

Diffraction - Réseau

Principe de Huygens - Fresnel:

1) Chaque point P d'une surface S atteinte par la lumière provenant d'une source primaire se comporte comme une source secondaire émettant une onde sphérique - ou ondelette d'amplitude en P proportionnelle à celle de l'onde incidente et à l'élément dS entourant le pt P .

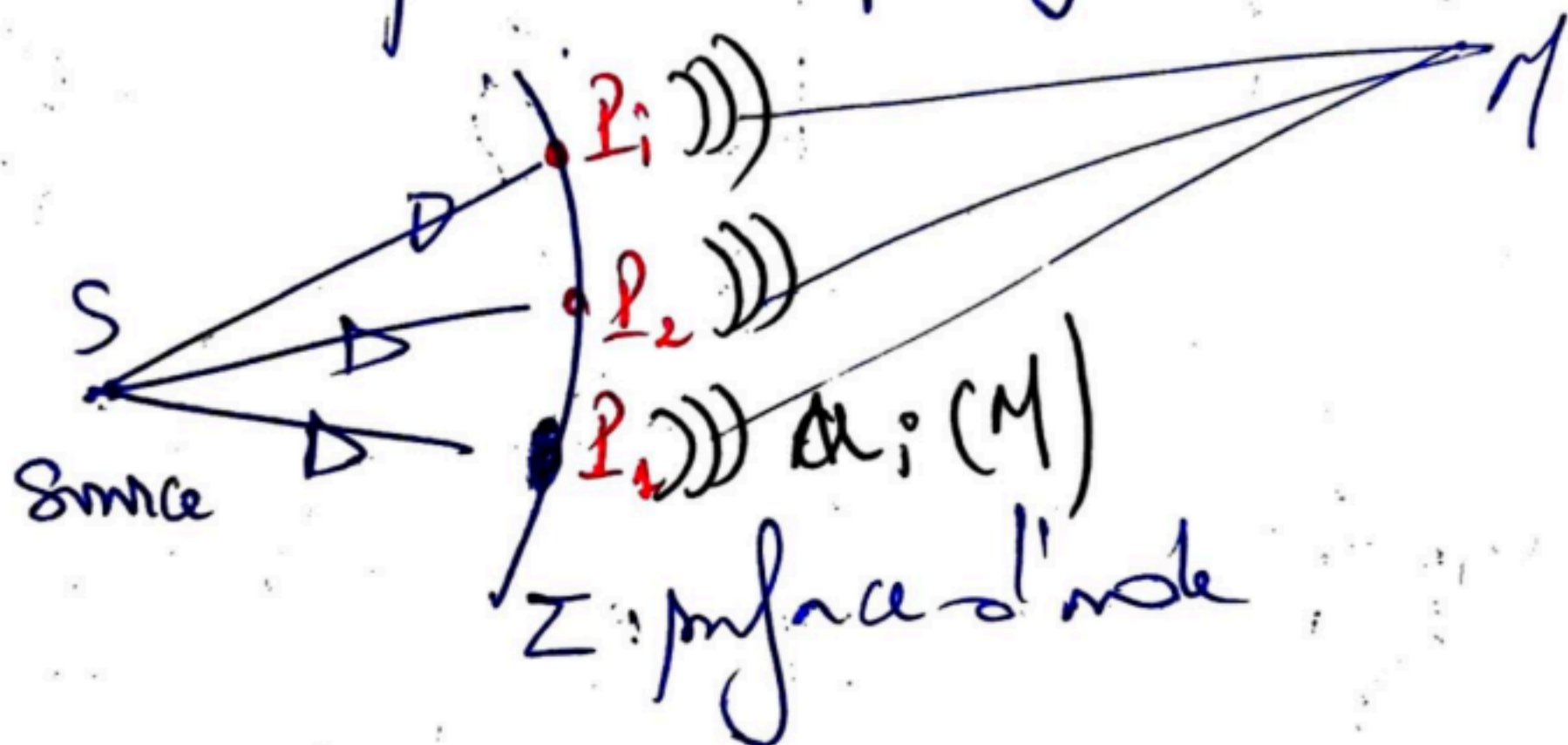
Les sources secondaires sont cohérentes et les différentes ondelettes émises P interfèrent entre elles: en un point M , les amplitudes P s'ajoutent.

2) La diffraction de Fraunhofer est observable lorsque la source primaire et l'écran d'observation sont très éloignés de la pupille diffractante de sorte que qu'on les considère à l'infini.

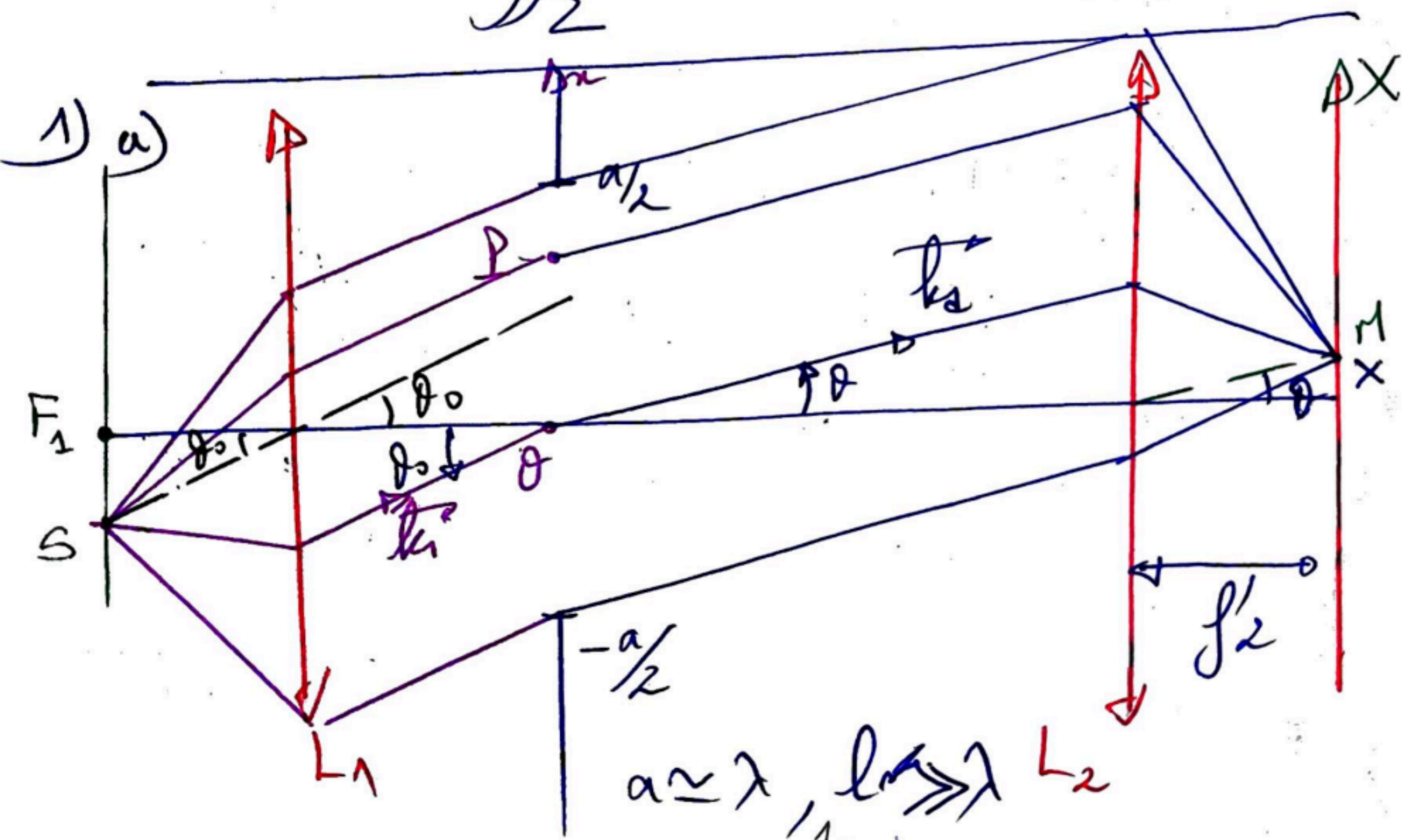
3) L'approximation de Gauss est l'approximation linéaire de l'optique géométrique. Les rayons envoyés par un objet situé au voisinage de l'axe optique du système ont des faibles inclinaisons et faibles incidences (rayons paraxiaux)

Réseaux

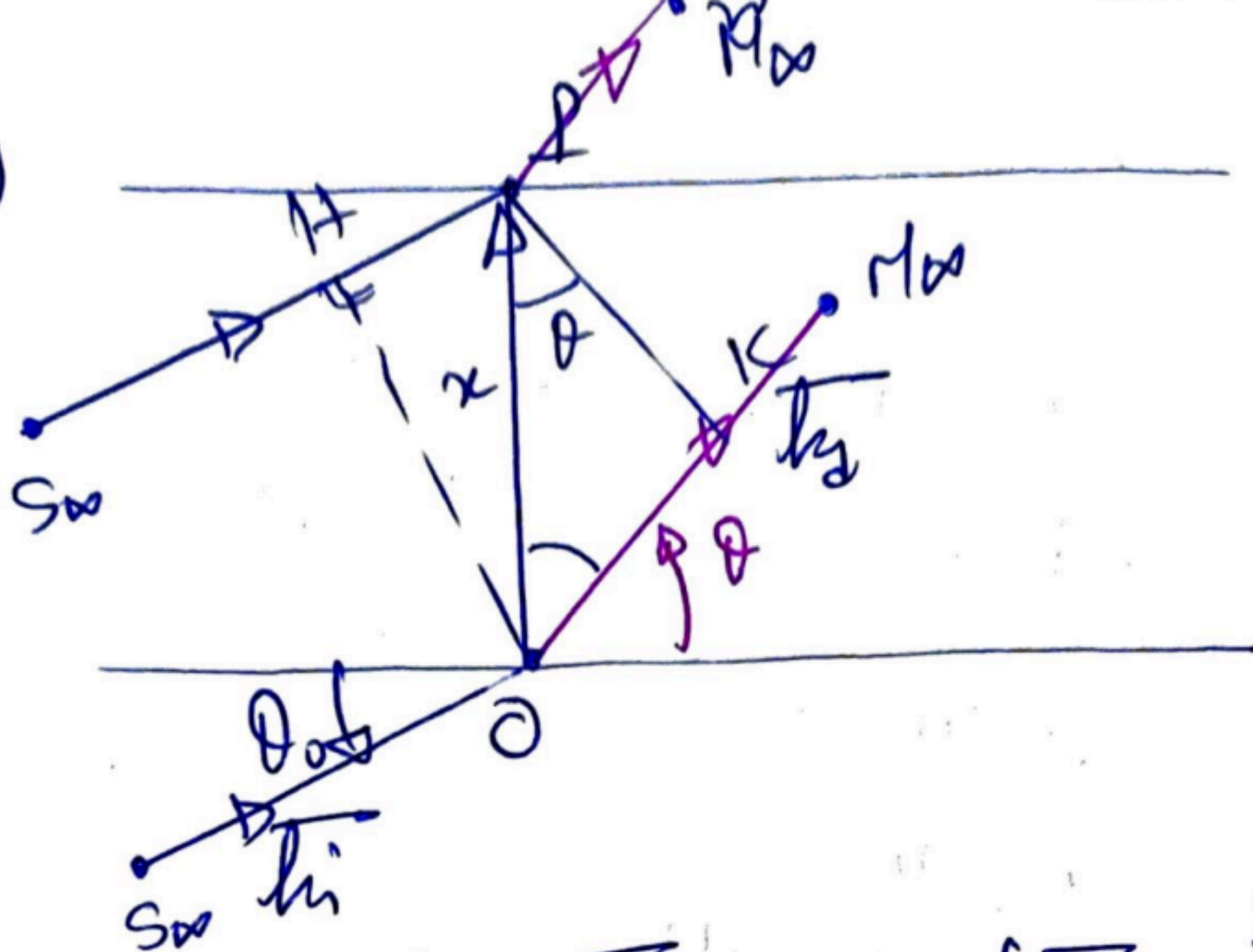
Diffraction : c'est un phénomène physique
qui n'est pas l'optique géométrique.



$$\underline{a}(r) = \alpha a_0 \int_{\Sigma} \underline{t}(P) \exp[i(\underline{k}_s - \underline{k}_i) \cdot \underline{OP}] d\Sigma$$



1b)



$$(\overline{A_H} - \overline{A_O}) \cdot \overline{OP} = \frac{2\pi}{\lambda} (\overline{u_d} - \overline{u_i}) \cdot \overline{OP}$$

$$\frac{2\pi}{\lambda} n [\sin \theta - \sin \theta_0]$$

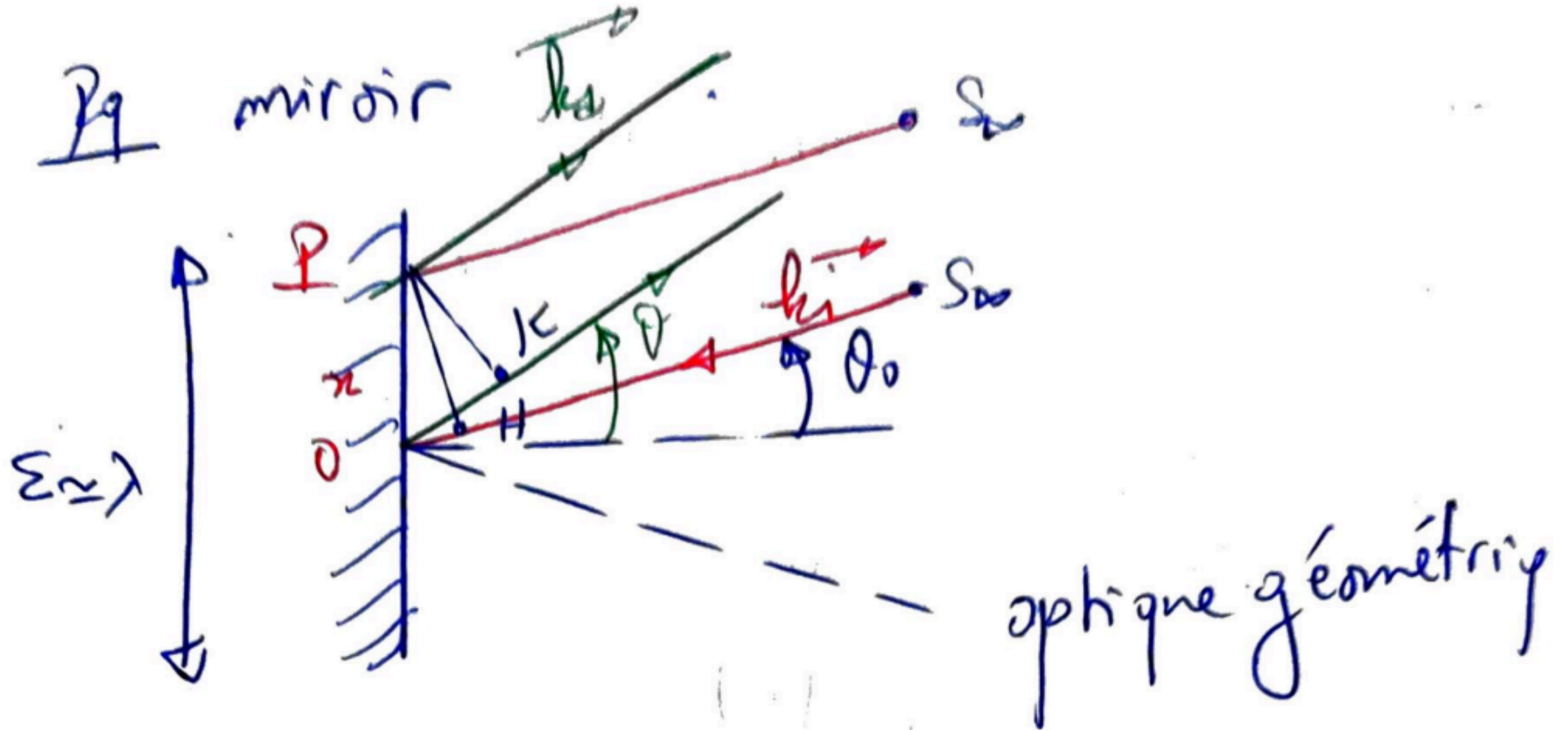
2^{de} méthode $CP = \frac{2\pi}{\lambda} \delta_{O/P}$

$$\delta_{O/P} = (S_{\infty} O M_{\infty}) - (S_{\infty} A M_{\infty})$$

$$= -PH + OK$$

$$= -n \sin \theta_0 + n \sin \theta$$

$$\underline{\delta_{O/P}} = n (\sin \theta - \sin \theta_0) \quad \text{---}$$



$$(S_{\infty} O \Gamma_{\infty}) - (S_{\infty} P \Gamma_{\infty}) = \delta_0 / l$$

$$= n (S_n \theta - S_n \theta_0)$$

Seite 1) b) $\frac{1}{4}l$ incidence normale $\theta_0 = 0$.

$$\underline{S} = \alpha S_0 \int_{-\frac{a}{2}}^{\frac{a}{2}} \exp\left[i \frac{2\pi}{\lambda} x(\sin\theta)\right] dx dy$$

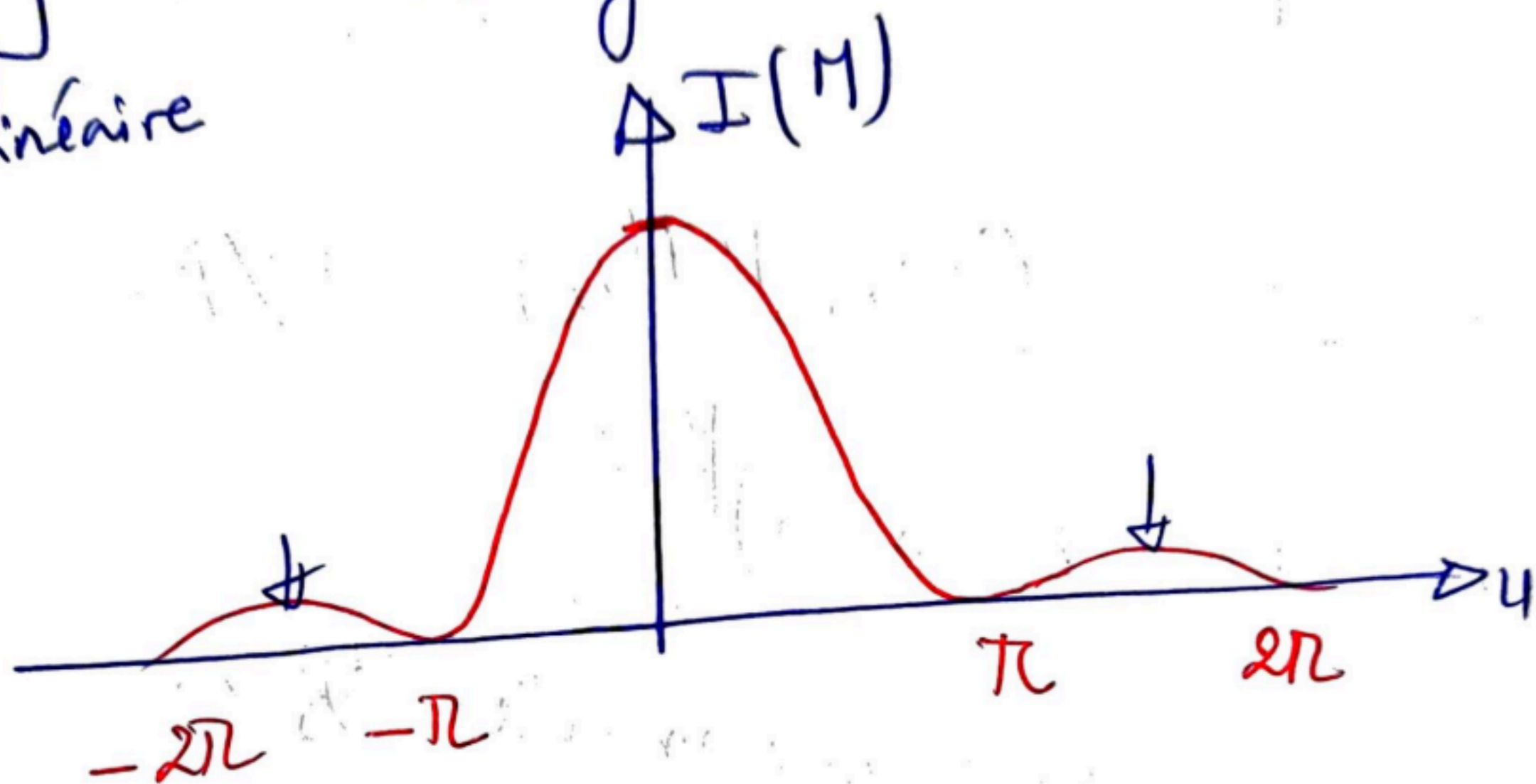
$$= \alpha S_0 \frac{\mathcal{L}\left\{ \sin\left(\frac{\pi}{\lambda} a \sin\theta\right) \right\}}{i \frac{2\pi}{\lambda} \sin\theta}$$

$$\underline{S(M)} = A_0 \operatorname{sinc} \left(\frac{\pi a \sin \theta}{\lambda} \right)$$

$$c) I = \underline{S} \underline{S}^* = I_0 \sin^2(u)$$

$$u = \frac{\pi a \sin \theta}{\lambda} \simeq \frac{\pi a \theta}{\lambda} = \frac{\pi a X}{\lambda f'_2}$$

$$X = f'_2 \theta \quad \begin{array}{l} \text{↪ angulaire} \\ \text{↪ linéaire} \end{array}$$

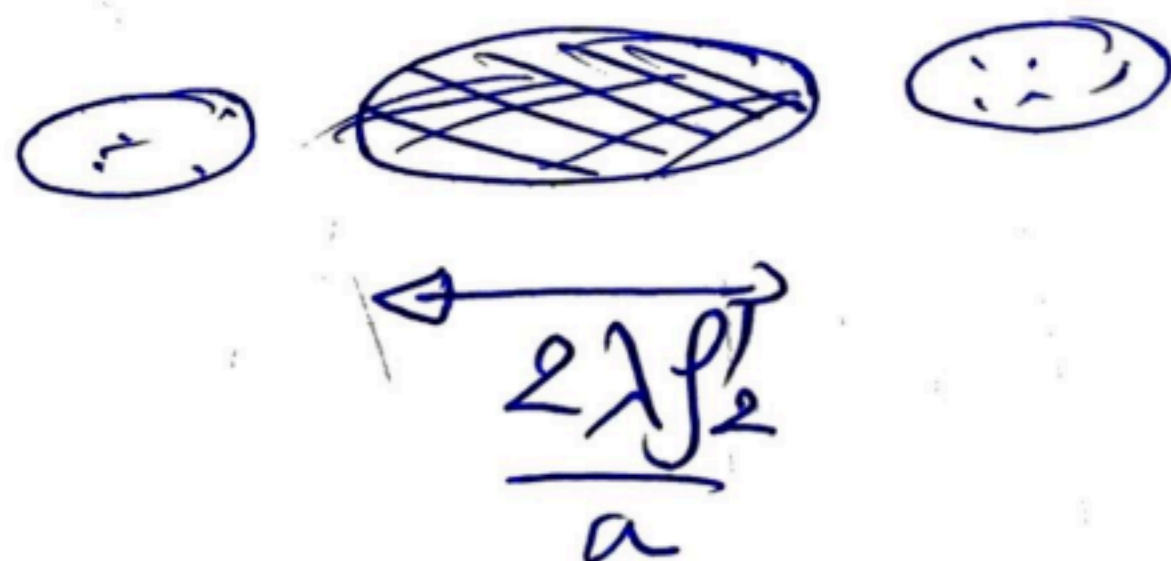


$$\left(\frac{\sin u}{u} \right) = 0 \Rightarrow u = k\pi = \frac{\pi a \theta}{\lambda}$$

$$\theta_{k_{\min}} = \frac{X_{\min}}{f'_2} = \frac{k\lambda}{a}$$

$$(\Delta \theta)_{\min} = \frac{2\lambda}{a}$$

$$(\Delta X)_{\min} = \frac{2\lambda f'_2}{a}$$



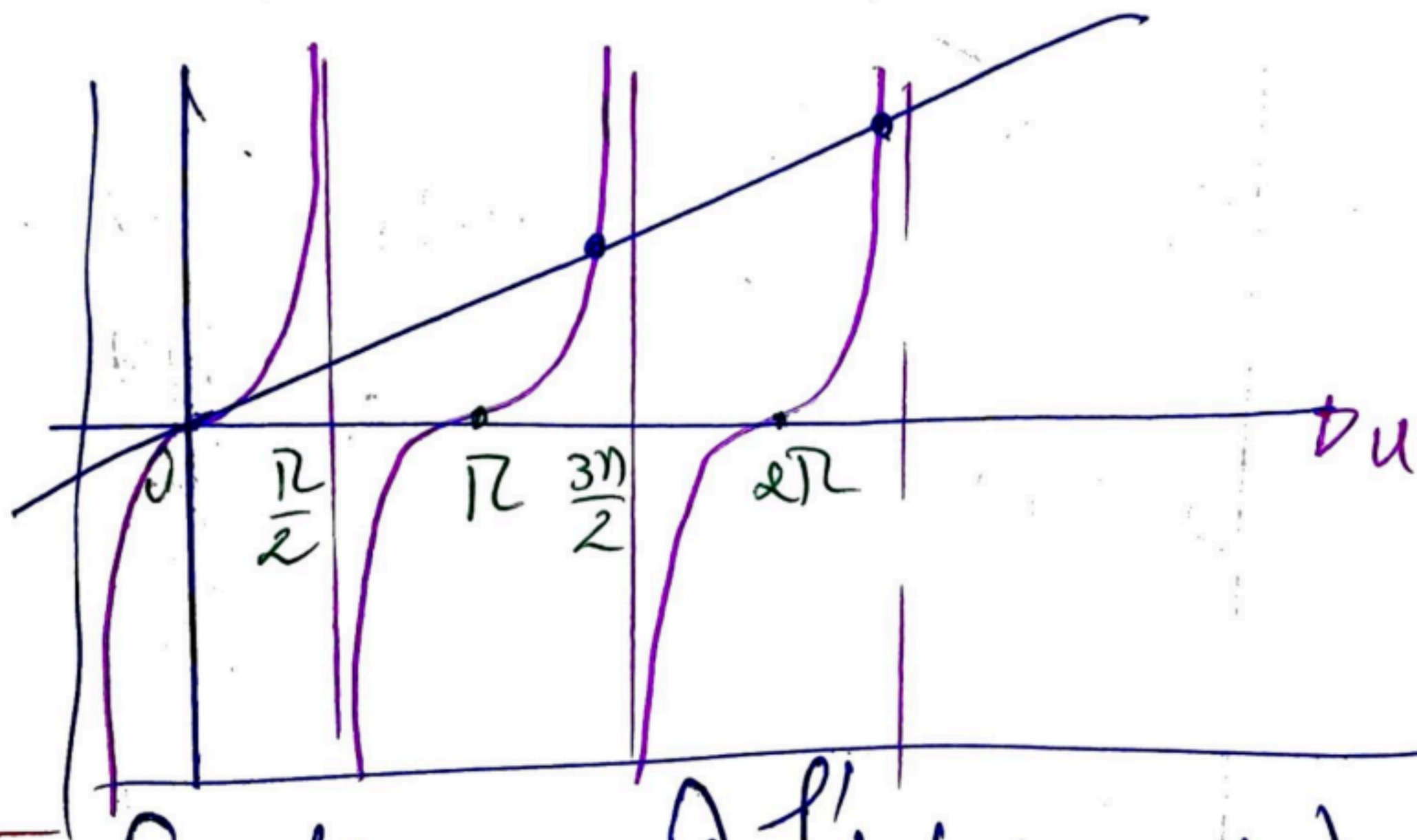
positions les maxima secondaire

$$\frac{d}{du} \left(\frac{\sin u}{u} \right)^2 = 0 \Rightarrow 2 \left(\frac{\sin u}{u} \right) \cdot \left(\frac{\sin u}{u} \right)' = 0$$

$$\Rightarrow \frac{4 \cos u - \sin u}{u^2} = 0$$

$$4 \cos u = \sin u \Rightarrow u \approx \left(k + \frac{1}{2} \right) \pi = \frac{\pi a x_{\max}}{\lambda f_2}$$

$$k \neq 0, \quad k \neq -1$$



$$x_{\max}^{\text{sec}} = \frac{\lambda f_2}{a} \left(k + \frac{1}{2} \right)$$

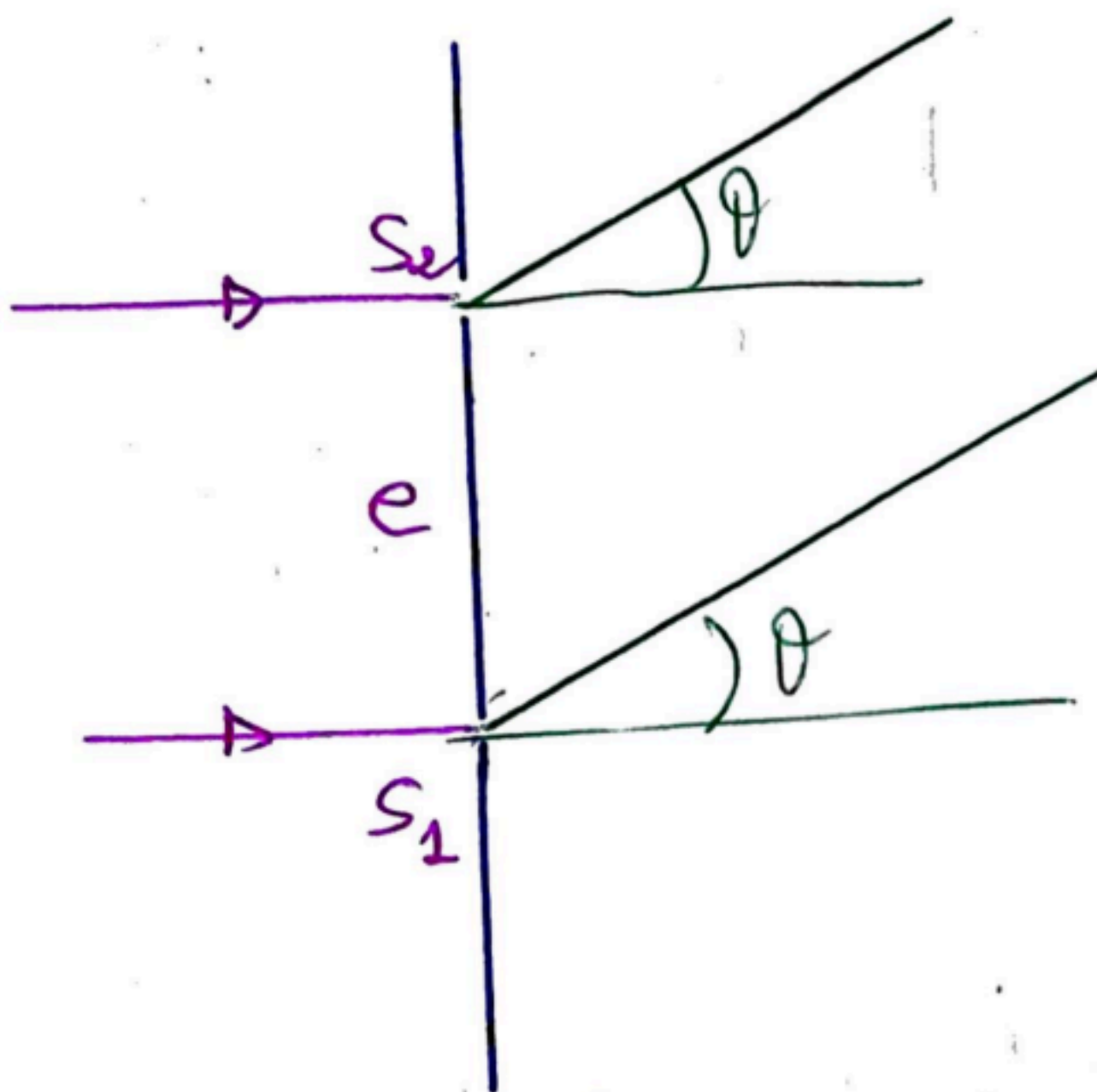
$$I_{\max, \text{sec}} = I_0 \sin^2 \left(\frac{u_{\max}}{a} \right)$$

$$= I_0 \frac{(\pm 1)^2}{\left(k + \frac{1}{2} \right)^2 \pi^2}$$

$$\frac{I_{m\gamma, \Delta}}{I_0} = \frac{4}{(2K+1)^2 \pi^2}$$

$$K=1 \quad \frac{I_{m\gamma, \Delta}}{I_0} = \frac{4}{\pi^2 9} = 4,5\%$$

$$K=2 \quad \frac{I_{m\gamma, \Delta}}{I_0} = \frac{4}{25 \pi^2} = 1,6\%$$



$$\begin{aligned} \delta(r) &= e \sin \theta \\ &= e \sin \theta \\ &= \frac{e \lambda}{\delta_2} \end{aligned}$$

$$\Phi = \frac{2\pi}{\lambda} \delta(r)$$

$$\underline{A}(r) = \underline{A}_1(r) + \underline{A}_2(r)$$

$$\underline{A}(r) = A_0 \operatorname{sinc}(u) (1 + e^{-i\Phi})$$

$$\underline{A}^*(r) = A_0 \operatorname{sinc}(u) (1 + e^{i\Phi})$$

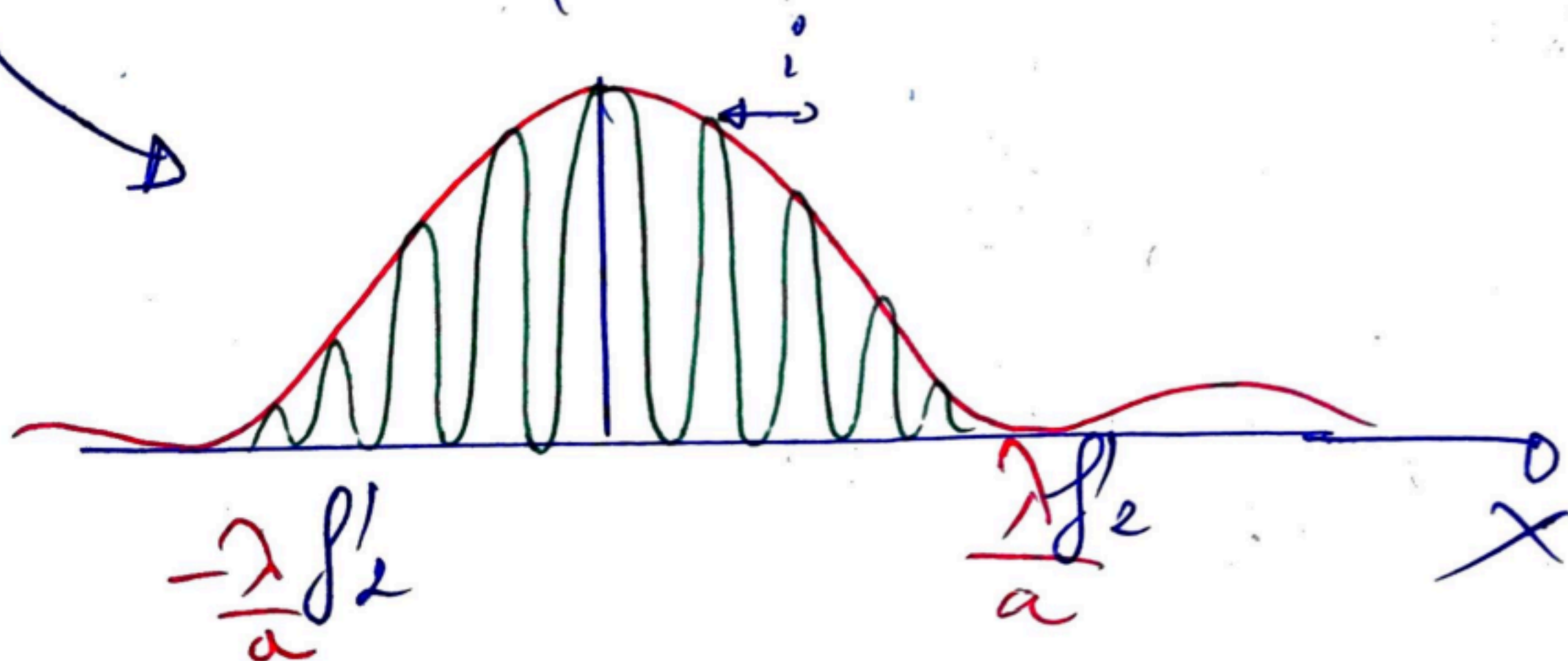
$$I = A_0^2 \text{sinc}^2(u) (1 + e^{-i\Phi}) (1 + e^{i\Phi})$$

$$I = 2I_0 \text{sinc}^2(l) (1 + \cos\Phi)$$

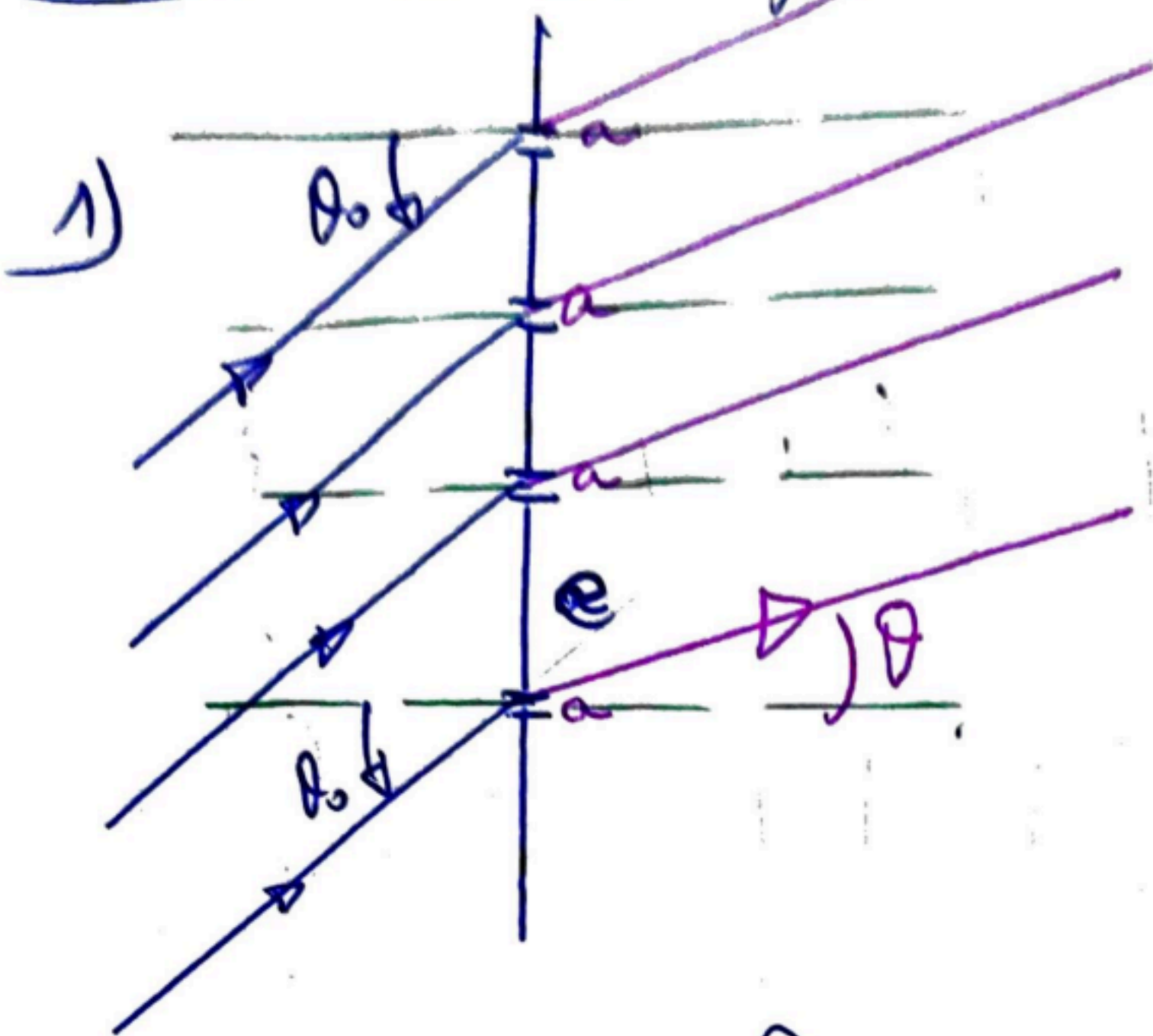
$$I = 2I_0 \text{sinc}^2\left(\frac{\pi a \theta}{\lambda}\right) \left(1 + \cos\left(\frac{2\pi e \theta}{\lambda}\right)\right)$$

$a \ll \lambda$ MP2 2001

$$I \approx 2I_0 \left(1 + \cos\left(\frac{2\pi e \theta}{\lambda}\right)\right)$$



II) Réseaux par transmission

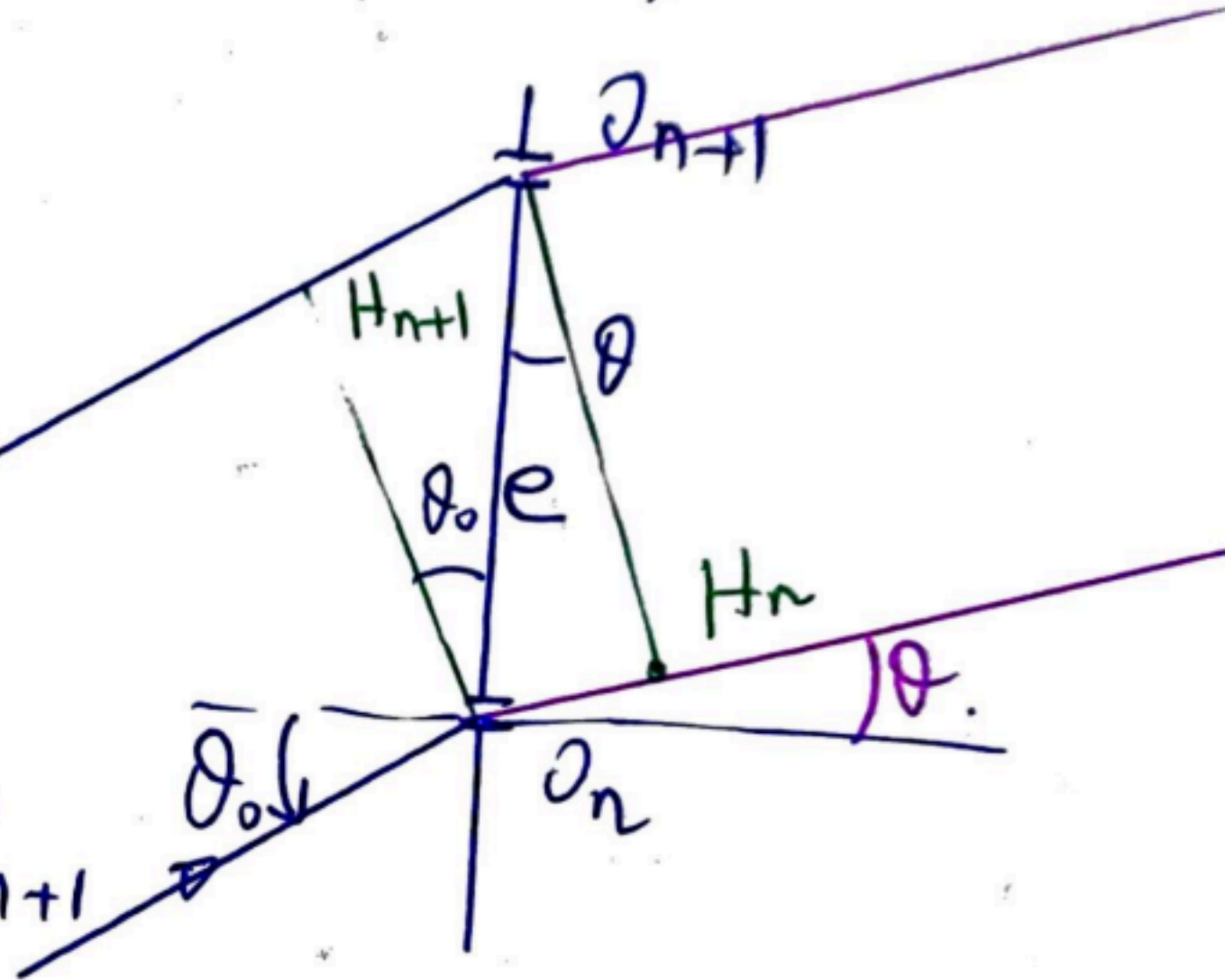


$e =$ le pas du réseau

$n = \frac{1}{e} =$ nbs de traits/mm.

2) $\Phi = \frac{2\pi}{\lambda} \delta_{O_n/O_{n+1}}$

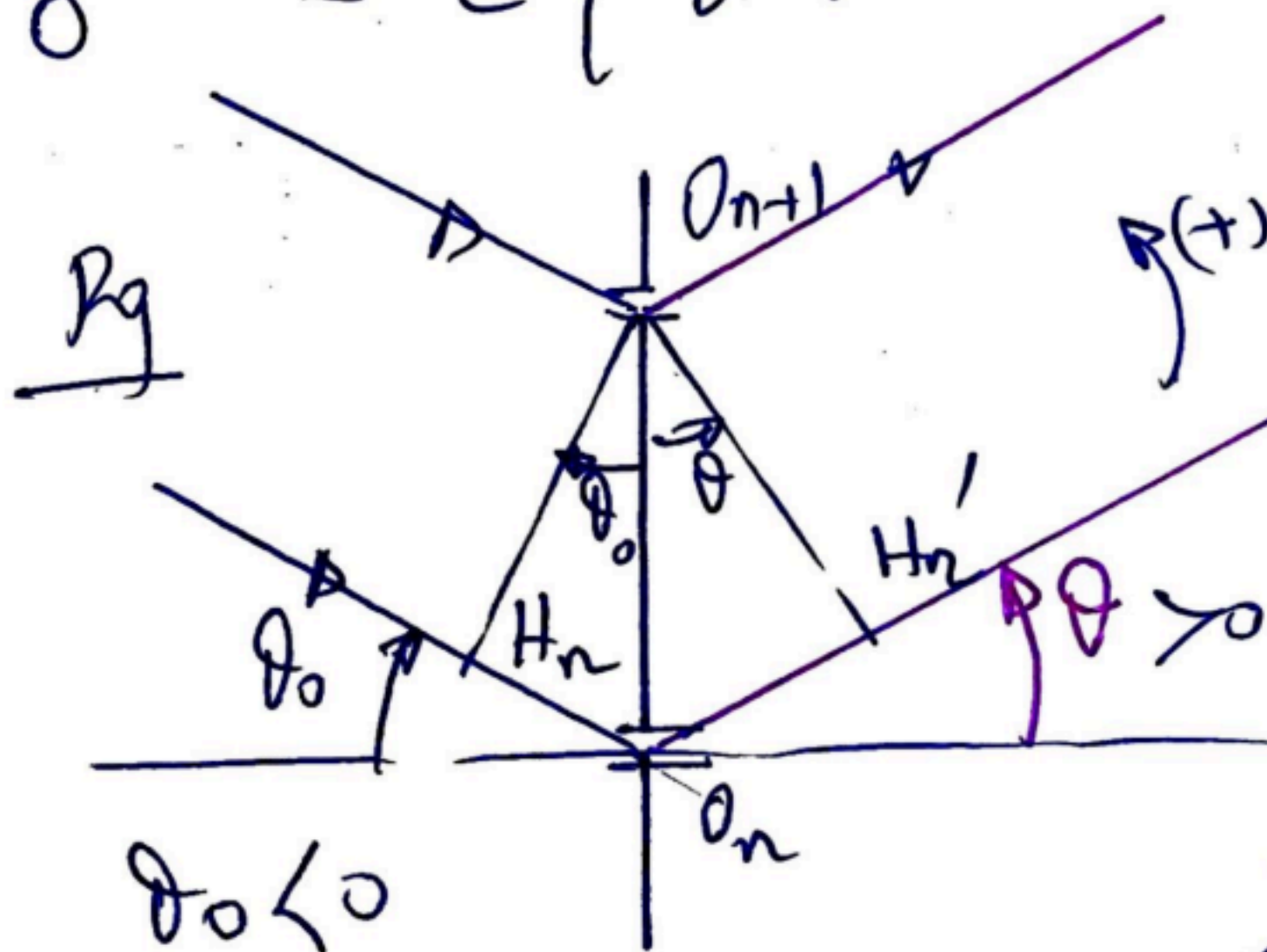
$\delta_{O_n/O_{n+1}} = O_n H_n - O_{n+1} H_{n+1}$



$\delta = e (\sin \theta - \sin \theta_0)$

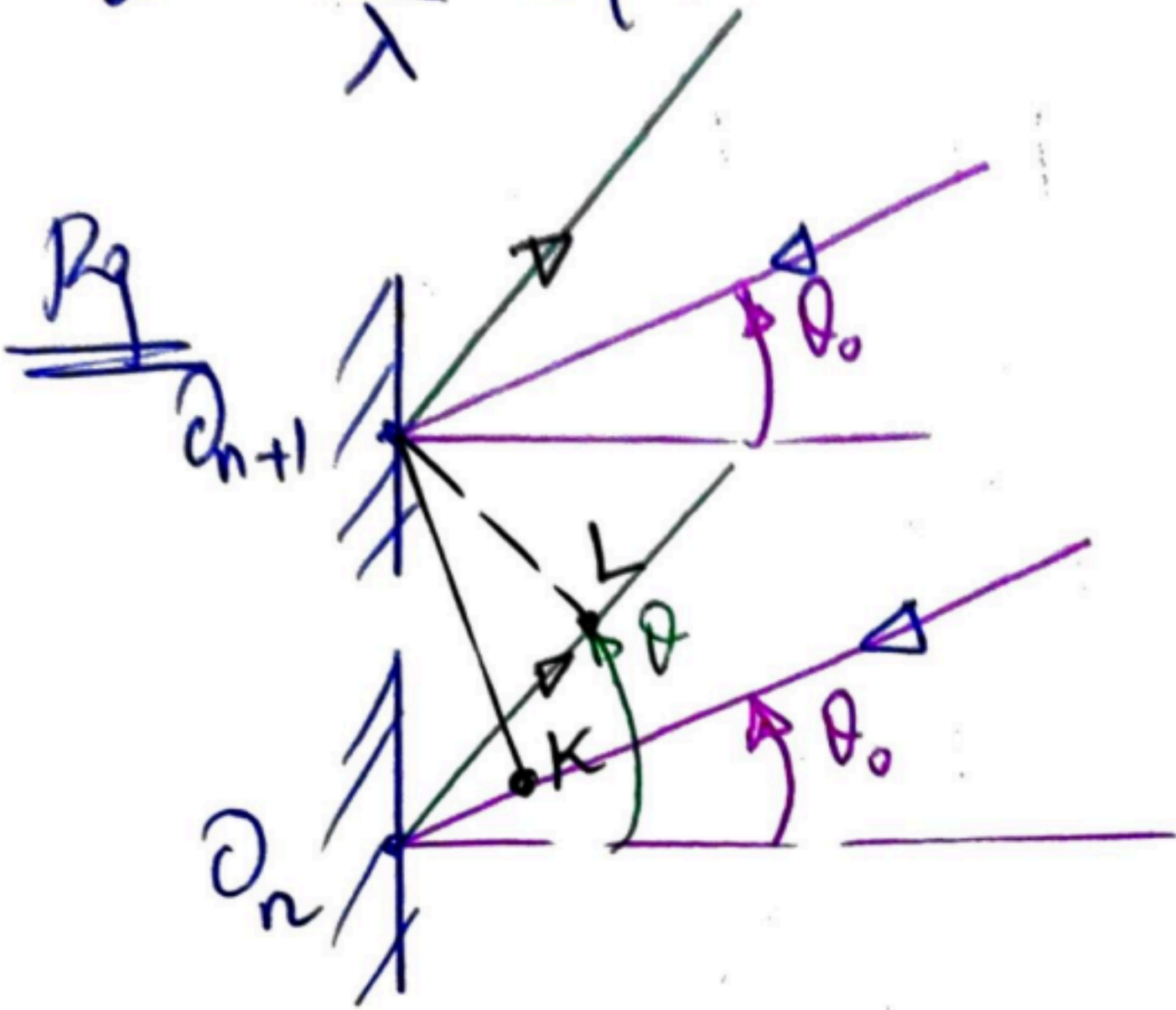
$\overline{O_n H_n} + \overline{O_n H'_n} = \delta$

$\delta = e (\sin \theta - \sin \theta_0)$



goniomètre

$$\Phi = \frac{2\pi}{\lambda} e (m\theta - n\theta_0) : \text{path transmission}$$



$$\delta_{n/n+1} = e (\sin \theta + \sin \theta_0)$$

ML2 2001

$$3) \underline{A}(\eta) = \underline{A}_1(\eta) + \underline{A}_2(\eta) + \underline{A}_3 + \dots + \underline{A}_N$$

$$= a_0 \sin c(u) \left[1 + e^{-i\Phi} + e^{-i2\Phi} + \dots + e^{-i(N-1)\Phi} \right]$$

$$= a_0 \sin c(u) \frac{1 - e^{-iN\Phi}}{1 - e^{-i\Phi}}$$

$$= a_0 \sin c(u) \frac{e^{-iN\Phi/2}}{e^{-i\Phi/2}} \frac{\sin(N\Phi/2)}{\sin(\Phi/2)}$$

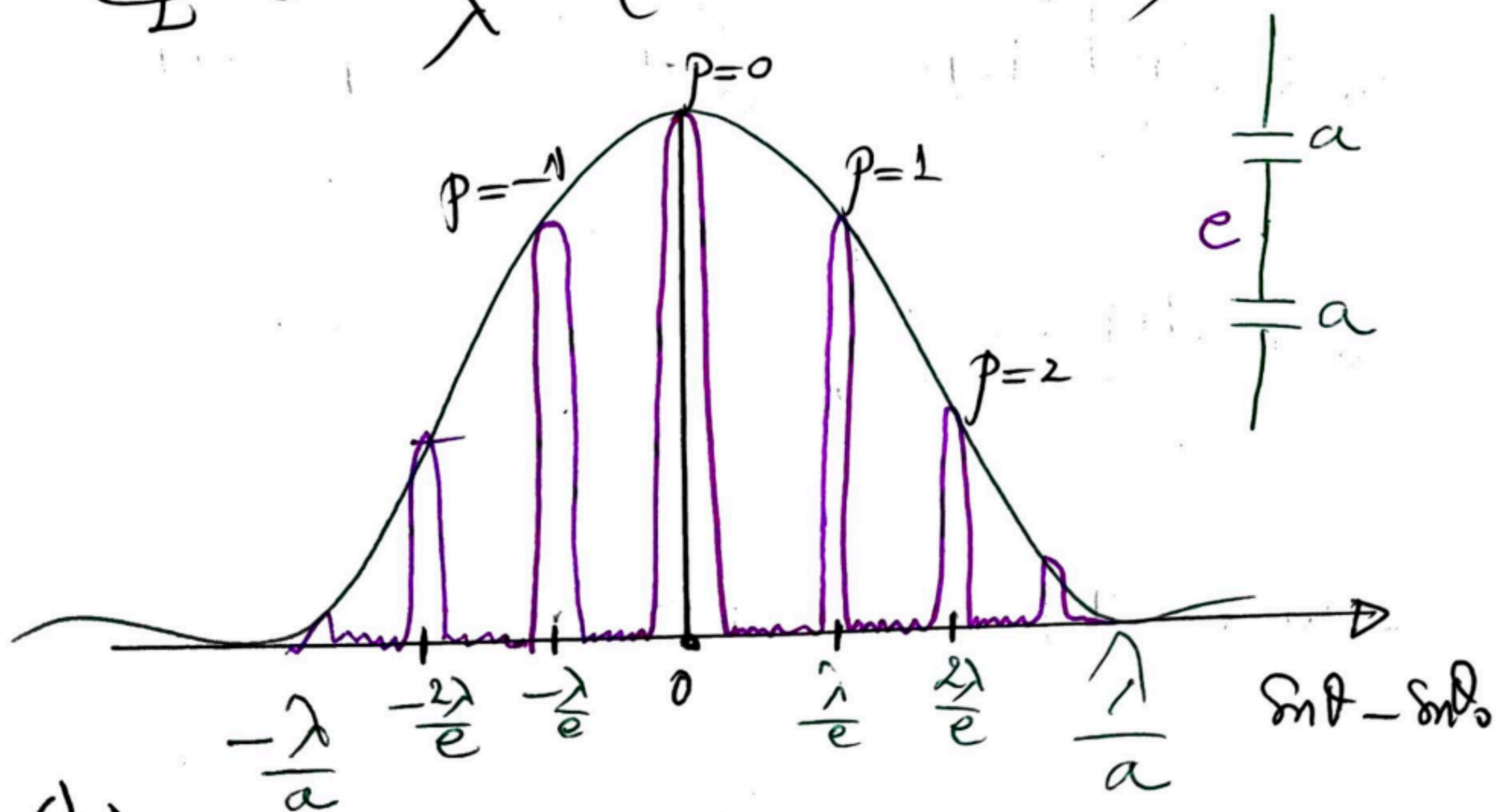
$$\underline{a} = a_0 \sin c(u) \left[\frac{\sin(N\Phi/2)}{\sin(\Phi/2)} \right] \exp\left(-\frac{i(N-1)\Phi}{2}\right)$$

$$I = I_0 N^2 \left(\frac{\sin u}{u} \right)^2 \left(\frac{\sin \left(\frac{N\Phi}{2} \right)}{N \sin \left(\frac{\Phi}{2} \right)} \right)^2$$

$\swarrow$ diffraction
 $\swarrow$ Interférence à Nondes

$$u = \frac{\pi a}{\lambda} (\sin \theta - \sin \theta_0)$$

$$\Phi = \frac{2\pi}{\lambda} e (\sin \theta - \sin \theta_0)$$



6a) pic principal

$$\sin \left(\frac{\Phi}{2} \right) = 0$$

$$\boxed{\Phi = 2\pi p} \quad \text{constructive}$$

$$\Phi = \frac{2\pi}{\lambda} e (\sin \theta - \sin \theta_0) = 2\pi p$$

$$\sin \theta_p - \sin \theta_0 = \frac{p\lambda}{e}$$

forme pour un métal sous incidence

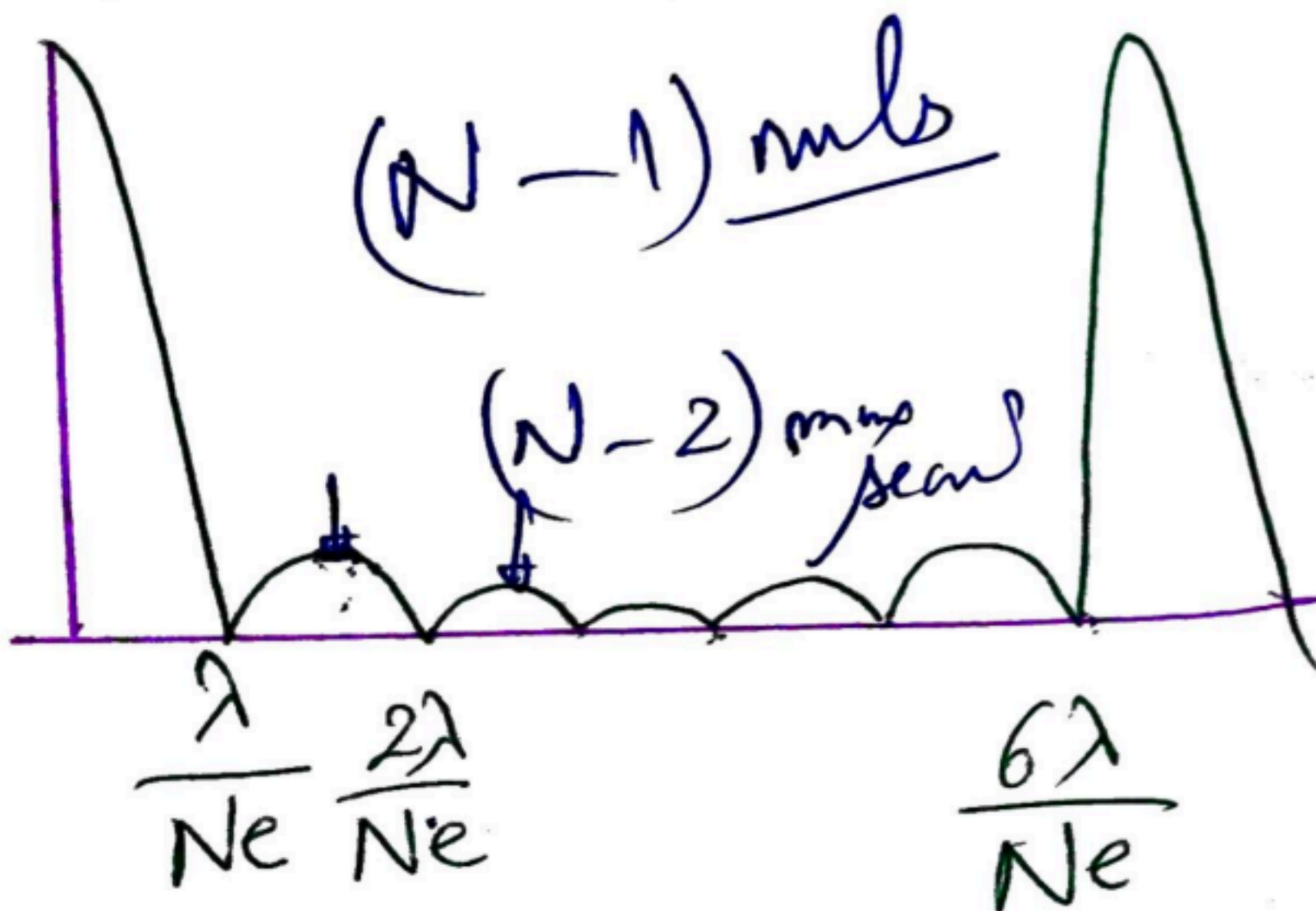
$$g) \sin\left(\frac{N\Phi}{2}\right) = 0; \quad \frac{N\Phi}{2} = K\pi$$

$$\Phi = \frac{2K\pi}{N}; \quad K \neq pN$$

$$\sin \theta_K - \sin \theta_0 = \frac{K\lambda}{Ne}$$

1^{re} max secondaire Φ

$$\underline{\underline{N=7}}$$



$$\Phi = \frac{(2K+1)\pi}{N} \quad \longleftrightarrow \quad \frac{N\Phi}{2} = \left(K + \frac{1}{2}\right)\pi$$

max, secondaire.

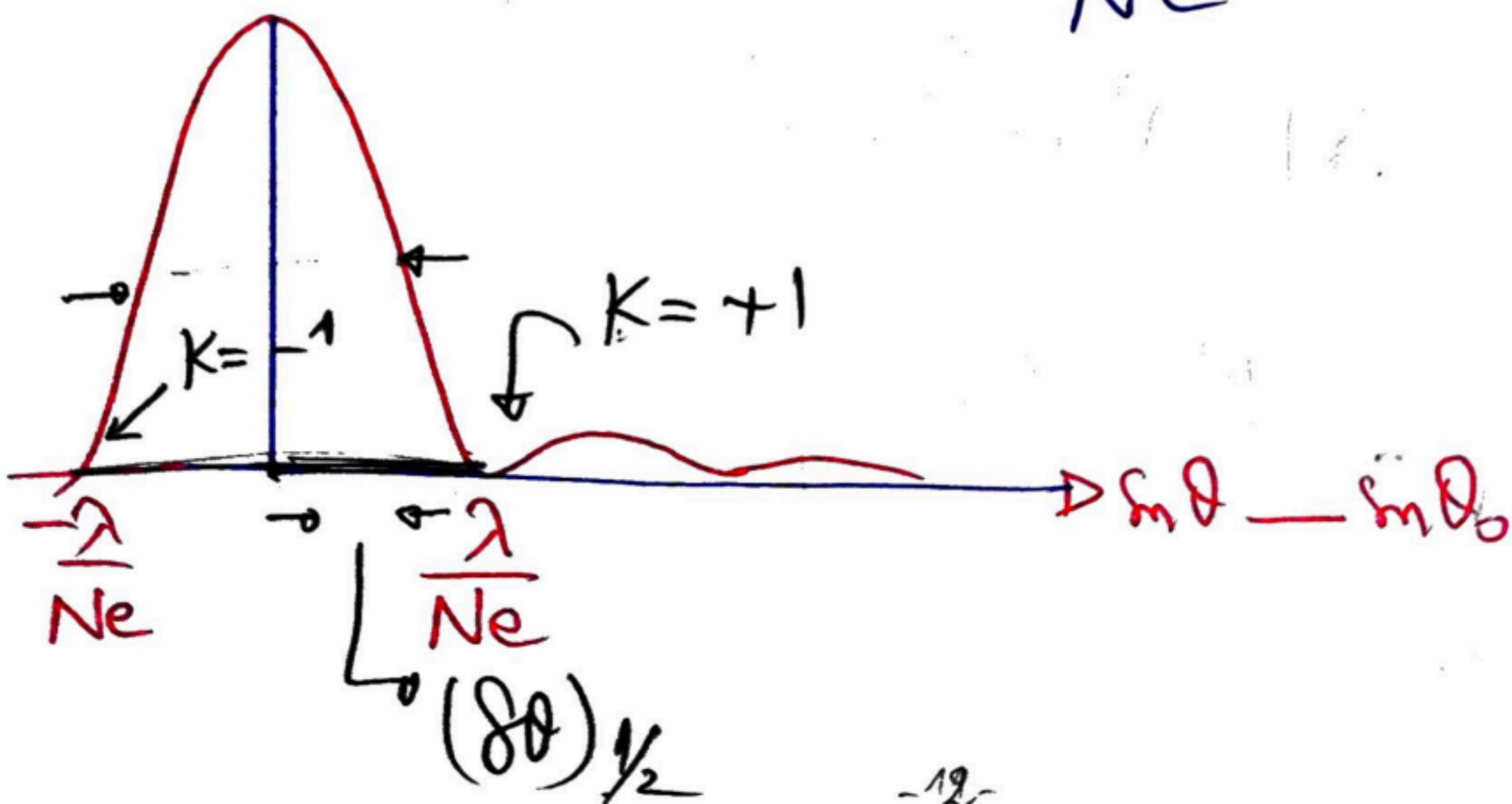
$$\frac{I_{\max, p}}{I_{0R}} = \left(\frac{\sin\left(\frac{N\Phi}{2}\right)}{N \sin\left(\frac{\Phi}{2}\right)} \right)^2$$

Résumé

$$= \left[\frac{\sin\left(\frac{(2K+1)\pi}{2}\right)}{N \frac{(2K+1)\pi}{2N}} \right]^2$$

$$\frac{I_{\max, p}}{I_{0R}} = \frac{1 \times 4}{(2K+1)^2 \pi^2}$$

$$\textcircled{d) c) \quad \sin \theta_K - \sin \theta_0 = \frac{K\lambda}{Ne}$$



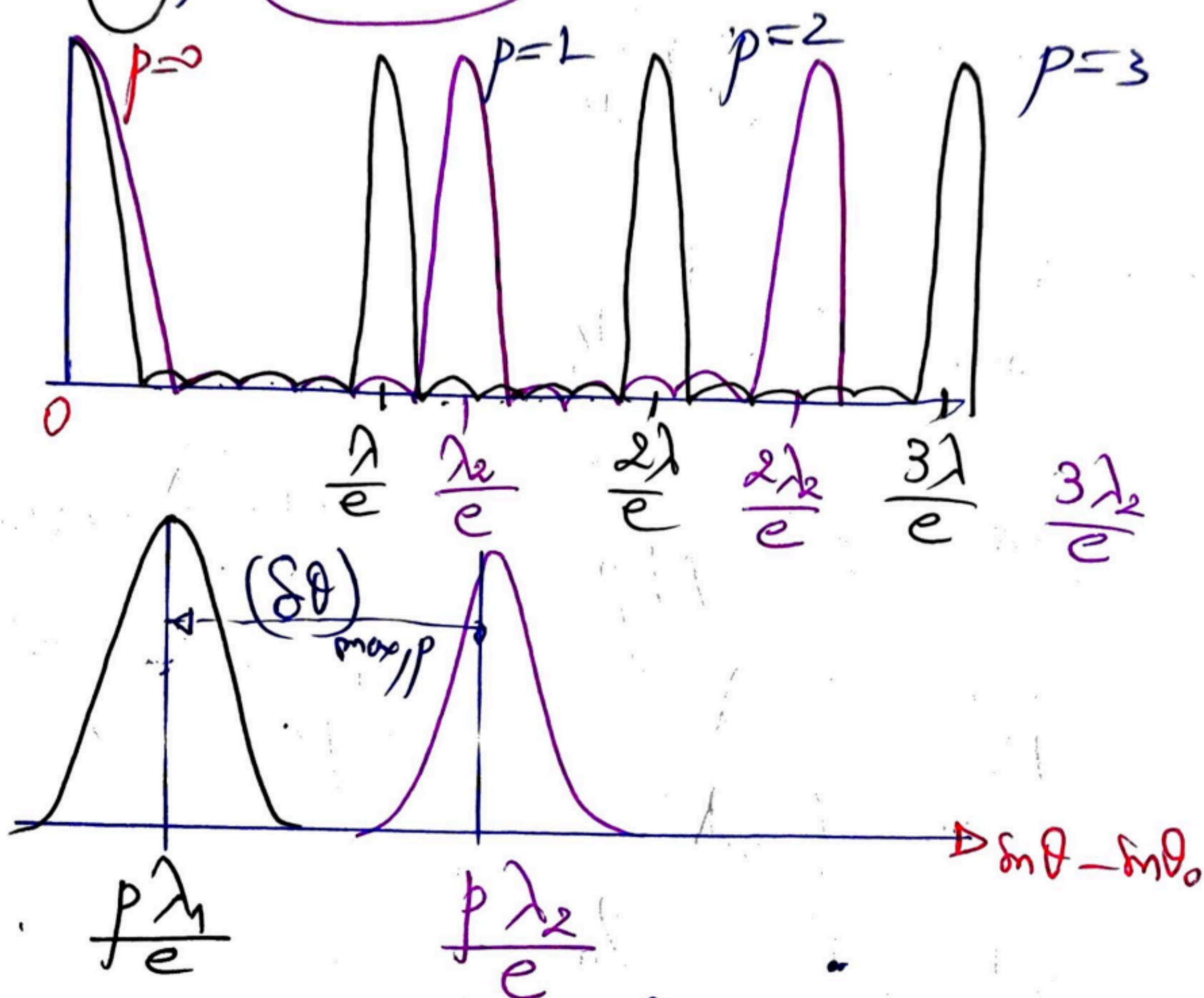
$$\cos \theta_k d\theta_k = \frac{\lambda}{Ne} dk$$

$$\delta k = 2$$

$$(\delta \theta)_{1/2} = \frac{\lambda \times 1}{Ne \cos \theta} ;$$

$$(\delta k)_{1/2} = 1$$

6) d) λ ; $\lambda + \Delta \lambda = \lambda_2$



$$\sin \theta_p - \sin \theta_0 = \frac{p \lambda}{e}$$

$$\cos \theta_p d\theta_p = \frac{p d\lambda}{e}$$

$$(\delta \theta)_{\max, p} = \frac{p \delta \lambda}{e \cos \theta}$$

c) critère de Rayleigh



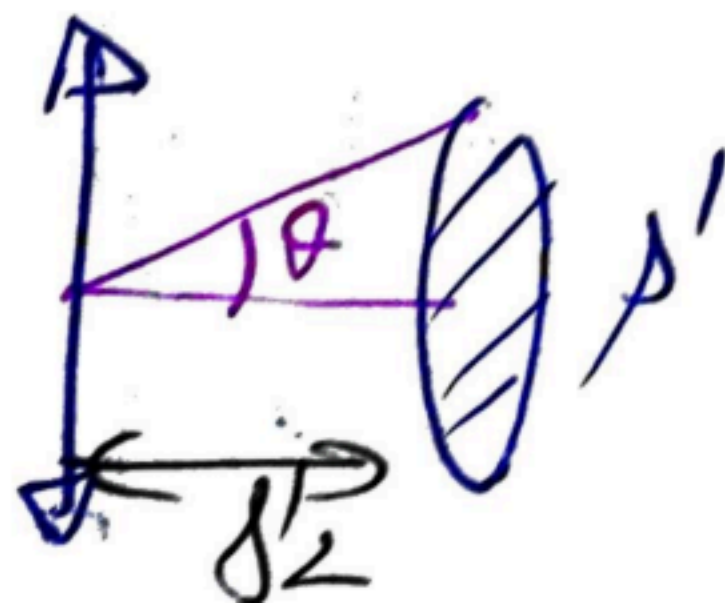
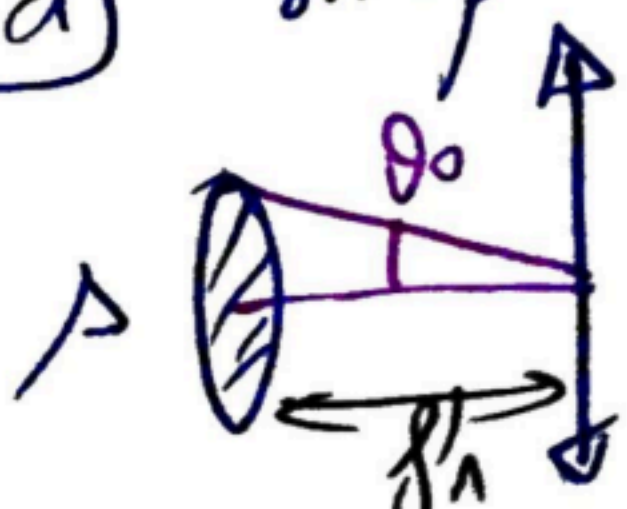
$$(\delta\theta)_{\max} \geq (\delta\theta)_{1/2}$$

$$\delta\lambda \geq \frac{\lambda}{Np} = (\delta\lambda)_{\text{résu}}$$

$$R_r = \frac{\lambda}{(\delta\lambda)_{\text{résu}}} = Np = pF$$

$N = F =$ la finesse.

$$7) a) \sin\theta_p - \sin\theta_0 = \frac{p\lambda}{e}$$



$$\sin \theta - \sin \theta_0 = \frac{p \lambda}{e} = d \sin \theta$$

$$\cos \theta d\theta = \cos \theta_0 d\theta_0 \Rightarrow \text{A Gauss.}$$

$$d\theta \approx d\theta_0 \Rightarrow \frac{A}{f_1'} = \frac{A'}{f_2'}$$

$$\boxed{A' = \frac{f_2'}{f_1'} A}$$



$$c) (\delta \theta)_{\max} = \frac{p \delta \lambda}{e \cos \theta} \approx \frac{p \delta \lambda}{e}$$

$$(\delta x)_{\max} \geq A' \quad \cdot \quad \frac{x}{f_1'} = \delta \theta \approx \frac{\delta x}{A'}$$

$$\frac{p f_2' \delta \lambda}{e} \geq \frac{f_2' A}{f_1'}$$

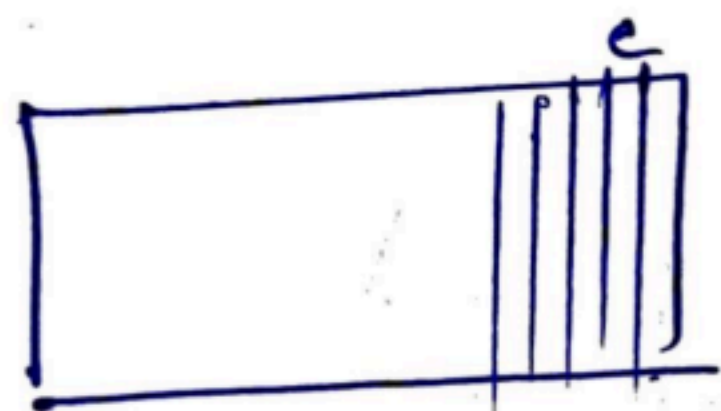
$$\delta \lambda \geq (\delta \lambda)_{\text{source}} = \frac{e A}{f_1' p}$$

$$e) \frac{A}{(\delta \lambda)_{\text{source}}} = R_{\text{source}} = \frac{\lambda p f_1'}{e p}$$

8) $p=1$ $R_r = Np$
 $R_s = \frac{\lambda f_1'}{e p}$

500 traits/mm = $n = \frac{1}{e} \Rightarrow$

$e = \frac{1}{500 \times 10^3} = 0,2 \times 10^{-5} = 2 \mu m$



$L = 2 \text{ cm}$

$L = Ne \Rightarrow$

$N = \frac{L}{e} = \frac{2 \times 10^{-2}}{2 \times 10^{-6}}$

$N = 10\,000$

$p = 0,1 \text{ mm}$

$\lambda = 589 \text{ nm} : (\text{Jaune})$

$f_1' = f_2' = 50 \text{ cm}^{-1}$

$R_r = 10\,000$; $R_s = \frac{589 \times 10^{-9} \times 50 \times 10^{-2}}{2 \times 10^{-6} \times 0,1 \times 10^{-3}}$

$R_s = 1472 \ll R_r$

$\frac{\lambda}{(\Delta\lambda)_s} \ll \frac{\lambda}{(\Delta\lambda)_r} \Rightarrow (\Delta\lambda)_{\text{sm}} \gg (\Delta\lambda)_{\text{re}}$

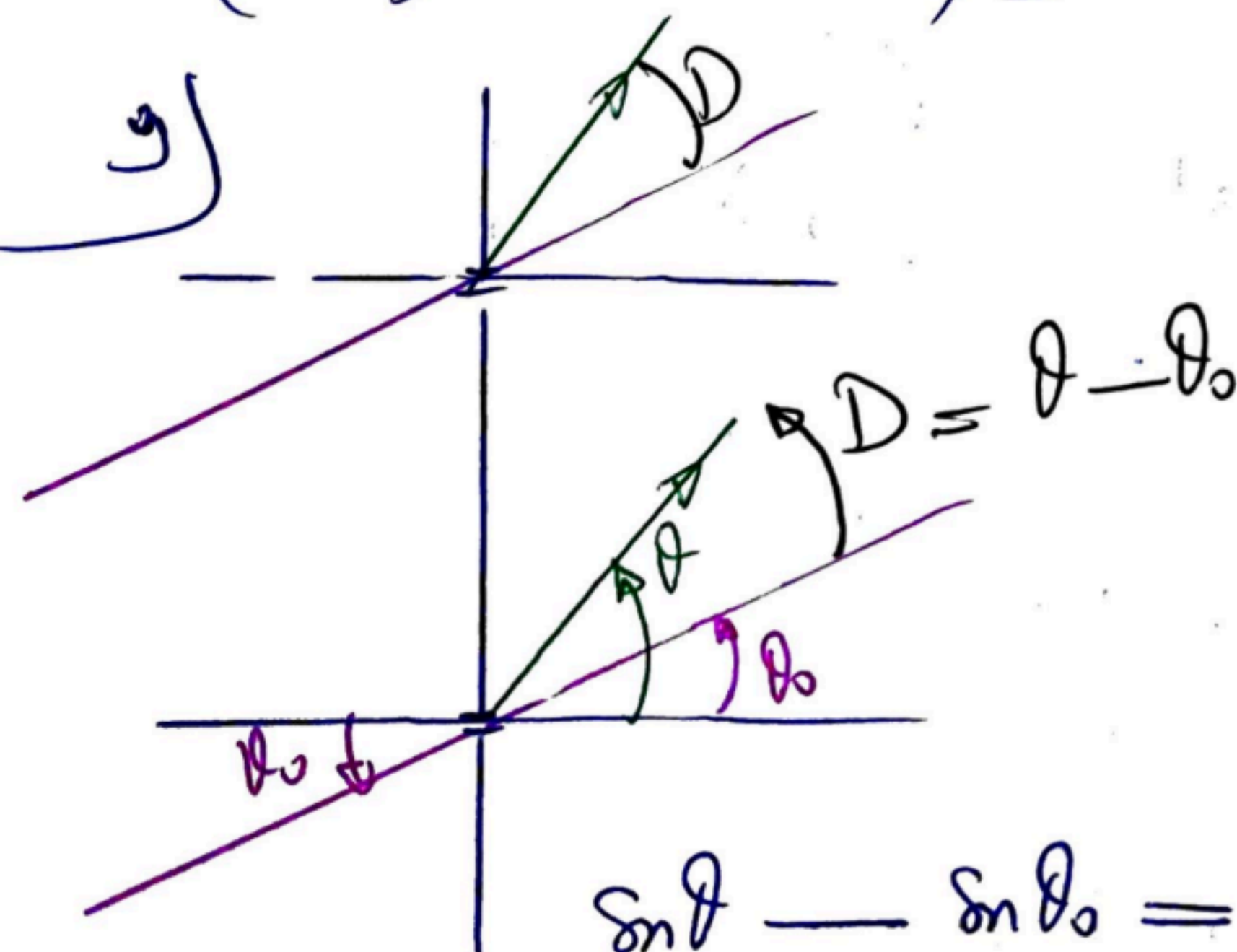
critère de résolution

$$(\Delta\lambda) > (\Delta\lambda)_{\text{smca}} \gg (\Delta\lambda)_{\text{résou}}$$

b) $\Delta\lambda = 0,6 \text{ nm}$

$$R_s = \frac{\lambda}{(\Delta\lambda)_{\text{smca}}} = 1472$$

$$(\Delta\lambda)_{\text{smca}} = \frac{\lambda}{R_s} = \frac{589}{1472} = 0,4 \text{ nm}$$



$$\frac{dD}{d\theta_0} = \frac{d\theta}{d\theta_0} - 1 = 0$$

$$d\theta = d\theta_0$$

$$\sin\theta - \sin\theta_0 = \frac{p\lambda}{e}$$

$$(\cos\theta - \cos\theta_0) d\theta = 0$$

$$\begin{cases} \theta = \theta_0 \longrightarrow D = 0 \\ \theta = -\theta_0 = \theta_{\text{min}} \end{cases}$$

$$D_m = \theta - \theta_0 = 2\theta_m$$

$$\sin \theta_m - \sin \theta_0 = \frac{p\lambda}{e} = 2 \sin \theta_m$$

$$2 \sin\left(\frac{D_m}{2}\right) = \frac{p\lambda}{e}$$

c) $p=2$; $D_{1m} = 51^\circ 40'$ $\lambda = 435,8 \text{ nm}$

$$1^\circ \rightarrow 60'$$

$$x \rightarrow 40'$$

$$x = \frac{40}{60} = \frac{2}{3} = \dots$$

$$D_{2m} = 66^\circ 12' \rightarrow \lambda' = ??$$

$$\frac{2 \sin\left(\frac{D_{1m}}{2}\right)}{2 \sin\left(\frac{D_{2m}}{2}\right)} = \frac{\lambda}{\lambda'}$$

$$\lambda' = \lambda \frac{\sin\left(\frac{D_{1m}}{2}\right)}{\sin\left(\frac{D_{2m}}{2}\right)}$$

$$e = \frac{p\lambda}{2 \sin\left(\frac{D_{1m}}{2}\right)} = 1 \mu\text{m}.$$

$$n = \frac{1}{10^{-3}} = 10^3 \text{ trau/mm}.$$

10) incidenza normale , $\theta_0 = 0$.

$$\sin \theta - \sin \theta_0 = \frac{p \lambda}{e} = \sin \theta$$

$$\cos \theta d\theta = \frac{p d\lambda}{e}$$

$$D_\lambda = \frac{d\theta}{d\lambda} = \frac{p}{e \cos \theta}$$

p : Charakterist
 $e \rightarrow n$

$$D_x = \frac{dx}{d\lambda} = \frac{p f_2}{e \cos \theta}$$

A Gau $\cos \theta \approx 1$

$$\frac{dx}{d\lambda} = \frac{2 \times 250}{1000}$$

$$= 0,5 \text{ mm/nm}$$

mm.nm⁻¹

$\Delta \lambda = 1 \text{ nm}$
 $0,5 \text{ mm}$

$$\Delta x_{\lambda \text{ min}} = 0,5 \times 0,6 = 0,3 \text{ mm} > 0,2 \text{ mm}$$

11) PC2 2013

$$n = \frac{\sin \left(\frac{A + D_m}{2} \right)}{\sin \left(\frac{A}{2} \right)}$$

$$dn \sin \left(\frac{A}{2} \right) = d \left(\sin \left(\frac{A + D_m}{2} \right) \right)$$

$$\ln \sin\left(\frac{A}{2}\right) = \cos\left(\frac{A+D_m}{2}\right) \sin\left(\frac{A+D_m}{2}\right)$$

$$\frac{dD_m}{d\lambda} = \frac{2 \sin(A/2)}{\cos\left(\frac{A+D_m}{2}\right)} \frac{dn}{d\lambda}$$

$$\frac{dD_m}{d\lambda} = \frac{2 \sin(A/2)}{\cos\left(\frac{A+D_m}{2}\right)} \left(-\frac{2\beta}{\lambda^3}\right) \int_a^{a+\varepsilon} (\sin u)^3 du$$

$$\Delta D_m = \int_{\lambda_1}^{\lambda_1 + \Delta \lambda} \frac{-4 \beta \sin(A/2)}{\cos(A + \frac{D_m}{2})} d\lambda = (\sin^3 a) \epsilon$$

19 $\sin\left(\frac{\Phi}{2}\right) = \frac{a_1}{2R}$; $2\eta = N\Phi + 2\beta$

$$a_1 = 2R \ln \left(\frac{\Phi}{2} \right)$$

$$\sin \beta = \frac{A}{2R}$$

$$\underline{A} = 2R \sin\left(\frac{N\Phi}{2}\right) \underline{A}$$

$$\underline{A} = a_1 \frac{\sin(N\Phi/2)}{\sin(\Phi/2)}$$

