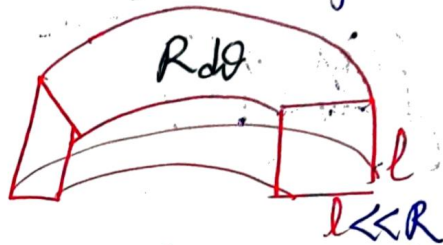


c) arlette boupe

$$T(\theta, t)$$

1) $\lambda_{Hg} \Rightarrow$: même instantané de la Temp

$$d\mathcal{L} = l^2 R d\theta$$



$$\frac{d\mathcal{L}}{dt} = \epsilon c (l^2 R d\theta) \frac{\partial T}{\partial t} = \left(\frac{\delta Q_{\text{rad}}}{dt} \right)_{\text{and}} - \frac{\delta Q_{\text{conv}}}{dt}$$

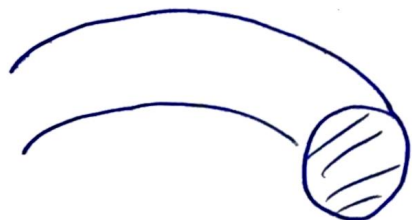
$$\left(\frac{\delta Q}{dt} \right)_{\text{and}} = [j(\theta, t) - j(\theta + d\theta, t)] l^2$$

$$= - \frac{\partial j(\theta, t)}{\partial \theta} l^2 d\theta$$

$$j_{\text{rad}} = \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial \theta} \right)$$

$$= + \frac{\lambda}{R} \frac{\partial}{\partial \theta} \left(\frac{\partial T(\theta, t)}{\partial \theta} \right) l^2 d\theta$$

$$\left(\frac{\delta Q_{\text{th}}}{dt} \right)_{\text{convect}} = h (T(\theta, t) - T_e) 4l R d\theta$$



Pg $d\mathcal{L} = \pi a^2 R d\theta$

$$dS_{\text{conv}} = 2\pi a R d\theta ; dS_{\text{and}} = \pi a^2$$

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} = \frac{1}{R^2} \frac{\partial^2 T}{\partial \theta^2} - \frac{4h}{l\lambda} (T(\theta, t) - T_e)$$

Req: $h \rightarrow 0$ $\frac{1}{R^2} \frac{\partial^2 T(\theta, t)}{\partial \theta^2} = \frac{1}{\rho c} \frac{\partial T}{\partial t}$

$$T(\theta, t) = f(\theta) g(t)$$

$$D_{th} = \frac{\lambda}{\rho c} \text{ m}^2/\text{s}$$

3) Stationnaire: $\frac{d^2 T}{d\theta^2} - \frac{R^2}{a^2} \underbrace{(T(\theta) - T_e)}_{\beta(\theta)} = 0$

$$a = \sqrt{\frac{\lambda l}{4h}} \rightarrow \text{en mètre}$$

$$\frac{d^2 \beta(\theta)}{d\theta^2} - \frac{R^2}{a^2} \beta(\theta) = 0 \quad \frac{1}{R^2} \quad 2023$$

$$\beta(\theta) = A_1 \cosh\left(\frac{R\theta}{a}\right) + A_2 \sinh\left(\frac{R\theta}{a}\right)$$

$$\beta(0) = A_1 = T_c - T_e = T_c - T_e$$

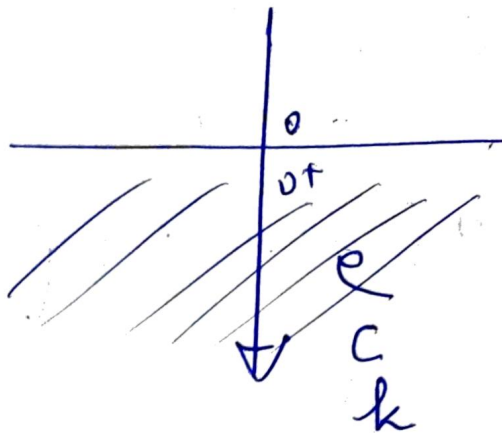
$$\begin{aligned} \beta(2\pi) &= A_1 \cosh\left(\frac{R \cdot 2\pi}{a}\right) + A_2 \sinh\left(\frac{R \cdot 2\pi}{a}\right) \\ &= T_c - T_e \end{aligned}$$

$$\beta(\theta) = (T_c - T_e) \frac{\cosh\left(\frac{(\pi - \theta)R}{a}\right)}{\cosh\left(\frac{\pi R}{a}\right)}$$

onde d'onde k sol. κ

$$J_{th} = -k_{grad} T$$

$$T(x,t) = T_0 + a \cos\left(\frac{2\pi t}{T'}\right)$$



$$\frac{\rho c}{k} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} ; u = \frac{T(x,t) - T_0}{a}$$

$$T(x,t) = a u(x,t) + T_0$$

$$\frac{\rho c}{k} \cdot \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} ; u = e^{(\alpha x + \beta t)}$$

$$\boxed{\frac{\rho c}{k} \beta = \alpha^2} ; \frac{T(x,t) - T_0}{a} = \cos(\omega t) = u(x,t)$$

$$u = e^{\beta t} \rightarrow \beta = i\omega$$

$$u = e^{\alpha x} e^{i\omega t}$$

$$\Rightarrow \alpha^2 = \frac{\rho c}{k} (i\omega) = \frac{(1+i)^2}{\left(\frac{2k}{\rho c \omega}\right)}$$

$$\delta = \sqrt{\frac{2k}{\rho c \omega}} \rightarrow \delta_{eff} = \sqrt{\frac{2}{\rho c \omega}}$$

$$\alpha = \pm \left(\frac{1+i}{\delta} \right)$$

$$u(x,t) = \exp \left[- \left(\frac{1+i}{\delta} \right) x + i\omega t \right]$$

$$u(x,t) = \exp \left(-\frac{x}{\delta} \right) \exp i(\omega t - \frac{x}{\delta})$$

$$T(x,t) = T_0 + a \exp \left(-\frac{x}{\delta} \right) \cos(\omega t - \frac{x}{\delta})$$

onde monoch, onde plane, progressante (on) +
atenuada (on); $\cos, \sin$

$$b) \lambda = 2\pi \sqrt{\frac{2k}{\rho c \omega}} = 2\pi \delta$$

→ comprimento de onda $\frac{1}{\delta} = \frac{2\pi}{\lambda}$

$$T(x,t) = T_0 + a \exp \left(-\frac{2\pi x}{\lambda} \right) \cos \left(2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \right)$$

comatave

$$\lambda = 2\pi \times \sqrt{\frac{2 \times 4,8 \times 385 \times 24 \times 3600}{3,1 \times 10^3 \times 870 \times 2\pi}}$$

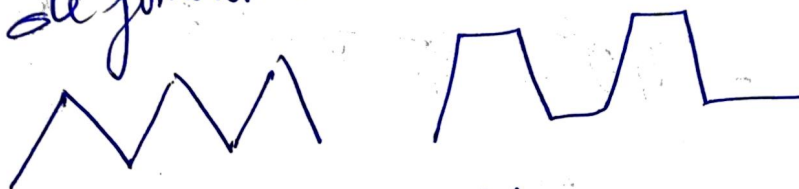
$$\boxed{\lambda = 16 \text{ m}}$$

$$\textcircled{d} \text{ kg} \quad \underline{k} = \frac{1}{8} - i \frac{1}{8} = k' - i k''$$

$$v_g = \frac{\omega}{k} = \omega \delta = \omega \frac{2k}{\sqrt{\rho c \omega}} = \frac{2k \omega}{\sqrt{\rho c}}$$

$v_g(\omega) =$ vitesse de propagation.

déformation



$$v_g(\omega) = \frac{d\omega}{dk'} = 2 v_g$$

$$\underline{v_g v_g = c^2} \quad \text{transparent}$$

c) 1^{er} Janvier: $T_{min} = -10^\circ\text{C}$
 1^{er} Juillet: $T_{max} = +30^\circ\text{C}$

$$\underline{\underline{n=0}}$$

$$T(2m, t = ??) = \text{et minimale}$$

$$T(2m, t) = T_0 + a \exp\left(-\frac{2\pi x^2}{16}\right) \times \cos\left[2\pi\left(\frac{t}{T_1} - \frac{2}{16}\right)\right]$$

$$T(2m, t) \text{ et minimale} \Rightarrow \cos\left[2\pi\left(\frac{t}{T_1} - \frac{1}{8}\right)\right] = 1$$

en effet

$$T(0, 0) = T_0 + a = -10^\circ\text{C}$$

$$T(0, \frac{l}{2}) = T_0 - a = +30^\circ\text{C}$$

$$2T_0 = 20^\circ\text{C} \rightarrow T_0 = 10^\circ\text{C}$$

$$2a = -40^\circ\text{C} \rightarrow a = -20^\circ\text{C}$$

$$\cos\left(2R\left(\frac{t}{T_1} - \frac{1}{8}\right)\right) = 1$$

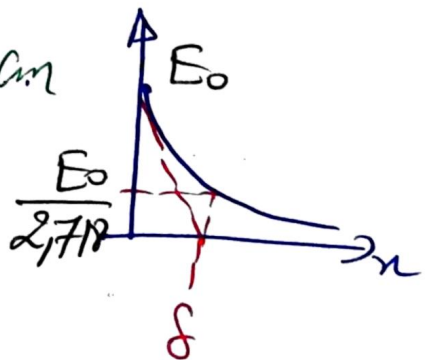
$$t = \frac{T_1}{8} = \frac{12}{8} = 1,5 \text{ mois}$$

$t = 15 \text{ Février}$

$$\frac{1}{e} = 0,37$$

$$2) \delta_j = \sqrt{\frac{2k T'_j}{\rho c 2R}} = 13 \text{ cm}$$

$$T'_j = 24 \times 3600 \text{ s}$$



$$\text{Sanné} = \sqrt{\frac{2k T_a}{\rho c 2R}} = 2,5 \text{ m}$$

d)

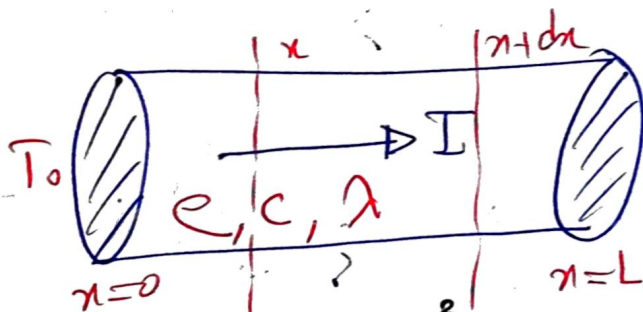
$$T_{\max} = T_0 + a \exp\left(-\frac{2\pi x}{\lambda}\right) \times (-1)$$

$$T_{\min} = T_0 + a \exp\left(-\frac{2\pi x}{\lambda}\right) \times (1)$$

$$\Delta T = -2a \exp\left(-\frac{2\pi x}{\lambda}\right) \leq 2^\circ\text{C}$$

$$x \geq 7.5 \text{ m}$$

Pb7



ML2
2015

$$p_v = \vec{j} \cdot \vec{E} = \frac{j^2}{\sigma} = \frac{I^2}{S^2 \sigma}$$

$$S[j(x) - j(x+dx)] + p_v(dx) = \frac{dU}{dt} = 0$$

$$\boxed{\frac{d^2 T(x)}{dx^2} + \frac{p_v}{\lambda} = 0}$$

$$\frac{dT(x)}{dx} = -\frac{p_v x}{\lambda} + C_1$$

$$T(x) = -\frac{p_v x^2}{2\lambda} + C_1 x + C_2$$

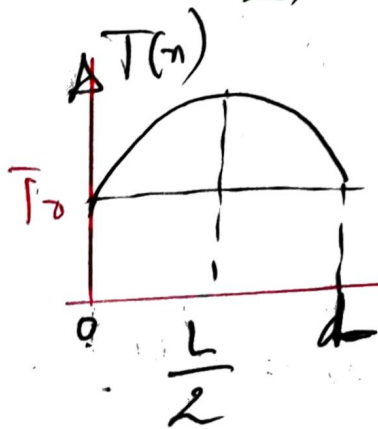
$$C_2 = T_0$$

$$T(L) = T_0 = C_2 + C_1 L - \frac{P_v L^2}{2\lambda}$$

$$C_1 = \frac{P_v L}{2\lambda}$$

$$T(x) = \frac{P_v}{2\lambda} (Lx - x^2) + T_0$$

$$T(x) = \frac{I^2}{2\lambda \gamma S^2} (Lx - x^2) + T_0$$



$$\frac{dT(x)}{dx} = \frac{P_v}{2\lambda} (L - 2x)$$

$$T\left(\frac{L}{2}\right) = T_F = 330K$$

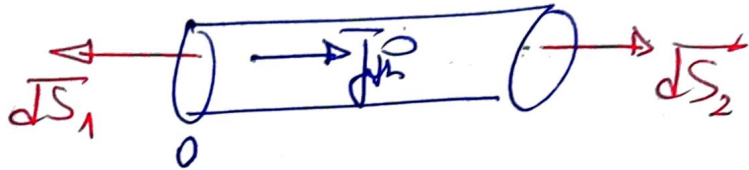
$$T_F = \frac{I^2}{2\lambda \gamma S^2} \left(\frac{L^2}{2} - \frac{L^2}{4} \right) + T_0$$

$$T_F = \frac{I^2}{8\lambda \gamma S^2} L^2 + T_0$$

$$3) \quad P_{th}(0) = - \iint \vec{f}(0) \cdot d\vec{S} = j(0) S = - \lambda \left. \frac{dT(x)}{dx} \right|_0 S$$

$$= - \lambda \frac{p_v}{2\lambda} L S = - \frac{R I^2}{2}$$

$$P_{th}(L) = - \iint \vec{f}(L) \cdot d\vec{S} = -j(L) S = \lambda \left. \frac{dT(x)}{dx} \right|_L S$$



$$P_{th}(L) = - \frac{p_v L S}{2} = - \frac{R I^2}{2}$$

$$\frac{\Delta U}{\Delta t} = P_{th}(0) + P_{th}(L) + P_{Joule} = 0$$

$$- \frac{R I^2}{2} + \left(- \frac{R I^2}{2} \right) + R I^2$$

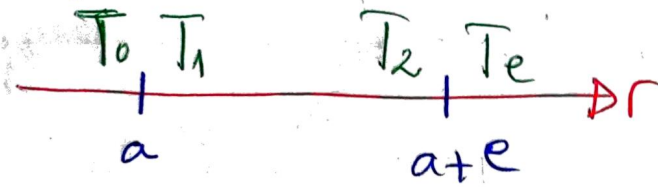
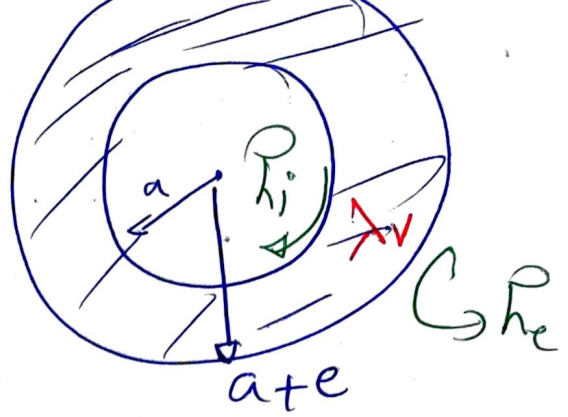
Pg Bilan entropique

$$\Delta S = \Delta S_{ech} + S_{créé} = 0$$

$$S_{créé} = - \left[\frac{P_{th}(0)}{T_0} \Delta t + \frac{P_{th}(L) \Delta t}{T_0} \right]$$

$$= + \frac{R I^2}{T_0} \Delta t > 0$$

Cylindre creux



$$R_{th\ conv_1} = \frac{1}{h_i 2\pi a L}$$

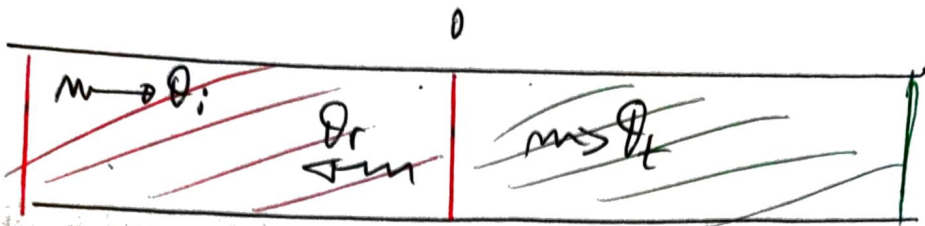
$$R_{th\ conv_2} = \frac{1}{2\pi L h_e (a+e)}$$

$$R_{th\ cond} = \frac{1}{2\pi \lambda_v L} \ln\left(\frac{a+e}{a}\right)$$

$$R_{th} = \frac{1}{2\pi L} \left[\frac{1}{h_i a} + \frac{1}{h_e (a+e)} + \frac{1}{\lambda_v} \ln\left(\frac{a+e}{a}\right) \right]$$

$$e \nearrow R_{th\ cond} \nearrow, R_{th\ conv_2} \searrow$$

$$\frac{\partial R_{th}}{\partial e} = 0$$



$$\theta_i(x, t) = \theta_{0i} \exp i(\omega t - \underline{k}_1 x)$$

$$\theta_r = \theta_{0r} \exp i(\omega t + \underline{k}_1 x)$$

$$\theta_t = \theta_{0t} \exp i(\omega t - \underline{k}_2 x)$$

$$k_1 = \frac{1-i}{\delta_1}$$

$$\Rightarrow \left\{ \begin{array}{l} \theta_{0i} + \theta_{0r} = \theta_{0t} \\ \left(+\lambda_1 \frac{\partial \theta_i}{\partial x} \right)_{0-} + \lambda_1 \left(\frac{\partial \theta_r}{\partial x} \right)_{0-} = +\lambda_2 \left(\frac{\partial \theta_t}{\partial x} \right)_{0+} \end{array} \right.$$

$$-\lambda_1 \cdot \underline{k}_1 \theta_{0i} + \lambda_1 \cdot \underline{k}_1 \theta_{0r} = \lambda_2 \underline{k}_2 \theta_{0t}$$

$$r = \frac{\frac{\lambda_1}{\delta_1} - \frac{\lambda_2}{\delta_2}}{\frac{\lambda_1}{\delta_1} + \frac{\lambda_2}{\delta_2}}$$

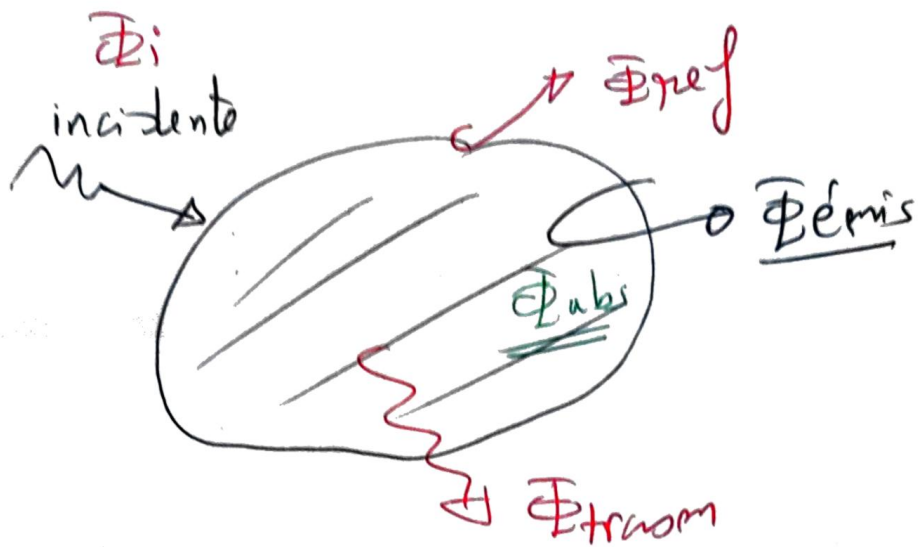
$$t = \frac{2 \lambda_1 / \delta_1}{\frac{\lambda_1}{\delta_1} + \frac{\lambda_2}{\delta_2}}$$

$$\frac{\lambda}{\delta} = \frac{\lambda}{\sqrt{2\lambda}} = \sqrt{\frac{\omega}{2}} \sqrt{\epsilon c \lambda}$$

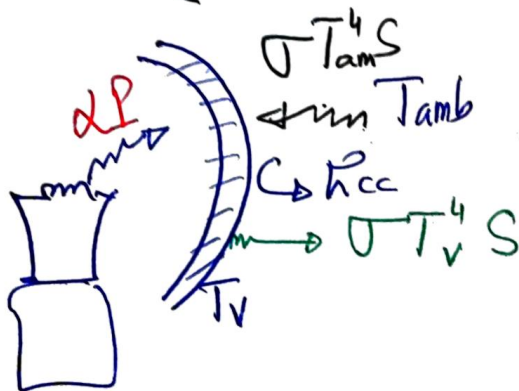
$$r = \frac{e_1 - e_2}{e_1 + e_2}$$

$$t = \frac{2e_1}{e_1 + e_2}$$

$$e = \sqrt{\epsilon c \lambda} = \text{effusivité thermique}$$



$$\Phi_{emis} = (\sigma T^4) S$$



$$\Phi_{emis} = \sigma (T_v^4 - T_{amb}^4) S;$$

CN → amb

$$\Phi_{th \text{ am du vitre} \rightarrow amb} = h_{cc} (T_v - T_{amb}) S$$

$$\alpha P = [h_{cc} (T_v - T_{amb}) + \sigma (T_v^4 - T_{amb}^4)] S$$

on pose $T_v = T_{amb} + \Delta T;$ $\Delta T \ll T_{amb}$

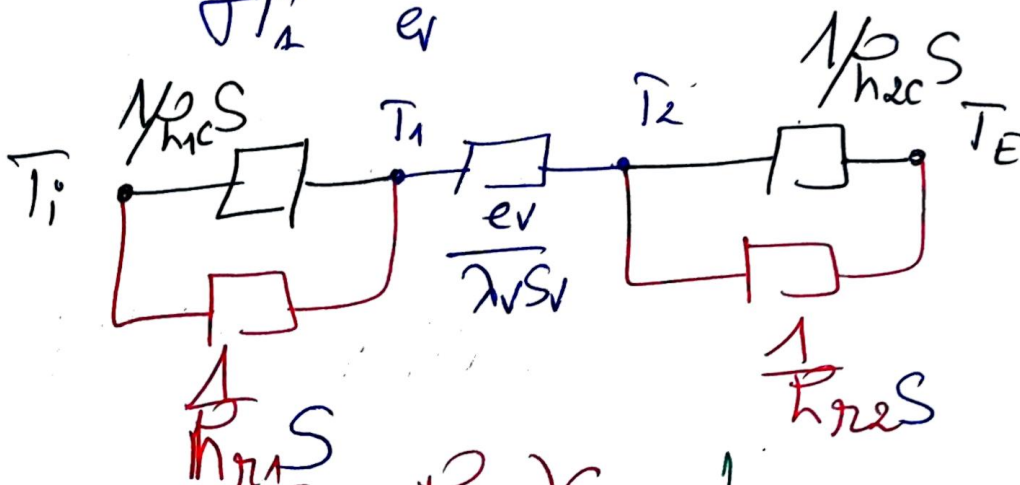
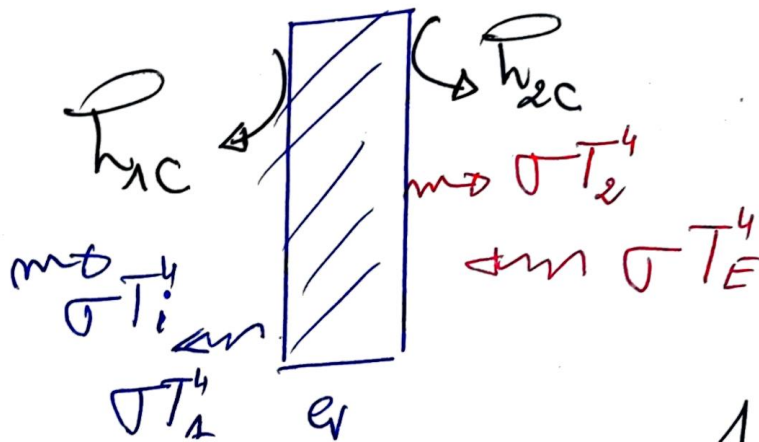
$$T_v^4 = T_{amb}^4 \left(1 + \frac{\Delta T}{T_{amb}} \right)^4 \simeq T_{amb}^4 \left(1 + 4 \frac{\Delta T}{T_{amb}} \right)$$

$$T_v^{ll} - T_{amb}^{ll} \approx 4 T_{amb}^3 \Delta T$$

$$\Phi_{th \text{ conduction}} = 4 \sigma T_{amb}^3 S (T_v - T_{amb})$$

on pose $R_{ry} = 4 \sigma T_{amb}^3$

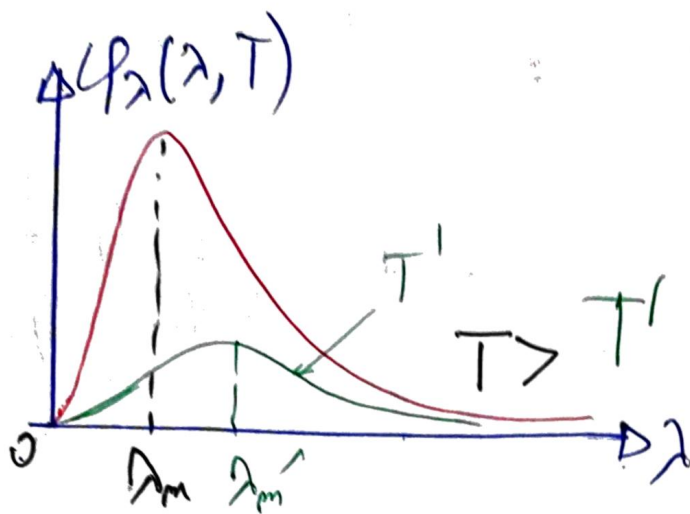
$$\alpha I = (h_{cc} + h_{ry}) S (T_v - T_{amb})$$



$$G_{11} = (h_{2c} + h_{r2}) S = \frac{1}{R_{11}}$$

$$R_{11} = \frac{1}{(h_{2c} + h_{r2}) S}$$

27/2
2016



Loi de Wien

$$\lambda_m T = \lambda'_m T' = 2898 \mu\text{m} \cdot \text{K}$$

$$\frac{C_\lambda}{T^5} = \text{cte}$$

soleil $T_s = 5870 \text{ K}$

$$\lambda_m \approx 95 \mu\text{m}$$

$\lambda \in \left[\frac{\lambda_m}{2}; 10\lambda_m \right]$ domaine spectral

$$\lambda \in [0,25 \mu\text{m}; 5 \mu\text{m}] \quad 93\%$$

$$\lambda \in \left[\frac{\lambda_m}{2}; 8\lambda_m \right] \quad 98\%$$

vitre ~~absorbe~~ $\lambda \geq 4 \mu\text{m}$

$\Rightarrow$ presque totalement transparent

Corps noirs: $T = 350 \text{ K}$

$$\lambda_m T \approx 3000 \mu\text{m} \cdot \text{K} \Rightarrow \lambda_m \approx \frac{3000}{35} = 85,7$$

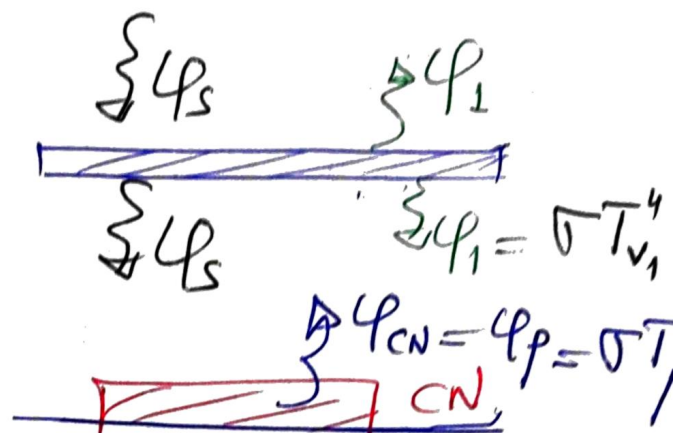
$$\lambda \in [4,3 \mu\text{m} - 85 \mu\text{m}]$$

Ex 1

vitre 1

$$q_s + q_p = 2q_1 + q_s$$

$$\boxed{2q_1 = q_p}$$



puisque: $q_s + q_1 = q_p = 2q_1$

$$\boxed{q_1 = q_s} \Rightarrow T_{v1} = \left(\frac{q_s}{\sigma}\right)^{1/4}$$

~~$$q_p = 2^{1/4} \left(\frac{q_s}{\sigma}\right)^{1/4}$$~~

$$q_p = 2q_s \Rightarrow T_p = \left(\frac{2q_s}{\sigma}\right)^{1/4}$$

$$T_p = 2^{1/4} T_{v1}$$

b) vitre $2q_1 = q_p$

puisque: $5q_s + q_1 S = q_p S + \frac{mc}{\Delta t} (\theta_f - \theta_i)$

$$m = 20 \text{ kg}$$

$$\Delta U = Q_v = mc_v (\theta_f - \theta_i)$$

$$\Delta H = Q_p = mc_p (\theta_f - \theta_i)$$

$$\text{rend} = \frac{mc \Delta \theta}{\Delta t (q_s S)}$$

2) cas de 2 vitres

vitre 2

$$2\varphi_2 = \varphi_1$$

vitre 1

$$2\varphi_1 = \varphi_2 + \varphi_p$$

plaque

$$\varphi_p = \varphi_1 + \varphi_s$$

$$\left\{ \begin{array}{l} 2\varphi_2 = \varphi_1 \\ 2\varphi_1 = \varphi_2 + \varphi_p \\ \varphi_p = \varphi_1 + \varphi_s \end{array} \right.$$

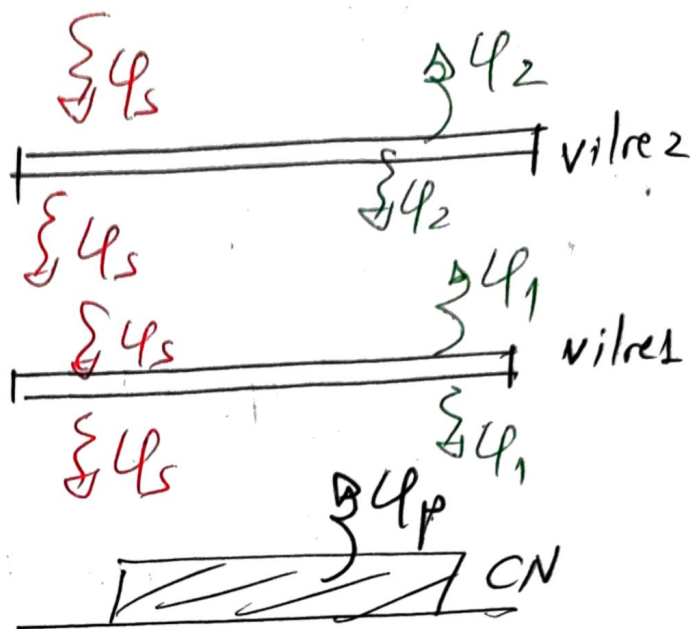
cas de n vitres

1	
n ^{ème} vitre	~~$2\varphi_n = \varphi_{n-1}$~~
n-1	~~$2\varphi_{n-1} = \varphi_n + \varphi_{n-2}$~~
n-2	~~$2\varphi_{n-2} = \varphi_{n-1} + \varphi_{n-3}$~~
1	
1	
1	
vitre 1	$2\varphi_1 = \varphi_2 + \varphi_p$
plaque	$\varphi_p = \varphi_1 + \varphi_s$

$$\varphi_n = \varphi_s$$

$$\varphi_{n-1} = 2\varphi_s$$

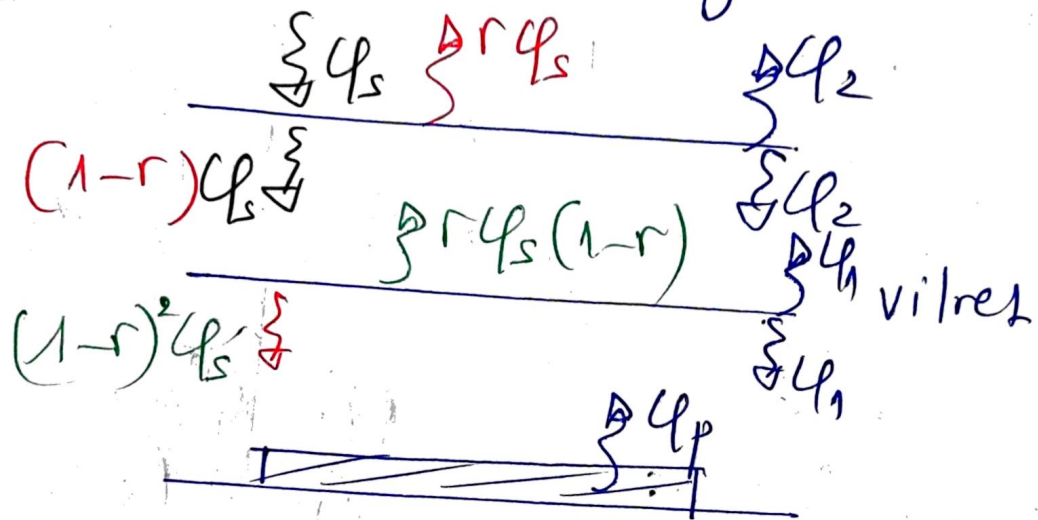
$$\varphi_1 = n\varphi_s$$



$$\begin{aligned} \varphi_2 &= \varphi_s \\ \varphi_1 &= 2\varphi_s \\ \varphi_p &= 3\varphi_s \end{aligned}$$

$$I_p = (n+1)I_s \Rightarrow T_p = \left(\frac{(n+1)I_s}{\sqrt{\quad}} \right)^{1/4}$$

3) My ces 2 vitres avec reflexion diverge



Bohen vitre 2

$$I_s + I_2 = 2I_2 + \underbrace{r I_s + (1-r) I_s}_{I_s}$$

$$\boxed{2I_2 = I_1}$$

vitre 1

$$(1-r) I_s + I_p + I_2 = 2I_1 + \underbrace{r(1-r) I_s + (1-r)^2 I_s}_{\quad}$$

$$\boxed{2I_1 = I_2 + I_p}$$

$$(1-r)^2 C_s + C_1 = C_p$$

$$\begin{cases} 2C_2 = C_1 \\ 2C_1 = C_2 + C_p \\ C_p = C_1 + (1-r)^2 C_s \end{cases}$$

$$C_2 = (1-r)^2 C_s \Rightarrow C_1 = 2(1-r)^2 C_s$$

$$C_p = (2+1)(1-r)^2 C_s$$

$$C_p = (n+1)(1-r)^n C_s$$

$$\frac{dC_p}{dn} = 0, \quad \frac{d \ln C_p}{dn} = 0$$

$$\ln C_p = \ln(n+1) + n \ln(1-r) + \ln C_s$$

$$\frac{d \ln C_p}{dn} = 0 \Rightarrow \frac{1}{(n+1)} + \ln(1-r) = 0$$

$$n = 11$$

E x 3

$$u_{em} = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$

$$u_\nu(\nu, T) = \frac{du}{d\nu} = \frac{8\pi R}{c^3} \frac{\nu^3}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

$\hookrightarrow \text{J. m}^{-3} \cdot \text{Hz}^{-1}$

$$u(T) = \int_0^\infty u_\nu(\nu, T) d\nu \quad ; \quad x = \frac{h\nu}{k_B T}$$

$$= \frac{8\pi R}{c^3} \left(\frac{h}{R}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

$$u(T) = \left(\frac{8\pi R}{c^3} \frac{h^4}{R^4} \frac{\pi^4}{15} \right) T^4 = a T^4$$

$u(T)$ = dépend de la Température 1^{er} loi de
Stefan

$$2) \quad \eta = \varphi(T) = \frac{c}{4} u(T)$$

$$\varphi(T) = \sigma T^4 = \frac{d\Phi}{dS}$$

$\text{Watt. m}^{-2} \text{ K}^{-4}$

$$u_\nu = \frac{c}{4} u_\nu$$

$$3) \quad \frac{du_\nu}{d\nu} = 0 \quad \frac{du_\nu}{d\nu} = 0 \Rightarrow e^{-x} = 1 - \frac{x}{3}$$

$T = 1800 \text{ K} \longrightarrow x = 287$

$$4) u_{\lambda}(\lambda, T) = \frac{du}{d\lambda} = \frac{du}{d\nu} \frac{d\nu}{d\lambda}$$

$$u_{\lambda} = u_{\nu} \left| \frac{d\nu}{d\lambda} \right|$$

$$\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$u_{\lambda} = \frac{8\pi k c}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

$$u_{\lambda} = \frac{c}{4} u_{\nu}$$

$$\frac{du_{\lambda}}{d\lambda} = 0,$$

$$\frac{du_{\nu}}{d\nu} = 0$$

$$e^{-x} = 1 - \frac{x}{5}$$

$$x = \frac{hc}{\lambda k_B T} = 4,964$$

$$\lambda_m T = 2898 \mu m K$$