}{\lambda} = \frac{2\pi \times 0,5 \cdot 10^{-10}}{0,5 \cdot 10^{-6}} \ll 1$$

dmc ARQS: $\vec{E} \approx E_0 \cos(\omega t) \vec{u}_z$

$$\frac{\|\vec{F}_{\text{mag}}\|}{\|\vec{F}_e\|} = \left| \frac{\vec{v} \wedge \vec{B}}{\vec{E}} \right| = \frac{v}{c} \ll 1$$

non relativiste

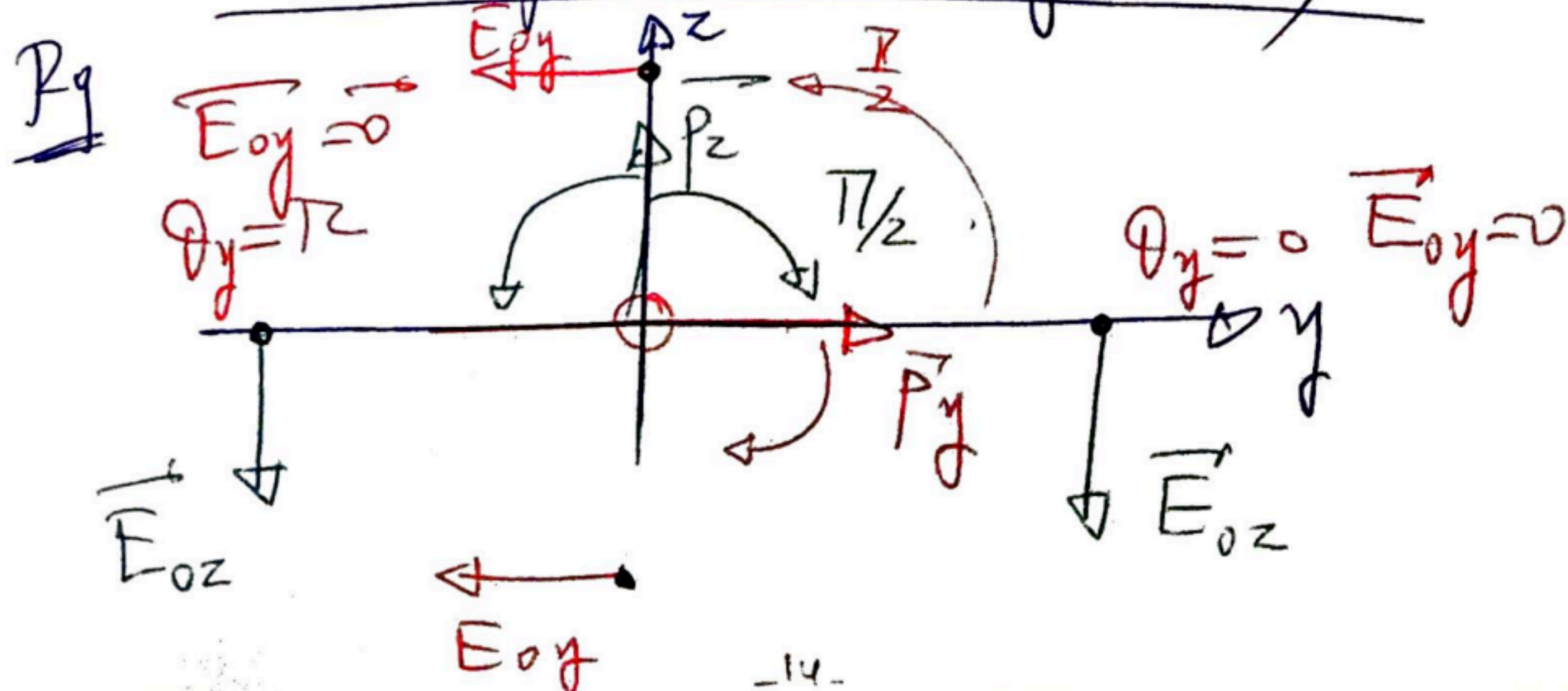
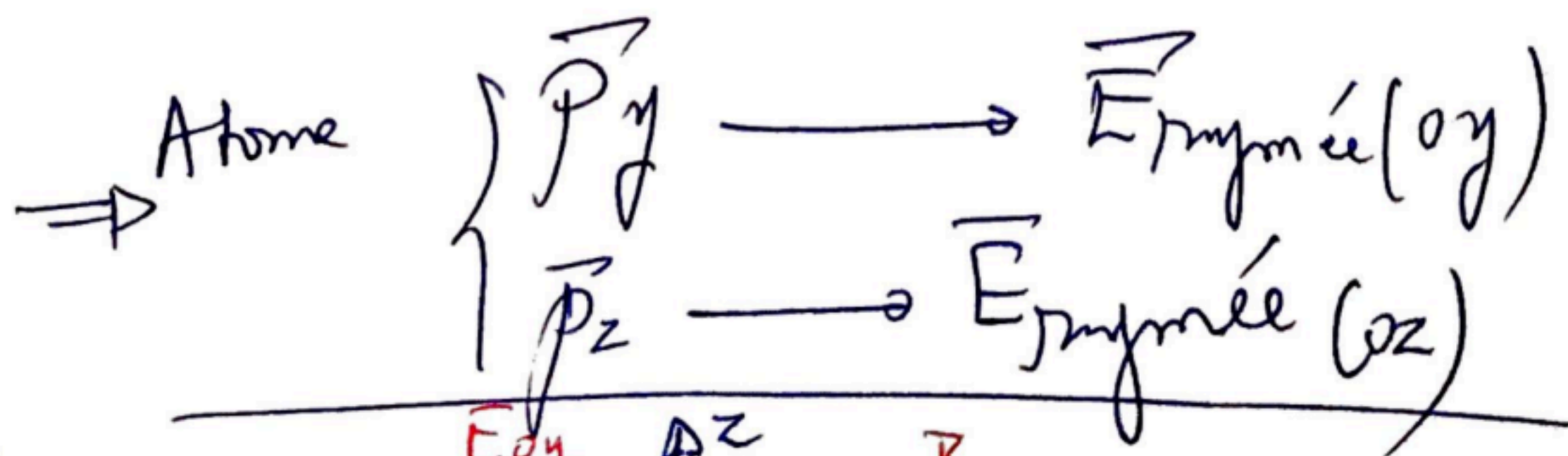
a) $\vec{E}_{\text{soleil}} = \sum \vec{E}_n \cos(n\omega t)$



$$\vec{p}_1, \vec{p}_2, \vec{p}_3, \dots, \vec{p}_n \Rightarrow$$

$$\bar{p}_{tot} = \sum \bar{p}_n$$

$$\underline{d)} \quad \vec{E}_{\text{orient}} = \begin{cases} E_y = \bar{E}_0 \cos(\omega t - kx - \varphi_1(t)) \\ E_z = \bar{E}_0 \cos(\omega t - kx - \varphi_2(t)) \end{cases}$$

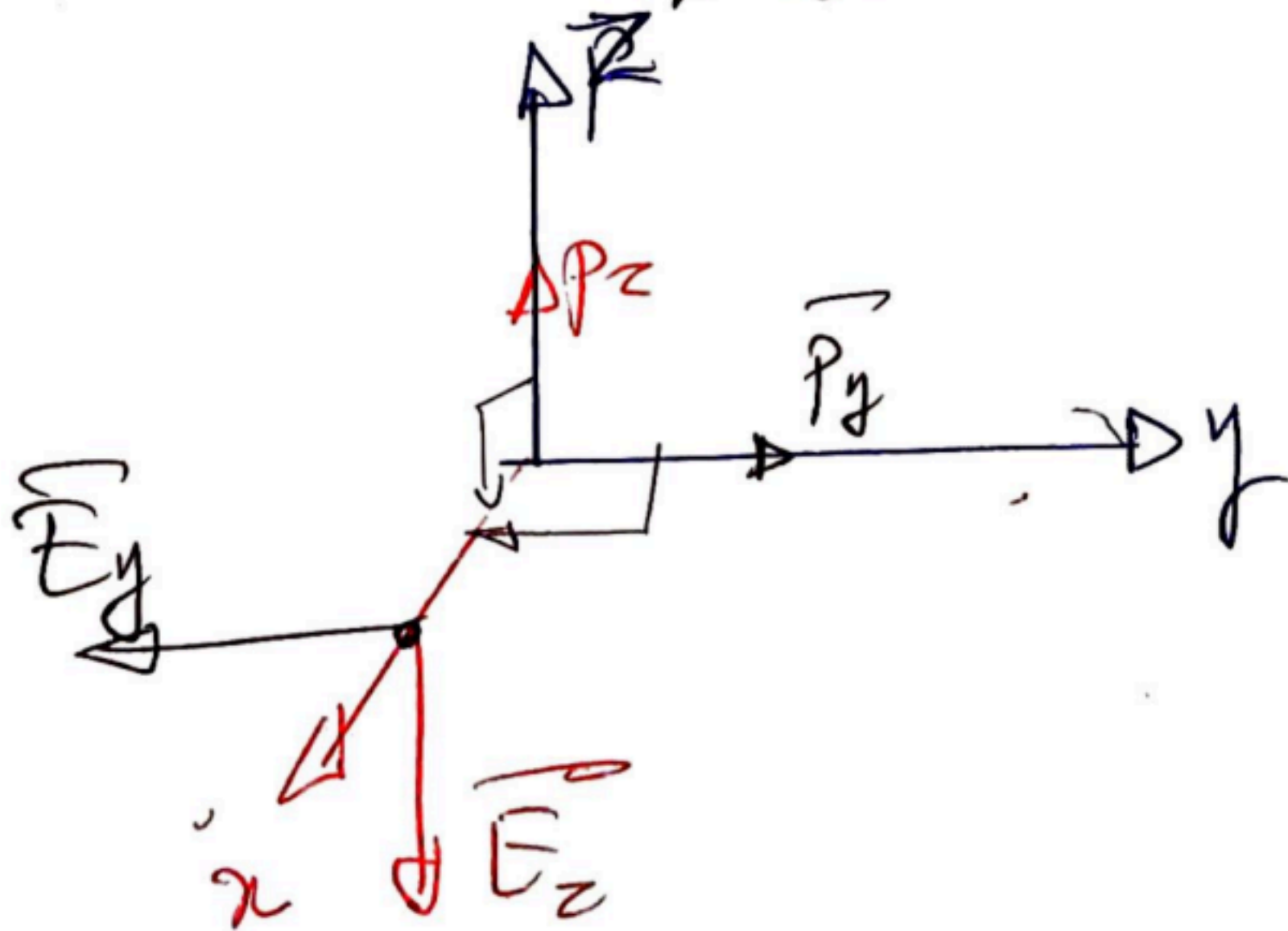


$$\vec{E}_{\text{rayonnée}} = \frac{\ddot{\vec{p}} \sin \theta}{4\pi \epsilon_0 c^2 r} \vec{e}_\theta$$

$$\theta = (\vec{p}, \vec{OM})$$

$$\theta = \frac{\pi}{2}; \quad \vec{E}_{\theta z} = \frac{\ddot{p}_z \times 1}{4\pi \epsilon_0 c^2 r} (-\vec{e}_z); \quad \eta \in (0, \pi)$$

$$\eta \in (0, \pi); \quad \vec{E}_{\theta y} = \frac{\ddot{p}_y \times 1}{4\pi \epsilon_0 c^2 r} (-\vec{e}_y)$$



$$2) \quad m \ddot{\vec{r}} = -e \vec{E} - m \omega_0^2 \vec{r} - m \Gamma \dot{\vec{r}}$$

$$a) \quad (oz): \quad m \ddot{z} = -e E - m \omega_0^2 z - m \Gamma \dot{z}$$

$$\ddot{z} + \Gamma \dot{z} + \omega_0^2 z = -\frac{e}{m} E$$

$$z(t) = Z_0 e^{i(\omega t + \varphi)}$$

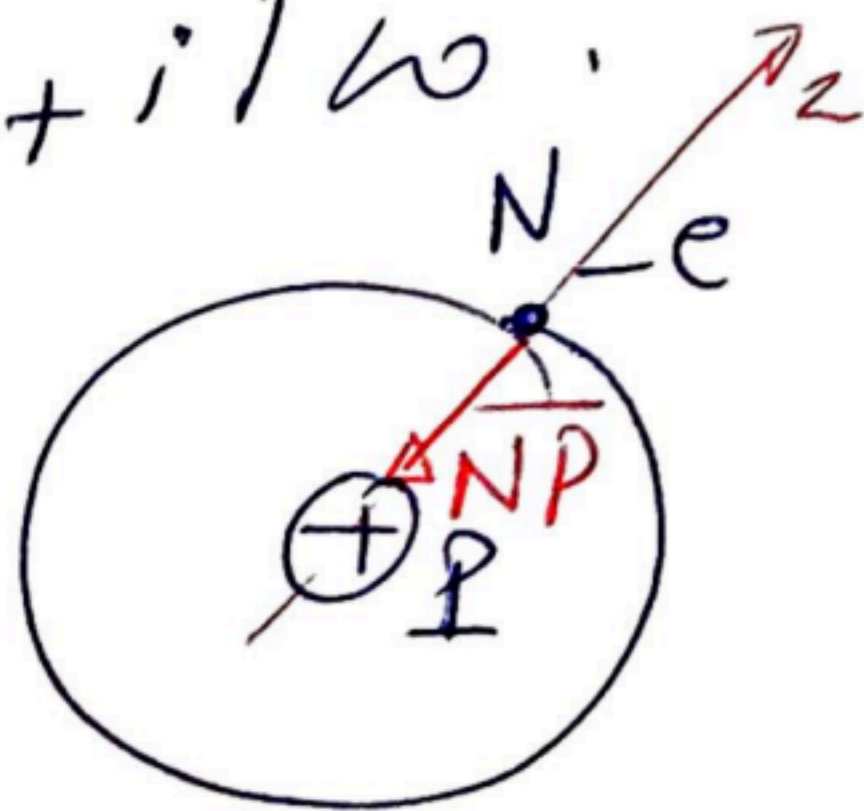
$$[-\omega^2 + \omega_0^2 + i\Gamma\omega] Z = -\frac{e}{m} E$$

$$Z = -\frac{e}{m} \frac{E(t)}{(\omega_0^2 - \omega^2) + i\Gamma\omega}$$

$$b) \quad \vec{p} = q N \vec{P} = +e(-z \vec{u}_z)$$

$$\vec{p} = -e z(t) \vec{u}_z$$

$$= +\frac{e^2}{m} \frac{E}{(\omega_0^2 - \omega^2) + i\Gamma\omega}$$



3) Larmor: $P_{\text{rayonée}} = \frac{e^2}{6\pi\epsilon_0 c^3} \ddot{a}^2(t)$

a) dém
$$P_{\text{ray}} = \int_0^\pi \frac{(\ddot{\rho})^2 \sin^2 \theta}{16\pi^2 \epsilon_0 c^3 r^2} r^2 \sin \theta d\theta \int_0^{2\pi} d\varphi$$

$$= \frac{(\ddot{\rho})^2 \times 2\pi \times 4}{16\pi^2 \epsilon_0 c^3 \times 3} = \frac{(\ddot{\rho})^2}{6\pi \epsilon_0 c^3}$$

or $\ddot{\rho} = -e \ddot{z}(t) = -e a(t)$

$$P_{\text{ray}} = \frac{e^2}{6\pi \epsilon_0 c^3} a^2(t)$$

$$\langle P_{\text{ray}} \rangle = \frac{e^2}{6\pi \epsilon_0 c^3} \langle a^2(t) \rangle$$

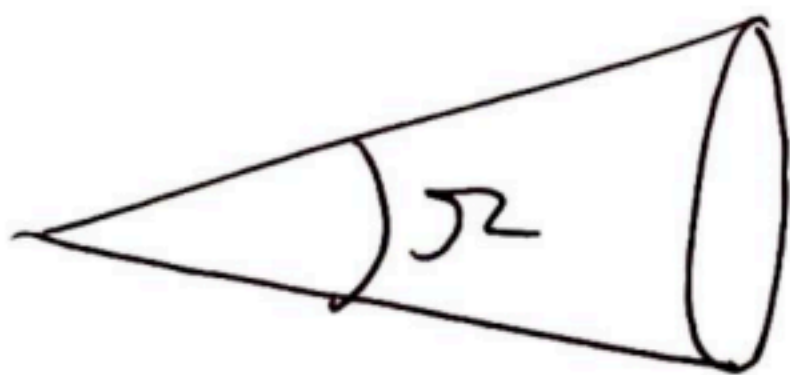
$$\ddot{z} = -\frac{e}{m} \frac{-\omega^2 E_0 e^{i\omega t}}{(\omega_0^2 - \omega^2) + i\Gamma\omega}$$

$$\langle a^2(t) \rangle = \frac{1}{2} \text{Re} \left[\underbrace{\ddot{z}}_{\downarrow e^{i\omega t}} \underbrace{\ddot{z}^*}_{\downarrow e^{-i\omega t}} \right]$$

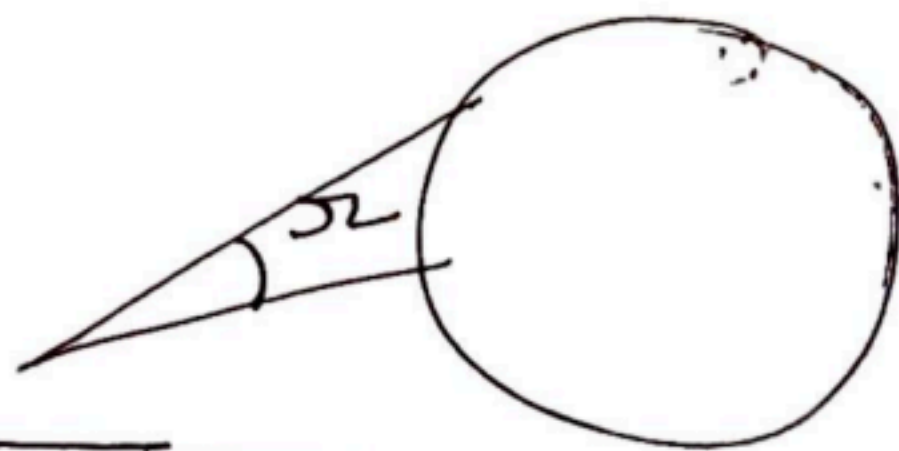
$$\langle \dot{a}^2 \rangle = \frac{1}{2} \text{Re} \left[\frac{e^2 E_0^2}{m^2} \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\Gamma \omega)^2} \right]$$

$$\langle P_{\text{avg}} \rangle = \frac{e^4 E_0^2 \omega^4}{12 \pi \epsilon_0 c^3 m^2 (\omega_0^2 - \omega^2)^2 + (\Gamma \omega)^2}$$

$$b) \sigma(\omega) = \frac{\langle P_{\text{avg}} \rangle}{I_0}$$



$$d\sigma = \frac{dS \cdot \vec{n}}{(P_1)^2}$$



$$\sigma = 4\pi$$

$$I_0 = \frac{\epsilon_0 c E_0^2}{2}$$

O.P.P.M

$$\vec{E}_i = E_0 \cos(\omega t - kn) \vec{u}_y$$

$$\vec{B}_i = \frac{E_0}{c} \cos(\omega t - kn) \vec{u}_z$$

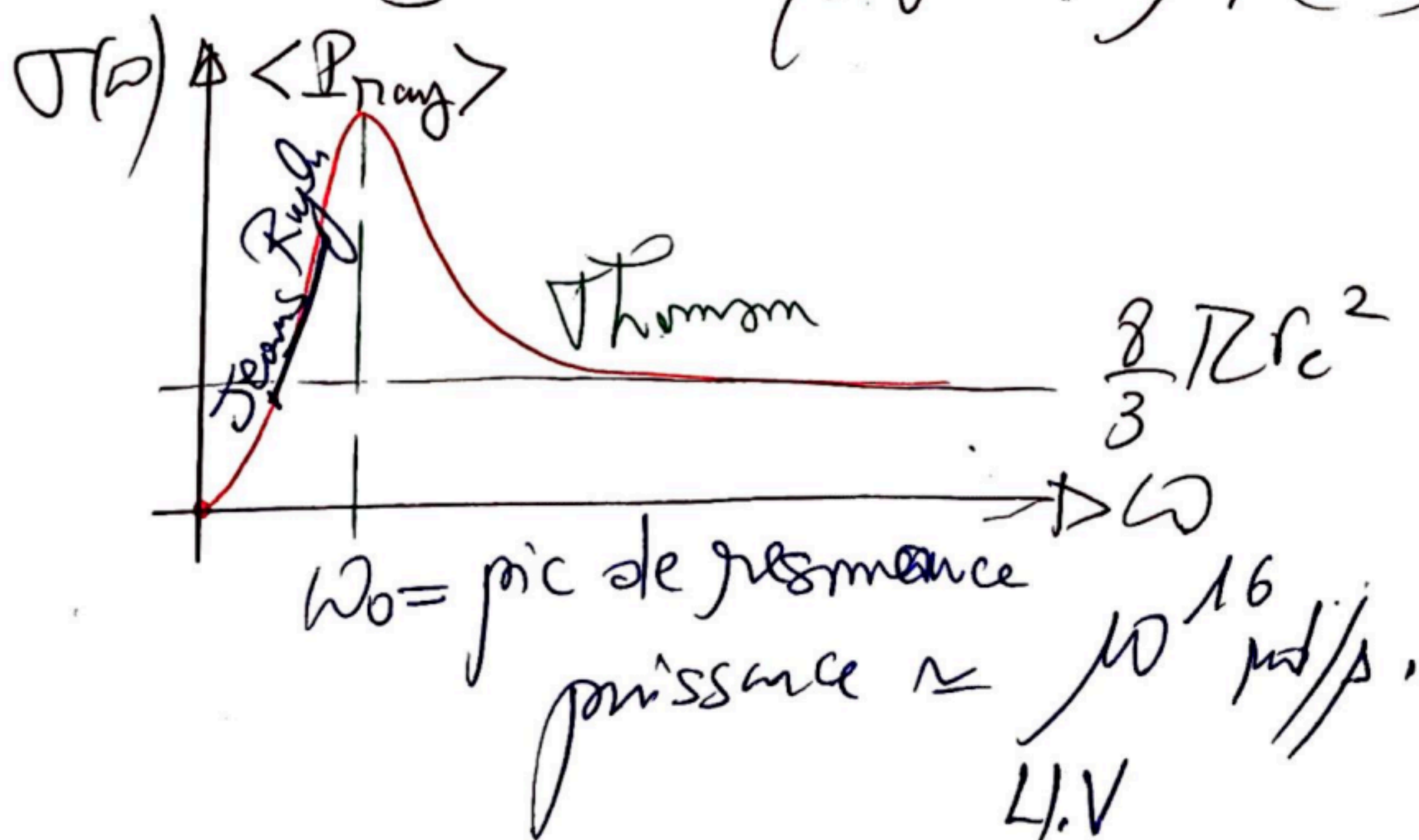
$$\vec{R}_i = \frac{E_0^2}{\mu_0 c} \cos^2(\omega t - kn) \vec{u}_n$$

$$\sigma(\omega) = \frac{e^4 E_0^2}{6\pi \epsilon^2 c^4 m^2} \cdot \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\Gamma\omega)^2}$$

$\left(\frac{\text{Watt}}{\text{Watt} \cdot \text{m}^{-2}} \right)$

$$c) m \cdot c^2 = \frac{e^2}{4\pi \epsilon r_e}$$

$$\sigma(\omega) = \frac{8}{3} \pi r_e^2 \cdot \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\Gamma\omega)^2}$$



$$4) \langle P_{\text{ray}} \rangle = \frac{8}{3} \pi r_e^2 \cdot \frac{I_0 \omega^4}{(\omega_0^2 - \omega^2)^2}$$

$$= \frac{e^4 E_0^2}{12\pi \epsilon c^3 m^2} \cdot \frac{\omega^4}{(\omega_0^2 - \omega^2)^2}$$

$$\omega_0 = 2,3 \cdot 10^{16} \text{ rad/s} \longrightarrow \lambda_0 = 0,08 \mu\text{m} \in \text{UV}$$

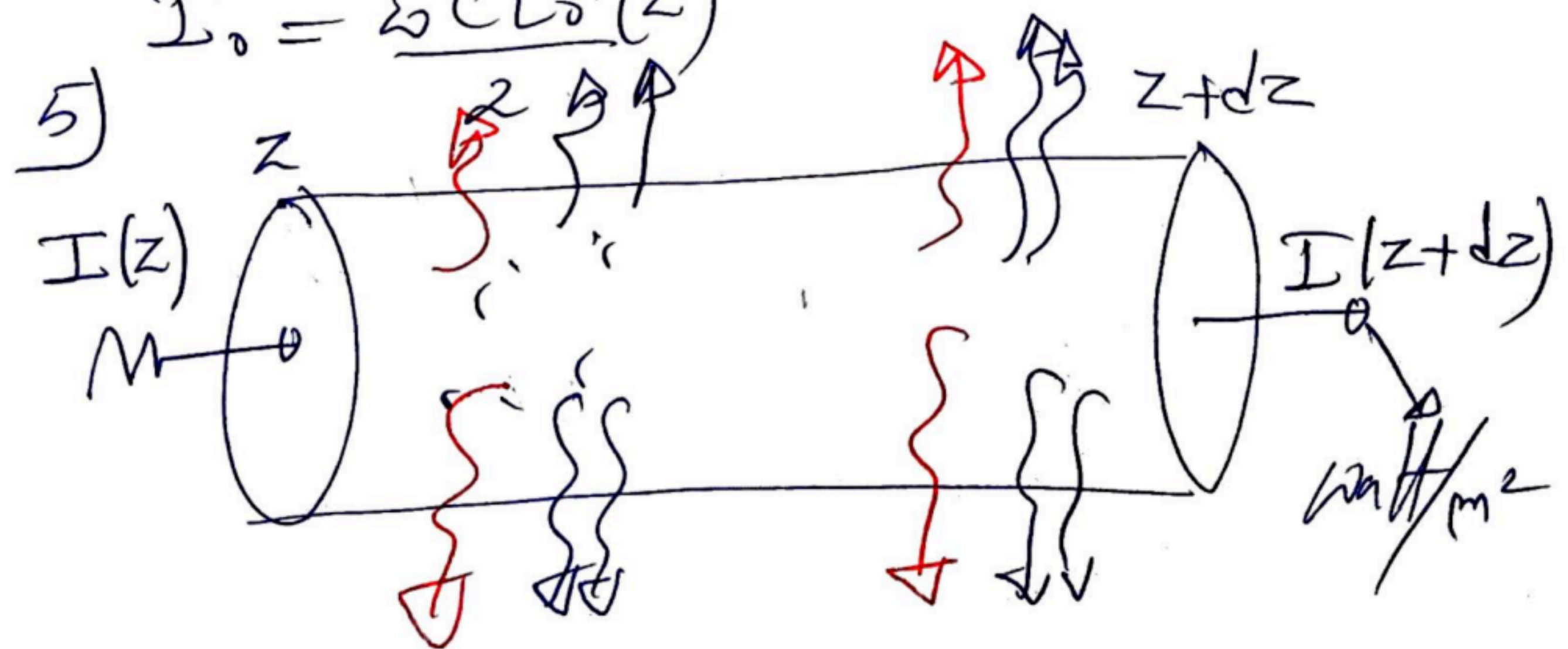
$$\left(\frac{\omega_0}{\omega_{\text{max}}} \right)^2 = \left(\frac{2\pi c}{\lambda_0} \right)^2 \left(\frac{\lambda_{\text{min}}}{2\pi c} \right)^2$$

$$= \left(\frac{\lambda_{\text{min}}}{\lambda_0} \right)^2 = \left(\frac{0,32}{0,08} \right)^2 = \gg 1$$

$$\langle P_{\text{ray}} \rangle \simeq \frac{8}{3} \pi r_e^2 I_0 \frac{\omega^4}{\omega_0^4}$$

$$\frac{\langle P_{\text{radiat}} \rangle}{\langle P_{\text{ray}} \rangle} = \left(\frac{\omega_0}{\omega_R} \right)^4 = 30.$$

$$I_0 = \varepsilon_0 c E_0^2(z)$$



$$\text{watt/m}^3 \leftarrow P_{\text{diffusée}} = n \langle P_{\text{ray}} \rangle$$

$$= n \frac{8}{3} \pi r_e^2 I(z) \frac{\omega^4}{\omega_0^4}$$

Böhlen

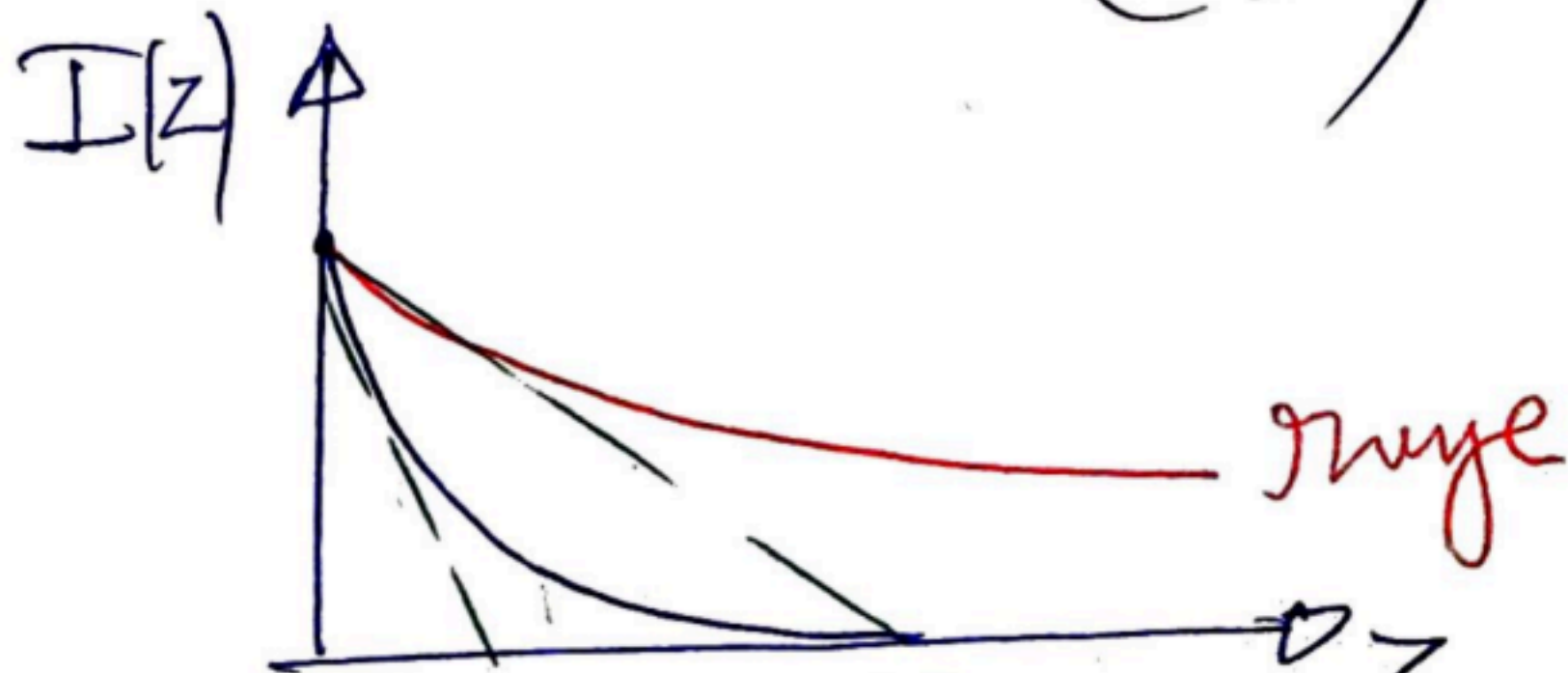
$$I(z)S = I(z+dz)S + \frac{8}{3} \pi r_e^2 n I(z) \left(\frac{\lambda_0}{\lambda} \right)^4 S dz$$

$$\frac{I(z) - I(z+dz)}{I(z)} = \frac{8}{3} \pi r_e^2 n \left(\frac{\lambda_0}{\lambda} \right)^4 dz$$

$$\frac{+ dI(z)}{I(z)} = - \left[\left(\frac{8}{3} \pi r_e^2 \frac{\lambda_0^4}{\lambda^4} \right) n \right] dz$$

$$\frac{dI(z)}{I(z)} = - \frac{dz}{H_\lambda}$$

$$I(z) = I_0 \exp\left(-\frac{z}{H_\lambda}\right) \quad H_\lambda = \frac{3 \lambda_0^4}{8 \pi r_e^2 \lambda^4 n}$$



$$H_{\lambda_B} = 320 \text{ km} \quad H_{\lambda_{\text{raye}}} \approx 3300 \text{ km}$$

$$n = \frac{N}{V} ; \quad \text{G-Parrish: } PV = \frac{m}{M} RT$$

$$P V = \frac{m_a \cdot N}{m_a C_V A} R T$$

$$P = \frac{N}{V} \cdot \left(\frac{R}{C_V A} \right) T = n \cdot k_B T = P$$

$k_B = \text{Boltzmann}$

$$E_c = \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)$$

$$\langle E_c \rangle = \frac{3}{2} m \langle v_x^2 \rangle, \quad \left\langle \frac{1}{2} m v_x^2 \right\rangle = \frac{1}{2} k_B T$$

$$= \frac{3}{2} k_B T$$

$$U = N \frac{3}{2} k_B T,$$

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{3N k_B}{2}$$

$$C_p = C_V + R = C_V + N k_B$$

$$\gamma = \frac{5}{3}$$