

$$\frac{\delta Q_{th}}{\delta t} = \Phi_{th} = \iint \vec{J}_{th} \cdot d\vec{S}$$

$$I = \iint \vec{J}_e \cdot d\vec{S}$$



$$-\oint \vec{J} \cdot d\vec{S} = \frac{\delta Q_{th}}{\delta t}_{regue}$$

$$\delta E_c + \delta E_p + \delta U = \delta W + \delta Q$$

$$\frac{j(n, t)}{T(n, t)} = F(n, t) \quad ; \quad F(n + \Delta n, t) = \frac{j(n + \Delta n, t)}{T(n + \Delta n, t)}$$

$$F(n, t) - F(n + \Delta n, t) = \frac{\partial F}{\partial n} \Delta n$$

$$\delta U = \left(\frac{\partial U}{\partial T} \right)_V \delta T + \left(\frac{\partial U}{\partial V} \right)_T \delta V$$

$$\chi_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T ; \quad \chi_S = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_S$$

Seite Pb1

$$1) a), b), c) d) \Rightarrow R_E = \frac{R_{DV} \cdot R_{\eta}}{R_{DV} + R_{\eta}}$$

$$\frac{Q}{\Delta t} = \Phi = \frac{T_1 - T_E}{R_E}$$

$\downarrow$
Kw/h

$$\Delta t = 24 \text{ h}; \quad Q = \left(\frac{T_1 - T_E}{R_E} \right) \Delta t$$

$$2) \underline{T_V = 18^\circ \text{C}}$$

$$\Phi_{\text{voisin}} = (T_V - T_1) G_V$$

$$\Phi_{\text{sortant}} = \frac{T_1 - T_E}{R_E}$$

$$\Phi_{\text{stocké}} = \frac{C d T_1 (H)}{dt} = \frac{dU}{dt}$$

$$\Phi_{\text{voisin}} = \Phi_{\text{stocké}} + \Phi_{\text{sortant}}$$

$$G_V (T_V - T_1) = C \frac{dT_1}{dt} + \frac{T_1 - T_E}{R_E}$$

(2)

$$C \frac{dT_1}{dt} + \left(\frac{1}{R_E} + G_V \right) T_1 = \frac{T_E}{R_E} + G_V T_V$$

$$\frac{dT_1}{dt} + \left(\frac{1 + R_E G_V}{R_E C} \right) T_1 = \underbrace{\left(\frac{T_E + R_E G_V T_V}{R_E C} \right)}_{\frac{T_{1l}}{\tau}}$$

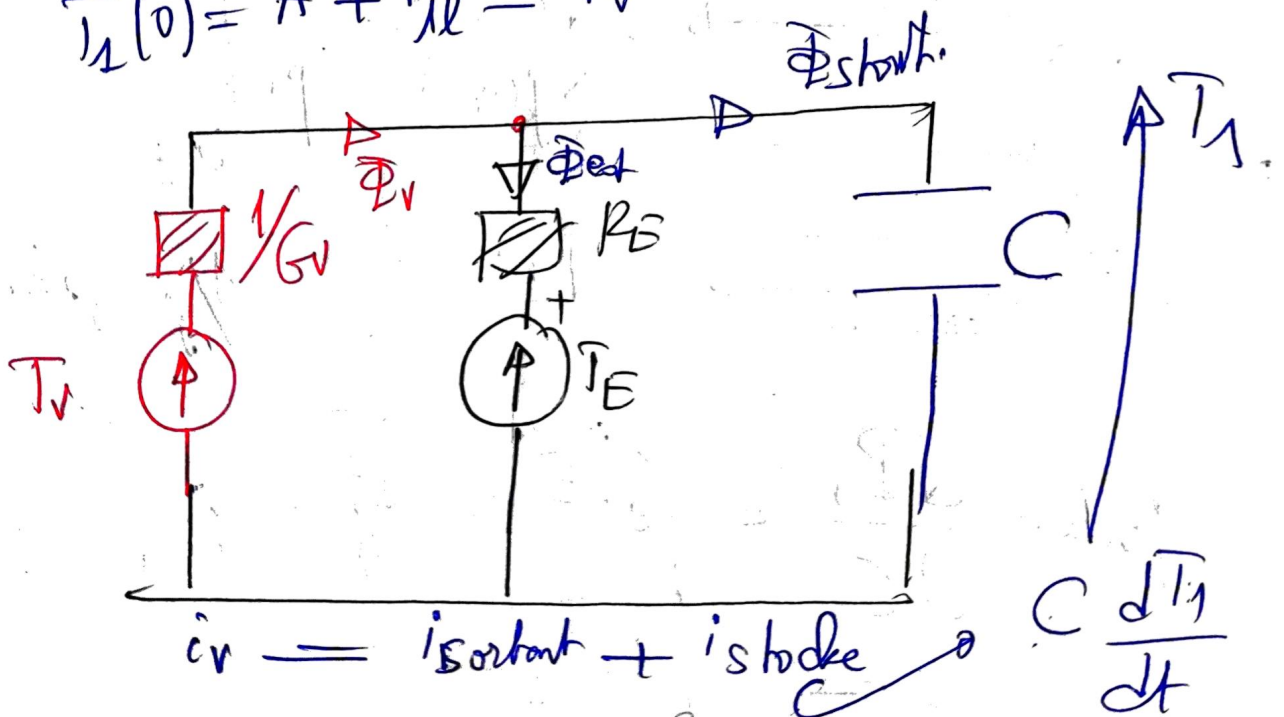
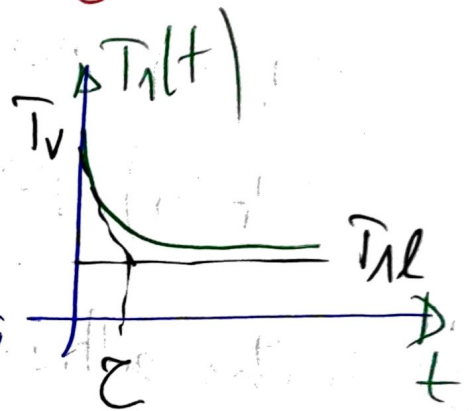
$$\tau = \frac{R_E C}{1 + R_E G_V}$$

$$\frac{T_{1l}}{\tau}$$

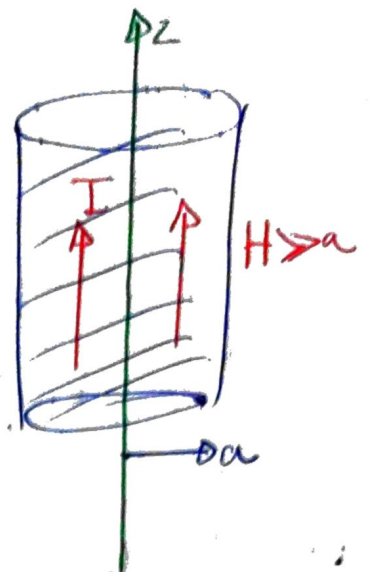
$$\frac{dT_1(t)}{dt} + \frac{T_1(t)}{\tau} = \frac{T_{1l}}{\tau}$$

$$T_1(t) = A e^{-t/\tau} + T_{1l}$$

$$T_1(0) = A + T_{1l} = T_V$$



$$I = \iint_{\substack{L \\ r dr d\theta}} \vec{j}_e \cdot d\vec{S} = j_e \cdot \pi a^2$$

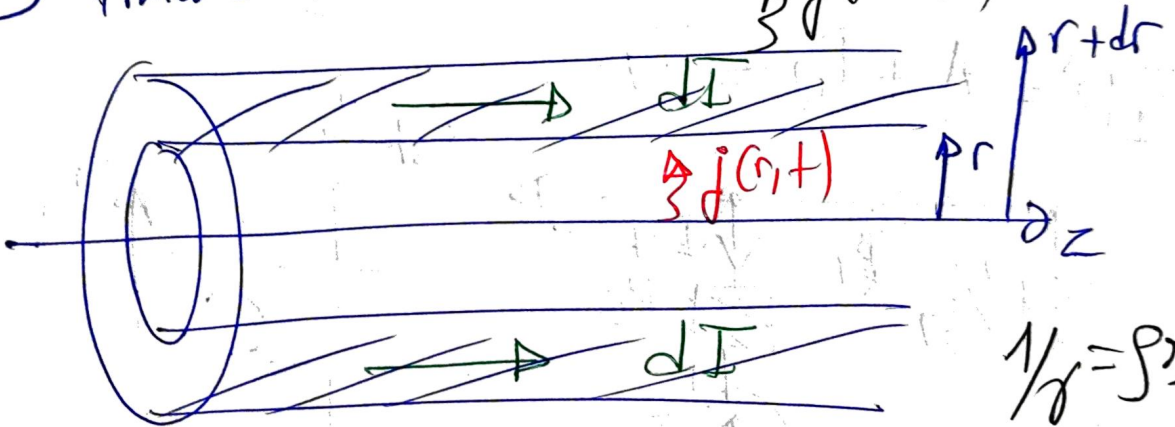


$$R_e = \frac{1}{\gamma} \frac{H}{\pi a^2}$$

1) a) densité globale + transmise --- $\vec{j}_{th}(r,t)$

- $(\nabla \cdot \vec{u}_r \vec{u}_z)$ P.S.I. $\Rightarrow \vec{j}_{th} \in \Rightarrow \vec{j}_{th}(r,t) = j(r,t) \vec{u}_z$
- $(\nabla \cdot \vec{u}_r \vec{u}_\theta)$ P.S.I. $\Rightarrow \vec{j}_{th} \in$

b) tranche entre r et $r+dr$ $\Delta j(r+dr, t)$



$$1/\gamma = \rho_{résist}$$

$$P_{mch} = \frac{dP_J}{dz} = \vec{j}_e \cdot \vec{E} = \frac{j_e^2}{\gamma} = \frac{1}{\gamma} \frac{I^2}{(\pi a^2)^2}$$

$$P_{mch} = \frac{R_e I^2}{\pi a^2 H}$$

$$\frac{dU}{dt} = \frac{\delta W}{dt} + \frac{\delta Q_{\text{th}}}{dt} + dP_{\text{prod}}$$

$$\star dP_{\text{prod}} = p_v \cdot d\tau$$

$$d\tau = r \, d\theta \, dz \rightarrow$$

$$\int_0^{2\pi} d\theta \int_0^H dz \, r \, dr = d\tau$$

$$d\tau = (2\pi r H) dr$$

$$\bullet \delta W = 0 \quad (\text{Indéformable})$$

$$\bullet \frac{\delta Q_{\text{th}}}{dt} = j(r, t) 2\pi H r - j(r+dr, t) 2\pi H (r+dr)$$

$$= 2\pi H \left[- \frac{\partial (r j(r, t))}{\partial r} dr \right]$$

$$\bullet \frac{dU}{dt} = \rho_c \frac{\partial T(r, t)}{\partial t} d\tau ; \quad \rho_c = \Gamma$$

capacité calorifique
masse
 $J \cdot K^{-1} \cdot kg^{-1}$

capacité calorifique
volume
 $J \cdot K^{-1} \cdot m^{-3}$

$$\rho_c \frac{\partial T}{\partial t} = - \frac{1}{r} \frac{\partial (r j(r, t))}{\partial r} + p_v$$

— $\text{div } \vec{j}_{\text{th}}$ —

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} = + \Delta T(r, t) + \frac{P_v}{\lambda}$$

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{P_v}{\lambda}$$

avec $P_v = \frac{R I^2}{\pi a^2 H}$

2) Permanent : $\frac{\partial T(r, t)}{\partial t} = 0 \Rightarrow T(r)$

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT(r)}{dr} \right) + \frac{P_v}{\lambda} = 0$$

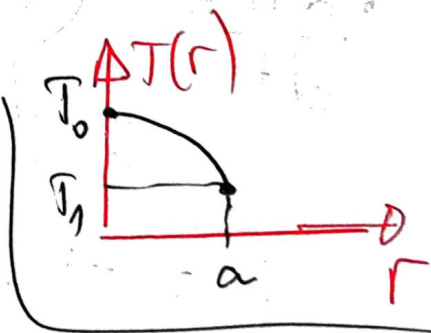
$$\cancel{r} \frac{dT(r)}{dr} = - \frac{P_v}{2\lambda} r^2 + \frac{C_1}{r}$$

$$T(r) = - \frac{P_v}{4\lambda} r^2 + C_1 \ln r + C_2$$

$$r \in [0, a] \Rightarrow \underline{C_1 = 0}$$

$$T(0) = T_0 = C_2$$

$$T(r) = T_0 - \frac{R I^2}{4\pi a^2 H \lambda} r^2$$



$$3a) \quad T_1 = T_0 - \frac{R I^2}{4\pi H \lambda}$$

$$\lambda = \frac{R I^2}{4\pi H (T_0 - T_1)} = 235 \text{ watt} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$$

b)

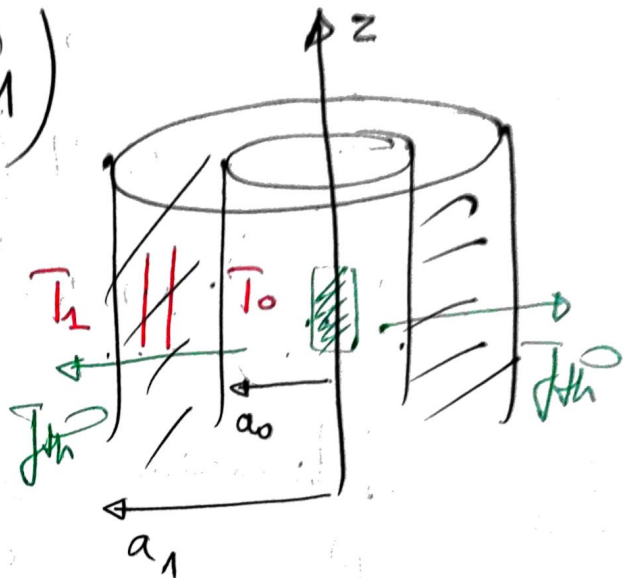
$$a_0 = r_{\text{int}}$$

$$a_1 = r_{\text{ext}}$$

$$T_0 = T_{\text{int}} = 24,4^\circ\text{C}$$

$$T_1 = T_{\text{ext}} = \text{fluide}$$

$$P_{\text{cal}} = R I^2 = 10^3 \text{ watts.}$$



Stationnaire + $(\rho = 0) \Rightarrow$ conservation de flux thermique $\Rightarrow \Phi = \text{cte}$

$$\Phi(r) = \Phi(r+dr) = P = \left(-\lambda \frac{dT(r)}{dr} \right) 2\pi r H$$

$$\int_{T_1}^{T_0} dT(r) = \frac{-P}{2\pi \lambda H} \int_{r_{\text{int}}}^{r_{\text{ext}}} \frac{dr}{r}$$

$$T_i - T_e = \frac{P}{2\pi\lambda H} \ln\left(\frac{r_e}{r_i}\right)$$

$$V_i - V_e = I R_e$$

$$T_i - T_e = \Phi R_{th}$$

$$\Rightarrow R_{th} = \frac{1}{2\pi\lambda H} \ln\left(\frac{r_e}{r_i}\right)$$

$$\underline{AN}: \lambda = \frac{P}{2\pi H (T_i - T_e)} \ln\left(\frac{r_e}{r_i}\right)$$

$$= 239 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$$

Pg : Stadimaire $\frac{dH}{dt} \rightarrow$

$\oplus$
 $P_r \rightarrow 0$

$$\Rightarrow \boxed{\Phi(r) = \Phi(r+dr)}$$

$$j(r) 2\pi H r = j(r+dr) 2\pi H (r+dr)$$

$$\frac{d}{dr} (r j(r)) = 0 ; \quad j(r) = -\lambda \frac{dT(r)}{dr}$$

$$\frac{d}{dr} \left(r \frac{dT(r)}{dr} \right) = 0$$

cylindre creux

$$T(r) = C_1 \ln r + C_2$$

$$T_i = T(r_i) \Rightarrow T(r) = C_2 \ln\left(\frac{r}{r_i}\right) + T_i$$

$$T_e = T(r_e) \Rightarrow T_e = C_1 \ln\left(\frac{r_e}{r_i}\right) + T_i$$

$$C_1 = \frac{T_e - T_i}{\ln\left(\frac{r_e}{r_i}\right)}$$

$$T(r) = \frac{T_e - T_i}{\ln\left(\frac{r_e}{r_i}\right)} \ln\left(\frac{r}{r_i}\right) + T_i$$

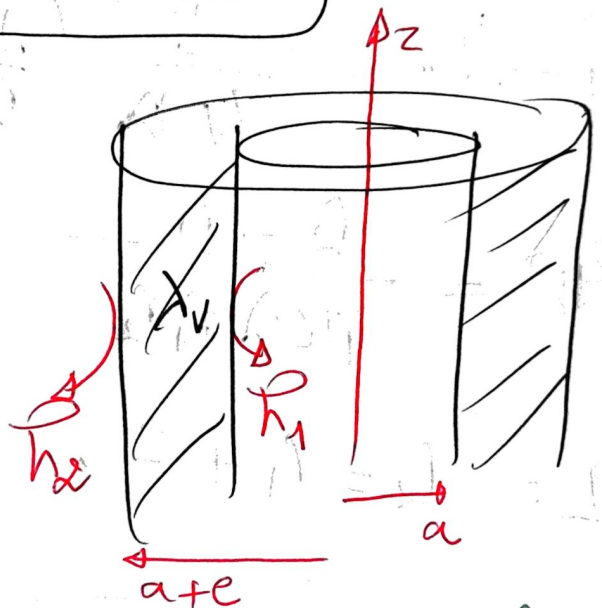
Pg 2

$$R_{\text{tot}} = \frac{1}{h_1 \times 2\pi a L}$$

$$+ \frac{1}{2\pi \lambda L} \ln\left(\frac{a+e}{a}\right)$$

$$+ \frac{1}{h_2 \times 2\pi L (a+e)}$$

$$\frac{\partial R_{\text{tot}}}{\partial e} = 0 ;$$



$$e \rightarrow R_{\text{th}}^{\text{cond}}$$

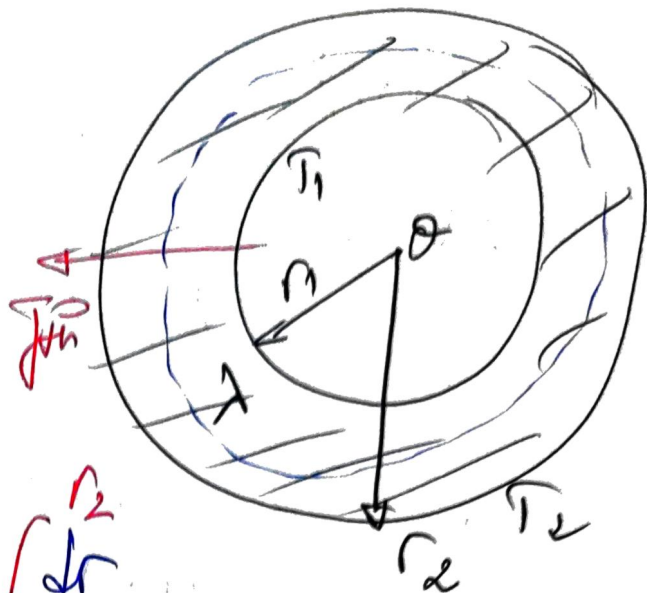
$$e \rightarrow R_{\text{th}_2}^{\text{conv}}$$

Ex 5

$$\Phi = j(r) 4\pi r^2$$

$$= -\lambda \frac{dT(r)}{dr} 4\pi r^2$$

$$\int_{r_1}^{r_2} dT(r) = \frac{-\Phi}{4\pi\lambda} \int_{r_1}^{r_2} \frac{dr}{r^2}$$



$$V_1 - V_2 = \frac{\Phi}{4\pi\lambda} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \Phi P_{th}$$

$$P_{th} = \frac{r_2 - r_1}{4\pi\lambda r_1 r_2}$$

By: $r_2 = r_1 + \epsilon$, $\epsilon \ll r_1$

$$P_{th} \approx \frac{\epsilon}{4\pi\lambda r_1^2} = \frac{\epsilon}{\lambda S}$$

2) a) $\int_{r_1}^{r_2} dT(r) = \frac{-\Phi}{4\pi\lambda} \int_{r_1}^{r_2} \frac{dr}{r^2}$

$$T(r) - T_1 = \frac{\Phi}{4\pi\lambda} \left(\frac{1}{r} - \frac{1}{r_1} \right)$$

$$T(r) - T_1 = \frac{r_1 r_2}{r_2 - r_1} \left(\frac{1}{r} - \frac{1}{r_1} \right) (T_1 - T_2)$$

$$-\lambda \frac{dT(r)}{dr} = \frac{\lambda r_1 r_2}{(r_2 - r_1)} \frac{1}{r^2} (T_1 - T_2)$$

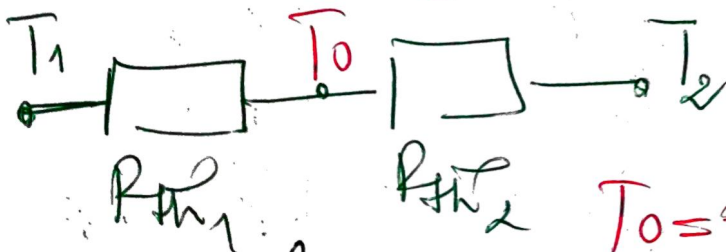
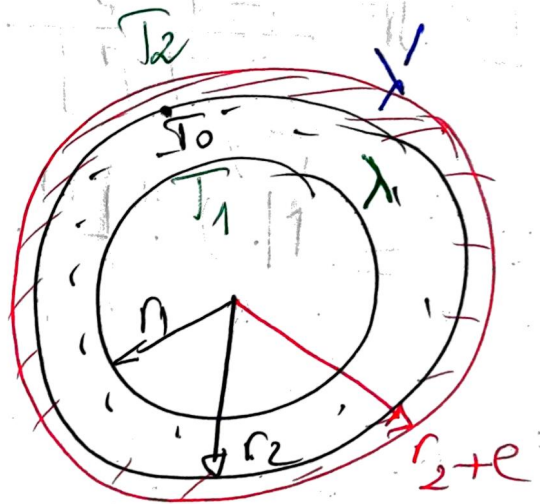
$$\dot{q}_{th}(r) = \frac{A}{r^2}$$

$r \rightarrow \dot{q}_{th}(r) \rightarrow$

$$S = (4\pi r^2) \rightarrow$$

$$3) R_{th1} = \frac{r_2 - r_1}{4\pi\lambda r_1 r_2}$$

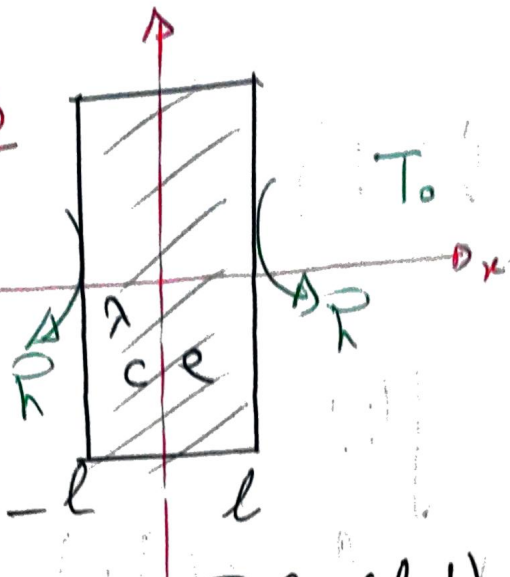
$$R_{th2} = \frac{e}{4\pi\lambda' (r_2 + e) r_2}$$



Gillmann

$$T_0 = \frac{G_{th1} T_1 + G_{th2} T_2}{G_{th1} + G_{th2}}$$

Pb3
 T_0



$$T_1 = T(x, 0) > T_0$$

$$T(x, t) = ??$$

$$a = \frac{\lambda}{\rho c}$$

$$\Delta \Phi_{\text{conv}} = h (T(l, t) - T_0) dS$$

$$1) \quad \frac{\rho c}{\lambda} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} ; \quad \theta(x, t) = T(x, t) - T_0$$

$$a = \frac{\lambda}{\rho c} \rightarrow \text{m}^2/\text{s}$$

$$2) \quad a) \quad \theta(x, t) = f(x) g(t)$$

$$\frac{1}{a} f(x) g'(t) = f''(x) g(t)$$

$$\frac{g'(t)}{g(t)} = \frac{a f''(x)}{f(x)} = -K$$

$\downarrow -\frac{a k^2}{l^2}$

$$\frac{dg(t)}{dt} - K g(t) = 0 \Rightarrow g(t) = g_0 e^{Kt}$$

$$K = -\frac{a k^2}{l^2}$$

$$g(t) = g_0 \exp\left(-\frac{a k^2}{l^2} t\right)$$

$$b) \frac{d^2 f(x)}{dx^2} + \frac{k^2}{l^2} f(x) = 0$$

$$f(x) = f_0 \cos\left(\frac{kx}{l}\right) + f_1 \sin\left(\frac{kx}{l}\right)$$

$$\theta(x, t) = \left[f_0 \cos\left(\frac{kx}{l}\right) + f_1 \sin\left(\frac{kx}{l}\right) \right] e^{-t/\tau}$$

$$\theta(0, 0) = T_1 - T_0 = (f_0 f_0)$$

$$\theta(x, t) = \theta(-x, t) \Rightarrow f_1 = 0$$

$$\theta(x, t) = (T_1 - T_0) \cos\left(\frac{kx}{l}\right) e^{-t/\tau}$$

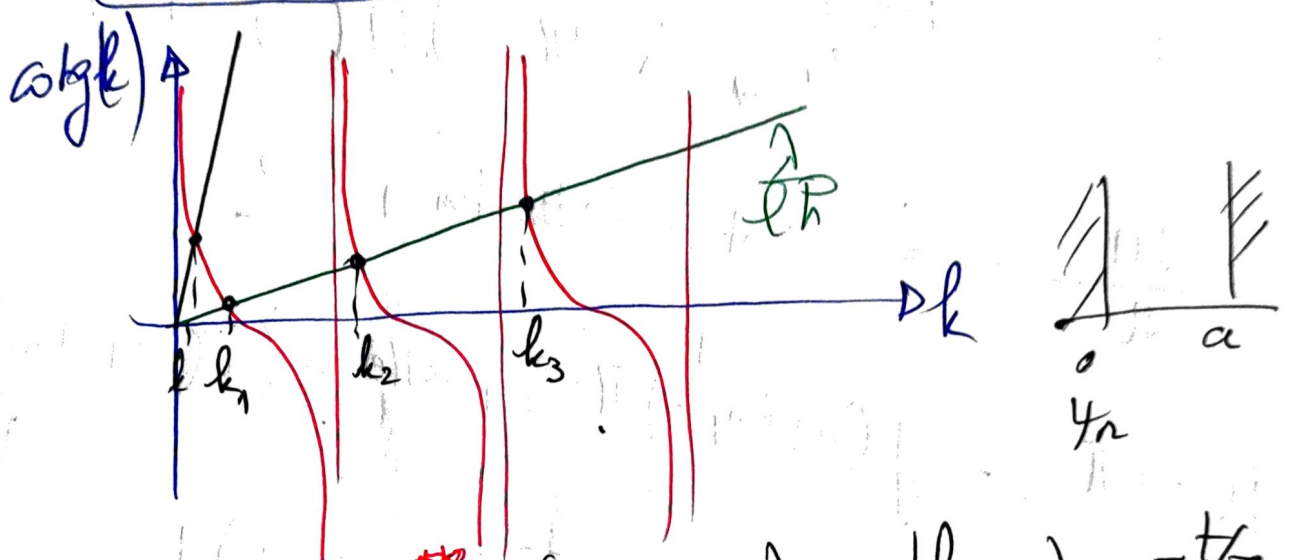
$$T(x, t) = (T_1 - T_0) \cos\left(\frac{kx}{l}\right) e^{-t/\tau} + T_0$$

$$3) \Phi_{\text{th}}^{\text{Anrechnung}} = \Phi_{\text{th}}^{\text{Anrechnung}} \\ \underline{\underline{n=l}}$$

$$R[(T(l,t) - T_0)]S = -\lambda \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=l} S$$

$$R \cos(k) = \lambda \frac{k}{l} \sin(k)$$

$$\cot(k) = \frac{\lambda}{lR} k$$



$$T(x,t) = \sum_{j=1}^{\infty} (T_1 - T_0) \cos\left(\frac{k_j x}{l}\right) e^{-t/\tau_j}$$

$$4) a) \cot(k_1) = \frac{\lambda}{lR} k_1 = 40 \quad k_1 \approx \frac{1}{k_1}$$

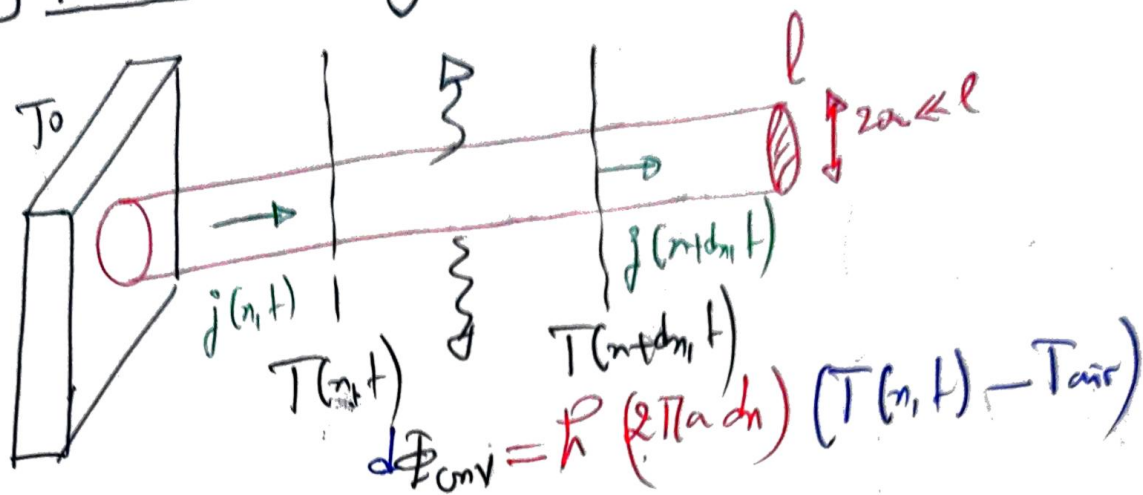
$$k_1 = \frac{1}{\sqrt{40}}$$

$$b) x \in [0, l) \Rightarrow \frac{AN}{15} \quad 0.98 \leq \cos\left(\frac{k_1 x}{l}\right) \leq 1$$

$$T(x,t) \approx T_0 + (T_1 - T_0) e^{-t/\tau_1}$$

Phy

A) Ailette de refroidissement



$$1) \frac{dU}{dt} = 0 = \Phi_{ent} - \Phi_{sortie} + \underline{0}$$

$$j(x,t) \pi a^2 = j(x+dx,t) \pi a^2 + 2\pi a h (T(x,t) - T_{air}) dx$$

$$-\frac{dj(x)}{dx} \pi a^2 dx = 2\pi a h (T(x,t) - T_{air}) dx$$

$$+\lambda \frac{d^2 T(x)}{dx^2} = \frac{2h}{a} (T(x) - T_{air})$$

$$\frac{d^2 T(x)}{dx^2} - \frac{2h}{\lambda a} (T(x) - T_{air}) = 0$$

$\theta(x)$

fg dimensions : λ

$$[Force] = [masse] [accélération]$$

$$= M \cdot \frac{L}{T^2} = M \cdot L \cdot T^{-2}$$

$$[Pression] = [Force] [vitesse]$$

$$= (M L T^{-2}) \cdot (L T^{-1}) = M L^2 T^{-3}$$

$$[|\vec{f}|] = \frac{M L^2 T^{-3}}{L^2} = M \cdot T^{-3}$$

$$[\lambda] = \frac{[f]}{[grad T]} = \frac{M T^{-3}}{L^{-1} \cdot \theta}$$

$$= M \cdot \theta^{-1} \cdot L^{+1} \cdot T^{-3}$$

MP2 \$2012
5/100

$M = \text{masse}$

$L = \text{Longueur}$

$T = \text{Temps}$

$\theta = \text{température}$

$$\frac{d^2 \theta(x)}{dx^2} - \frac{2\hbar}{\lambda a} \theta(x) = 0$$

$$2) a) A = \sqrt{\frac{2\hbar}{\lambda a}}; \frac{d^2 \theta(x)}{dx^2} - A^2 \theta(x) = 0$$

$$\theta(x) = \alpha e^{-Ax} + \beta e^{Ax}$$

$$\bullet \theta(0) = \alpha + \beta = T_0 - T_{\text{air}} = \theta_0$$

$$\bullet \underline{x=l} \quad \Phi_{\text{th}}^{\text{cond}}(x=l^-) = \Phi_{\text{th}}^{\text{conv}}(x=l^+)$$

$$-\lambda \left(\frac{d\theta}{dx} \right)_l = \hbar \theta(l)$$

$$-\lambda [-\alpha A e^{-Al} + \beta A e^{Al}] = \hbar [\alpha e^{-Al} + \beta e^{Al}]$$

$$\alpha (\hbar - \lambda A) e^{-Al} + \beta (\hbar + \lambda A) e^{Al} = 0$$

$$\left. \begin{aligned} & -(\alpha e^{-Al})_m + \beta e^{Al} = 0 \quad (2) \\ & \alpha + \beta = \theta_0 \quad (1) \end{aligned} \right\}$$

$$(2) + (1)(-e^{Al})$$

$$-\alpha m e^{-Al} - \alpha e^{Al} = -\theta_0 e^{Al}$$

$$\alpha = \frac{(T_0 - T_{\infty}) e^{Al}}{e^{Al} + m e^{-Al}}$$

$$(1) (e^{-Al} m) + (2) \Rightarrow$$

$$\beta (m e^{-Al} + e^{Al}) = \theta_0 m e^{-Al}$$

$$\beta = \frac{(T_0 - T_{\infty}) m e^{-Al}}{e^{Al} + m e^{-Al}}$$

$$\theta(n) = \frac{(T_0 - T_{\infty}) [e^{A(l-n)} + m e^{-A(l-n)}]}{e^{Al} + m e^{-Al}}$$

$$b) l \rightarrow \infty \quad e^{-Al} \rightarrow 0, \quad e^{-A(l-n)} \rightarrow \infty$$

$$\theta(n) = (T_0 - T_{\infty}) e^{-An}$$

$$\lim_{n \rightarrow \infty} \theta(n) = 0$$

$$T(n=l) = T_{\infty}$$

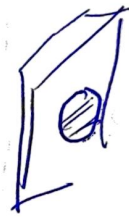
$$3) a) \quad P_{\text{évaporée}} = \int_0^l \frac{\lambda}{R} (T(x) - T_{\text{air}}) 2\pi a \, dx + \frac{\lambda}{R} \pi a^2 (T(l) - T_{\text{air}})$$

$$= - \lambda \left(\frac{dT(x)}{dx} \right)_{0+} \pi a^2$$

$$= - \lambda \frac{(-A) [e^{Al} - m e^{-Al}]}{e^{Al} + m e^{-Al}} (T_0 - T_{\text{air}}) \pi a^2$$

$$P_{\text{eva}} = \lambda A \frac{e^{Al} + m e^{-Al}}{e^{Al} - m e^{-Al}} (T_0 - T_a) \pi a^2$$

$$b) e_t = \frac{P_{\text{évaporée}}}{P_0} \leftarrow \text{sans ailette}$$

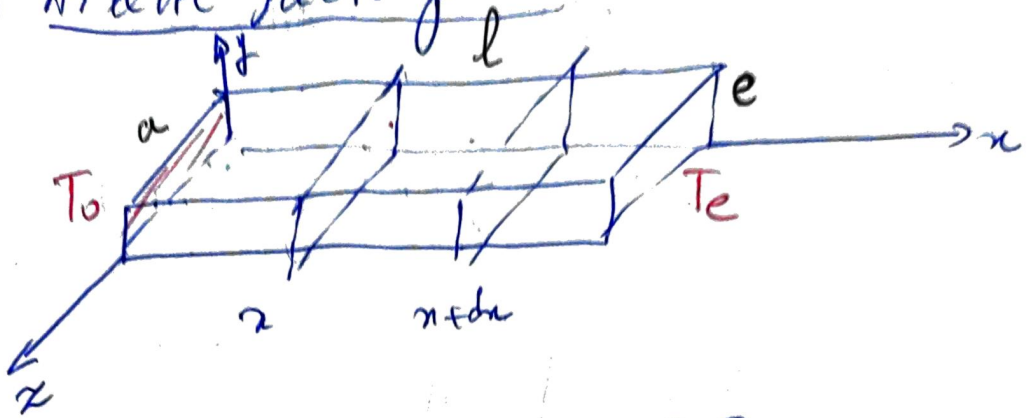


$$P_0 = \frac{\lambda}{R} (T_0 - T_{\text{air}}) \pi a^2$$

$$e_t = \left(\frac{\lambda A}{R} \right) \frac{e^{Al} - m e^{-Al}}{e^{Al} + m e^{-Al}}$$

$$e_{\infty} = \frac{\lambda A}{R} > 1$$

B) Arête rectangulaire



$$\frac{dU}{dt} = \frac{\delta Q_x}{dt} - \frac{\delta Q_{x+dx}}{dt} - \frac{\delta Q_{conv}}{dt}$$

$$\bullet \frac{dU}{dt} = \rho c \frac{\partial T(x,t)}{\partial t} (ae) dx$$

$$\bullet \left(\frac{\delta Q_{th}}{dt} \right)_{recu} = [j(x,t) - j(x+dx,t)] ae =$$

$$= \left[- \frac{\partial j(x,t)}{\partial x} dx \right] ae$$

$$\frac{\delta Q_{conv}}{dt} = h(T(x,t) - T_e)(a+e) dx$$

$$\rho c \frac{\partial T}{\partial t} (ae) = - \frac{\partial j}{\partial x} (ae) - \frac{2h(T(x,t) - T_e)}{x(a+e)}$$

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - \frac{2h^2 (a+e)}{ae} (T(x,t) - T_e)$$

Steady state $\frac{\partial T}{\partial t} \rightarrow 0$

$$\frac{d^2 T(x)}{dx^2} - \frac{1}{L^2} \underbrace{(T(x) - T_e)}_{\theta(x)} = 0$$

$$\theta(x) = \alpha \cosh\left(\frac{x}{L}\right) + \beta \sinh\left(\frac{x}{L}\right)$$

$$\bullet \theta(0) = \alpha = (T_0 - T_e)$$

$$\bullet j_{th}^{cond}(x=l^-) = j_{th}^{conv}(x=l^+)$$

$$- \lambda \left[\frac{\alpha}{L} \sinh\left(\frac{l}{L}\right) + \beta \frac{1}{L} \cosh\left(\frac{l}{L}\right) \right] = h \left[\alpha \cosh\left(\frac{l}{L}\right) + \beta \sinh\left(\frac{l}{L}\right) \right]$$



$$\theta(x=0) = (T_0 - T_e) = \theta_0$$

$$\theta(x=L) = (T_1 - T_e) = \theta_1$$

$$\theta(x) = \alpha \cosh\left(\frac{x}{L}\right) + \beta \sinh\left(\frac{x}{L}\right)$$

$$\boxed{\theta_0 = \alpha}$$

$$\theta_1 = \theta_0 \cosh\left(\frac{l}{L}\right) + \beta \sinh\left(\frac{l}{L}\right)$$

$$\beta = \frac{\theta_1 - \theta_0 \cosh(l/L)}{\sinh(l/L)}$$

$$\theta(x) = \theta_0 \cosh\left(\frac{x}{L}\right) + \frac{(\theta_1 - \theta_0 \cosh(l/L)) \sinh(x/L)}{\sinh(l/L)}$$

$$\theta(x=0) = \theta_0$$

$x=l$ parce qu'elle est isolée: adiabatique
 $\left. \frac{dT(x)}{dx} \right|_l = 0$

$$\theta(x) = \alpha \cosh\left(\frac{x}{L}\right) + \beta \sinh\left(\frac{x}{L}\right)$$