

$$u_\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

$$u_\nu^{JR}(\nu, T) = A (k_B T)^\alpha \nu^\beta c^\gamma$$

Jeans Rayleigh: 

$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} K x^2$$

Déterminer α, β, γ

$$[u_\nu] = [k_B T]^\alpha [\nu]^\beta [c]^\gamma$$

$$[u_\nu] = \frac{[\text{énergie}]}{[\text{volume}] [\text{Freq}]} = \frac{[\text{Force}] [\text{distance}]}{[L]^3 T^{-1}}$$

$$= \frac{1}{T^2} \cdot \frac{L}{L^3} T = 1 \cdot L^{-1} \cdot T^{-1}$$

$$[k_B T] = [\text{énergie d'activation thermique}]$$

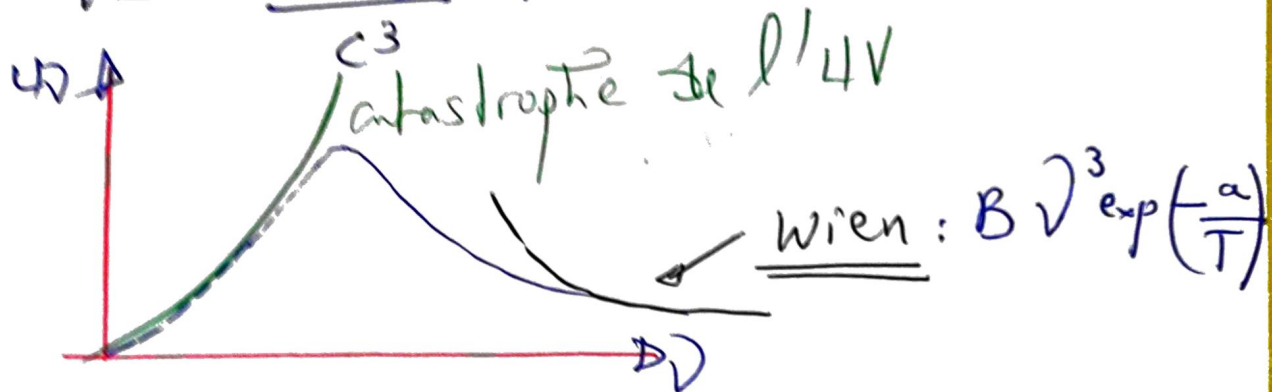
$$= 1 \frac{L}{T^2} L = 1 L^2 T^{-2}$$

$$[\nu] = [\text{temps}]^{-1} = T^{-1}$$

$$[c] = L T^{-1}$$

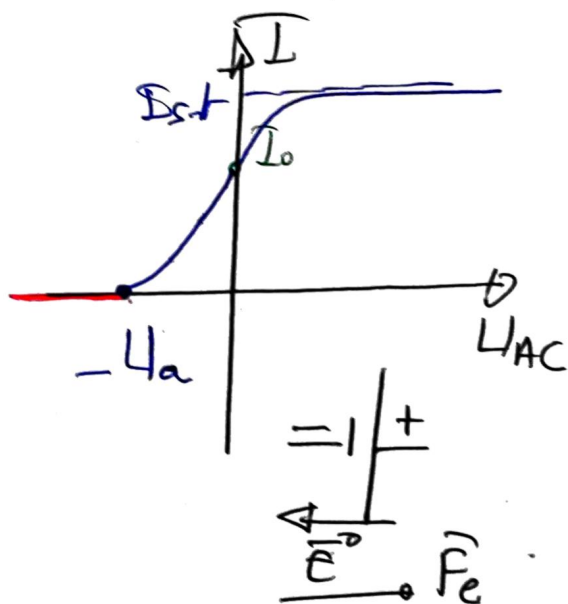
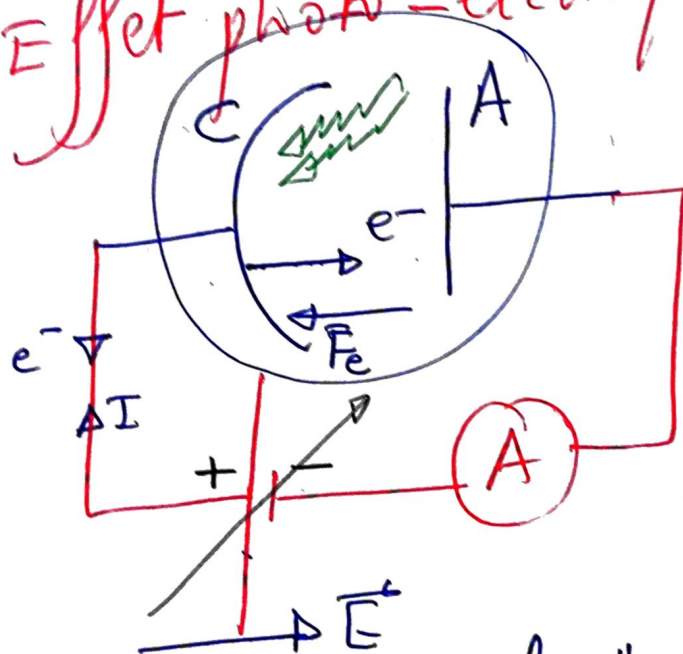
$$\left. \begin{array}{l} \alpha = 1 \\ \beta = 2 \\ \gamma = -3 \end{array} \right\} \underline{182} \underline{2012} \underline{1/20}$$

$$u\nu = A \frac{(k_B T)}{c^3} \nu^2 \quad ; \quad h\nu \ll k_B T$$

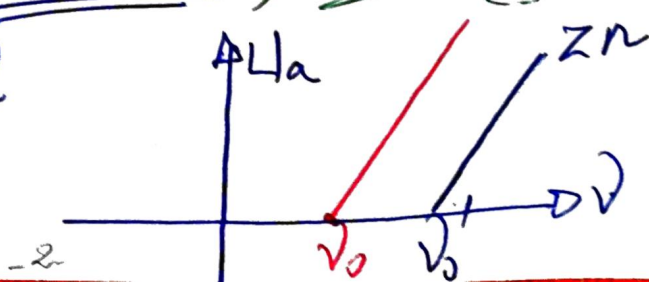


$$E_{\text{oscillateur harmonique}} = h\omega\left(n_x + \frac{1}{2}\right)$$

Effet photo-électrique 182 2021



$U_a =$ le potentiel d'arrêt, $\nu_A = 0$ CS
 $\nu_0 =$ fréquence seuil



Théorème de l'énergie mécanique
 $\Delta E = W(\text{Forces mécaniques}) = 0$

$$E_{\text{cathode}} = E_{\text{anode}}$$

$$(E_c + E_p)_{\text{cathode}} = (E_c + E_p)_{\text{anode}}$$

$$\frac{1}{2} m v_c^2 + (-e V_c) = \frac{1}{2} m v_A^2 + (-e V_A)$$

$$\frac{1}{2} m v_c^2 - \frac{1}{2} m v_A^2 = -e(V_A - V_c)$$

$$\frac{1}{2} m v_c^2 - \frac{1}{2} m v_A^2 = -e U_{Ac}$$

$$\boxed{\frac{1}{2} m v_c^2 = +e U_a}$$

$$E = h\nu = W_0 + \frac{1}{2} m v_c^2$$

$W_0 = \text{énergie d'extraction}$

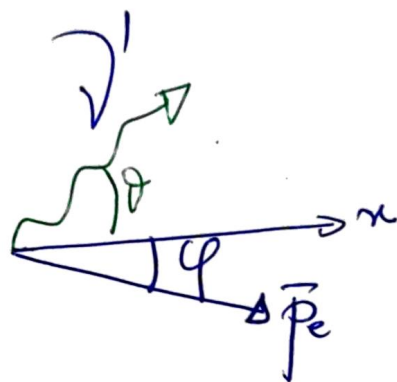
$$E - W_0 = e U_a$$

$$\frac{1}{2} m v_c^2 =$$

$$U_a = \frac{h\nu}{e} - \frac{W_0}{e} ; \text{ pente } \frac{h}{e}$$

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Effect Compton



$$(h\nu + 0) = (h\nu' + \frac{p_e^2}{2m})$$

Pq : photon $\vec{p} = \hbar \vec{k}$

$$E = \hbar \omega = \frac{\hbar}{2\pi} 2\pi \nu$$

$$\boxed{E = h\nu}$$

$$\vec{p} = \hbar \vec{k}$$

$$= \frac{\hbar}{2\pi} \cdot \frac{2\pi}{\lambda} \vec{u}$$

$$\lambda = cT = \frac{c}{\nu}$$

$$\boxed{\vec{p} = \frac{\hbar}{\lambda} \vec{u}}$$

$$\boxed{p_{\text{photon}} = \frac{E}{c} \vec{u}}$$

$$\underline{I_{\text{tot}}} = \sum \vec{F}_{\text{ext}} = \vec{0} = \frac{d\vec{p}}{dt}$$

Conservatⁿ de la qt de mv

$$\frac{h\nu}{c} \vec{u}_n + \vec{0} = \vec{p}_e + \frac{h\nu'}{c} \vec{u}'$$

$$\vec{p}_e = \frac{h}{c} (\nu \vec{u}_n - \nu' \vec{u}')$$

$$p_e^2 = \frac{h^2}{c^2} [\nu^2 + \nu'^2 - 2\nu\nu' \cos\theta]$$

$$\frac{\partial}{\partial \nu} p_e^2 = 2m h (\nu - \nu')$$

Sachant $\nu \approx \nu'$

$$p_e^2 = \frac{h^2}{c^2} 2\nu^2 (1 - \cos\theta)$$

$$p_e^2 = \frac{2h^2 \nu^2}{c^2} 2 \sin^2\left(\frac{\theta}{2}\right) = 2m h (\nu - \nu')$$

$$\nu - \nu' = \frac{2h \nu^2}{m c^2} \sin^2\left(\frac{\theta}{2}\right)$$

$$\frac{c}{\lambda} - \frac{c}{\lambda'} = \frac{2h}{m \lambda^2} \sin^2\left(\frac{\theta}{2}\right)$$

$$\boxed{\lambda' - \lambda = \frac{2h}{m c} \sin^2\left(\frac{\theta}{2}\right)}$$

$$\lambda_{DB} = \frac{h}{p} \sim \text{dimension systeme}$$

→ quantique

exple $E_c = 10 \text{ eV} = 10 \times 1,6 \times 10^{-19} \text{ J}$

$$E_c = \frac{p^2}{2m} \rightarrow p = \sqrt{2m E_c}$$

$$\lambda_{DB} = \frac{6,62 \times 10^{-34}}{\sqrt{2 \times 3,1 \times 10^{-31} \times 1,6 \times 10^{-18}}}$$

$$\lambda_{DB} = \dots \quad A: \text{atome} \Rightarrow \text{quantique}$$

• une personne $m = 70 \text{ kg}$; $v = 1 \text{ m/s}$.

$$p = mv = 70 \text{ kg} \cdot \text{m s}^{-1}$$

$$\lambda_{DB} = \frac{6,62 \times 10^{-34}}{70} \ll 1 \text{ m}$$

→ classique

Gaz monoatomique $\langle E_c \rangle = \frac{1}{2} m \langle v^2 \rangle$

$$= \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

$$= \frac{3}{2} m \langle v_x^2 \rangle = 3 \times \frac{1}{2} k_B T$$

$S = a \dim = \text{moment cinétique} \approx \hbar$
quantique

$S \gg \hbar$; classique

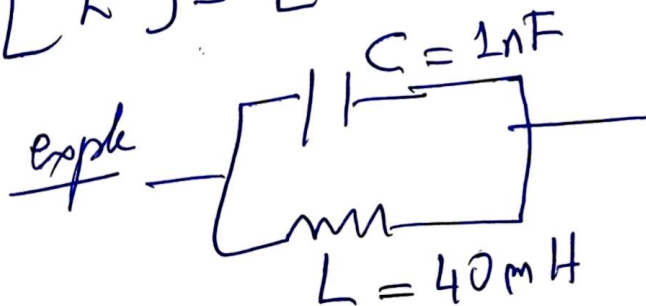
$\hbar = \text{moment cinétique de rotation de l'e-}$
autour de lui-même

$$\vec{m}_s = \pm \frac{1}{2} \hbar \vec{U}_z \quad \rho = \frac{1}{2}$$

$\rho = \text{demi-entier} = \text{Fermions}$

$\rho = \text{entier} = \text{Bosons}$

$$[L] = [\text{énergie}] [\text{Temps}] = E T$$



$$Q = 1 \text{ pC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T_0}$$

$$S = \left(\frac{1}{2} \frac{Q^2}{C} \right) 2\pi \sqrt{LC} \dots \rightarrow \hbar$$

$$[L] = [\text{Force}] [\text{distance}] [\text{Temps}]$$

$$[L] = p L$$

$p = \text{impulsion}$
 $= q \hbar \text{ de mvh.}$

$$[R] = M \frac{L}{T^2} L T = M L^2 T^{-1}$$

$$[R]^2 = (M L^2 T^{-1}) (E T) \\ = E M L^2$$

chute libre : $m = 70 \text{ kg}$
 $E = m g H$

$$L = H$$

$$S^2 = (m g H) m H^2 = R^2$$

$$H = 2 \mu m$$

proton $m = 10^{-27} \text{ kg}$

$$H = 7 \mu m$$

$$-\frac{\hbar^2}{2m} \Delta \psi + V \psi = E \psi = i \hbar \frac{\partial \psi}{\partial t}$$

$$\left[\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] + V \right] \psi = i \hbar \frac{\partial \psi}{\partial t}$$

$$\boxed{H \psi = i \hbar \frac{\partial \psi}{\partial t}}$$

$V =$ énergie potentielle ou potentiel (Joule)

• particule libre $V=0$

• à 1 dimension $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = i \hbar \frac{\partial \psi}{\partial t}$

• $\rho = \frac{dP}{dx} =$ densité de probabilité

$$[\rho] = L^{-1} \quad |\psi(x, t)|^2 = \rho$$

$$[\psi] = L^{-1/2}$$

• état stationnaire $\psi(x, t) = \psi(x)$

$$\hookrightarrow H \psi = E \psi = i \hbar \frac{\partial \psi}{\partial t}$$

$$|\psi\rangle, \langle\psi|$$

$$H \psi(x, t) = i \hbar \frac{\partial \psi}{\partial t}$$

$$-\frac{\hbar^2}{2m} \psi''(x) f(t) = i \hbar f'(t) \psi(x)$$

$$-\frac{\hbar^2}{2m} \frac{\psi''(x)}{\psi(x)} = i \hbar \frac{f'(t)}{f(t)} = i \hbar \omega$$

$\frac{\dot{p}}{p} = \frac{i \omega p}{p}$

$$+ i \hbar \frac{\partial \psi}{\partial t} = E \psi \Rightarrow$$

$$i \hbar f'(t) = E f(t)$$

$$\frac{f'(t)}{f(t)} = \frac{E}{i \hbar} = -i \omega$$

$$f^*(t) = \int \exp(-i \omega t)$$

$$\psi(x, t) = \psi(x) e^{-i \omega t}$$

$$-\frac{\hbar^2}{2m} \psi''(x) = E \psi(x)$$

$$\psi''(x) + \frac{2m E}{\hbar^2} \psi(x) \Rightarrow$$

$$\frac{2mE}{\hbar^2} = \frac{2m}{\hbar^2} (E_c + V) = \frac{2m}{\hbar^2} \left(\frac{p^2}{2m} \right) = \left(\frac{p}{\hbar} \right)^2$$

$$\frac{2mE}{\hbar^2} = k^2; \quad \boxed{p = \hbar k}$$

$$\psi''(x) + k^2 \psi(x) = 0.$$

$$\psi(x) = A e^{ikx} + B e^{-ikx}$$

$$\psi(x, t) = A e^{i(kx - \omega t)} + B e^{-i(kx + \omega t)}$$

$(\omega)^+$
moving

$(\omega)^-$
moving

2) $\psi(x, t) = A e^{i(kx - \omega t)}$

$$a) E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} = \hbar \omega$$

$$\omega = \frac{\hbar k^2}{2m}$$



$$b) v_g = \frac{\omega}{k} = \frac{\hbar k}{2m} = \frac{p}{2m} = \frac{v_{\text{particle}}}{2}$$

$$p = m v_p$$

$$v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m} = v_p$$

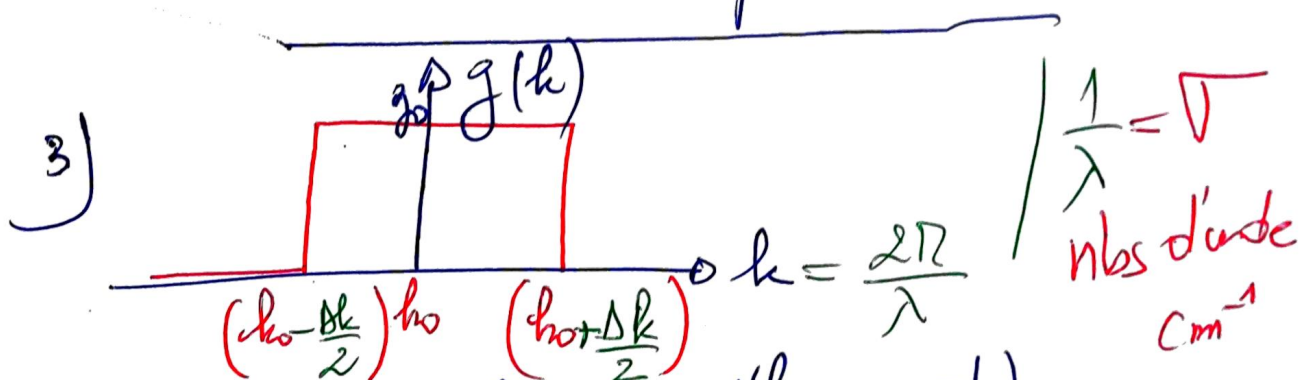
$$c) P = \int_{-\infty}^{+\infty} \rho(x) dx = A^2 \int_{-\infty}^{+\infty} dx \quad \text{diverge}$$

$P = \text{diverge}$: non localisé

onde plane $\langle u_{em} \rangle = 2 \langle u_e \rangle = 2 \times \left(\frac{\epsilon E_0^2}{2} \right)$

$$\langle u_{em} \rangle = \epsilon E_0^2$$

$$\langle L_{em} \rangle = \iiint_{\text{espace}} \epsilon E_0^2 d\tau \quad \text{diverge}$$



$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(k) e^{i(kx - \omega t)} dk$$

$$a) \psi(x, 0) = \frac{1}{\sqrt{2\pi}} \int_{k_1}^{k_2} g_0 e^{ikx} dk$$

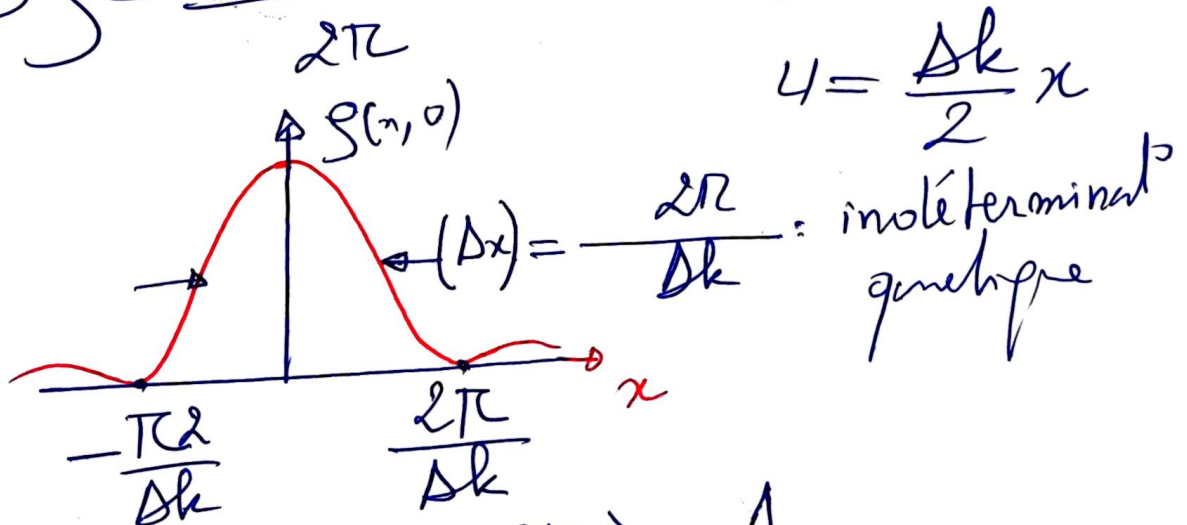
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$$g(k) = g_0; \quad k \in [k_1, k_2]$$

$$\begin{aligned} \psi(n, 0) &= \frac{g_0}{\sqrt{2\pi}} \left[\frac{e^{ikx}}{ix} \right]_{k_0 - \frac{\Delta k}{2}}^{k_0 + \frac{\Delta k}{2}} \\ &= \frac{g_0}{\sqrt{2\pi}} \frac{(e^{ik_0 x})}{ix} \left(2i \sin\left(\frac{\Delta k}{2} n\right) \right) \end{aligned}$$

$$\psi(n, 0) = \frac{g_0 \Delta k}{\sqrt{2\pi}} \operatorname{sinc}\left(\frac{\Delta k}{2} n\right) e^{ik_0 x}$$

$$b) S = \frac{g_0^2 (\Delta k)^2}{2\pi} \operatorname{sinc}^2\left(\frac{\Delta k}{2} n\right)$$



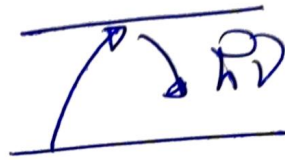
$$(\Delta n) \cdot (\Delta k) = 2\pi > \frac{1}{2}$$

$$(\Delta n) (\Delta p) \geq \frac{\hbar}{2}$$

$$v_g = \frac{\Delta \omega}{\Delta t} = \frac{\Delta \omega}{\Delta k}$$

$$\Delta \omega \Delta k = \Delta \omega \Delta t \geq \frac{1}{2}$$

$$\Delta \omega \sim \frac{1}{\tau}$$



$$\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$$

$$\text{c) } P=1 = \int_{-\infty}^{+\infty} \frac{g_0^2 (\Delta k)^2}{2\pi} \text{sinc}^2\left(\frac{\Delta k}{2} x\right) dx$$

$$= \frac{g_0^2 (\Delta k)^2}{2\pi} \frac{2}{\Delta k} \left(\int_{-\infty}^{+\infty} \text{sinc}^2(u) du \right)$$

$$1 = \frac{g_0^2 \Delta k}{\pi} \Rightarrow$$

$$g_0 = \frac{1}{\sqrt{\Delta k}}$$

$$\psi(x, 0) = \sqrt{\frac{\Delta k}{2\pi}} \text{sinc}\left(\frac{\Delta k}{2} x\right) e^{i k_0 x}$$

$$g(\omega(k)) = \omega(k_0) + (k - k_0) \left(\frac{d\omega}{dk} \right)_{k_0} + \dots$$

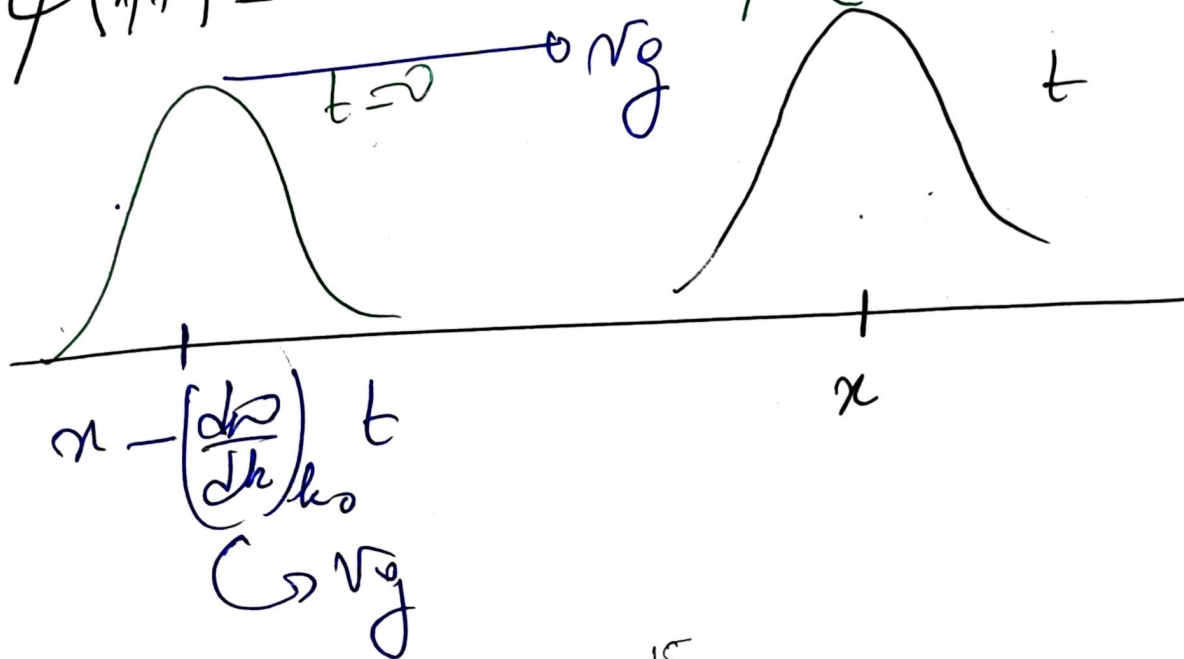
$$f(n) = f(n_0) + (n - n_0) f'(n_0) + \dots$$

$$\omega(k) = \omega_0 + (k - k_0) v_g$$

$$\psi(n, t) = \frac{1}{\sqrt{2\pi}} \int g(k) e^{i(kn - (\omega_0 + (k - k_0)v_g)t)} dk$$

$$= \frac{1}{\sqrt{2\pi}} e^{-i(\omega_0 t - k_0 v_g t)} \int_{-\infty}^{\infty} g(k) e^{i k (n - v_g t)} dk$$

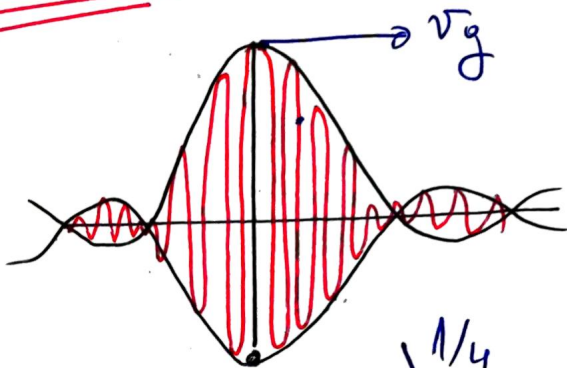
$$\psi(n, t) = e^{i(k_0 v_g - \omega_0)t} \psi(n - v_g t, 0)$$



$$\psi(n, t) = \left(\sqrt{\frac{\Delta k}{2\pi}} \right) \text{sinc} \left[\frac{\Delta k}{2} (n - v_g t) \right] \times e^{i k_0 (n - v_g t)} e^{i(\omega_0 t - k_0 n)}$$

$$\psi(n, t) = \sqrt{\frac{\Delta k}{2\pi}} \text{sinc} \left[\frac{\Delta k}{2} (n - v_g t) \right] e^{-i(\omega_0 t - k_0 n)}$$

$\Delta k \ll k_0$ ≈ 1



$$4) g(k) = \left(\frac{a^2}{2\pi} \right)^{1/4} \exp \left[-\frac{a^2}{4} (k - k_0)^2 \right]$$

$$a) [a][k] = 1 \Rightarrow [a] \frac{1}{L} = 1$$

$$[a] = \text{Longueur}$$

$$\frac{P_0}{T} = \exp \left(\frac{P_0}{T_0} \right) ; T(r) = C_1 \ln r + C_2$$

$$T(r) = C_1 \ln \left(\frac{r}{a} \right) + T_2$$

$$b) \psi(x, 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(k) e^{ikx} dk$$

$$= \left(\frac{a^2}{2\pi}\right)^{1/4} \frac{1}{\sqrt{2\pi}} \int e^{ikx} e^{-\frac{a^2}{4}(k-k_0)^2} dk$$

$$* -\frac{a^2}{4}(k-k_0)^2 + ikx - ik_0x + ik_0x$$

$$= -\frac{a^2}{4} \left[(k-k_0)^2 + 2i(k-k_0)\left(\frac{x}{a^2}\right) + \left(\frac{2ix}{a^2}\right)^2 \right]$$

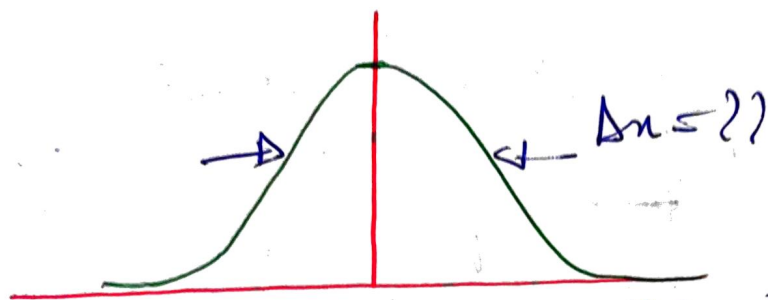
$$+ ik_0x - \frac{x^2}{a^2}$$

$$= -\frac{a^2}{4} \left[(k-k_0) - \frac{2ix}{a^2} \right]^2 + ik_0x - \frac{x^2}{a^2}$$

$$\psi(x, 0) = \left(\frac{a^2}{2\pi}\right)^{1/4} \frac{1}{\sqrt{2\pi}} e^{ik_0x} e^{-\frac{x^2}{a^2}} \sqrt{\frac{\pi}{a^2}}$$

$$= \left(\frac{2}{\pi a^2}\right)^{1/4} e^{(ik_0x - \frac{x^2}{a^2})}$$

$$S = |\psi|^2 = \left(\frac{2}{\pi a^2}\right)^{1/2} e^{-\frac{2x^2}{a^2}}$$



$$f(l) = \frac{1}{\sigma} \exp\left(-\frac{l^2}{b^2}\right)$$

$$\Delta l = \frac{b}{\sqrt{2}}$$

$$\exp\left(-\frac{2n^2}{a^2}\right)$$

$$= \exp\left[-\frac{n^2}{\left(\frac{a}{\sqrt{2}}\right)^2}\right]$$

$$\boxed{\Delta n = \frac{a/\sqrt{2}}{\sqrt{2}} = \frac{a}{2}}$$

$$|g(k)|^2 = \left(\frac{a^2}{2\pi}\right)^{1/2} \exp\left[-\frac{a^2}{2}(k-b)^2\right]$$

$$\approx \exp - \frac{(k-b)^2}{\left(\frac{\sqrt{2}}{a}\right)^2}$$

$$\Delta k = \frac{\sqrt{2}/a}{\sqrt{2}} = \frac{1}{a}$$

$$(\Delta n)(\Delta k) = \frac{1}{2}$$

$$\Delta n = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$$

$$\langle n \rangle = \int_{-\infty}^{+\infty} n |y|^2 dn$$

$$\langle n^2 \rangle = \int_{-\infty}^{+\infty} n^2 |y|^2 dn$$

5) a) $g(k) = \sqrt{b} \exp(-b|k-k_0|)$
longueur

b) $\psi(n,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \sqrt{b} \exp(-b|k-k_0|) e^{ikx} dk$

$$= \frac{\sqrt{b}}{\sqrt{2\pi}} \left[\int_{-\infty}^{k_0} e^{ikx} e^{b(k-k_0)} dk + \int_{k_0}^{+\infty} e^{ikx} e^{-b(k-k_0)} dk \right]$$

$$\Rightarrow \psi(n,0) = \frac{\sqrt{2}}{\sqrt{\pi}} b^{3/2} \frac{e^{ik_0 x}}{x^2 + b^2}$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x \cdot \frac{1}{4} | \psi(x, 0) |^2 dx = \frac{2}{\pi} b^3 \int_{-\infty}^{+\infty} \frac{x dx}{(x^2 + b^2)^2}$$

impair

$$\langle x^2 \rangle = \int_{-\infty}^{+\infty} x^2 | \psi(x, 0) |^2 dx \quad \underline{\underline{= 0}}$$

$$= \int_{-\infty}^{+\infty} x^2 \frac{2}{\pi} b^3 \frac{dx}{(x^2 + b^2)^2}$$

$$= \frac{2}{\pi} b^3 \frac{\pi}{2b} = b^2$$

$$\Delta x(0) = b$$

$$2) \Delta h = \frac{1}{b\sqrt{2}}; \quad \Delta x \cdot \Delta h = \frac{1}{\sqrt{2}} > \frac{1}{2}$$

