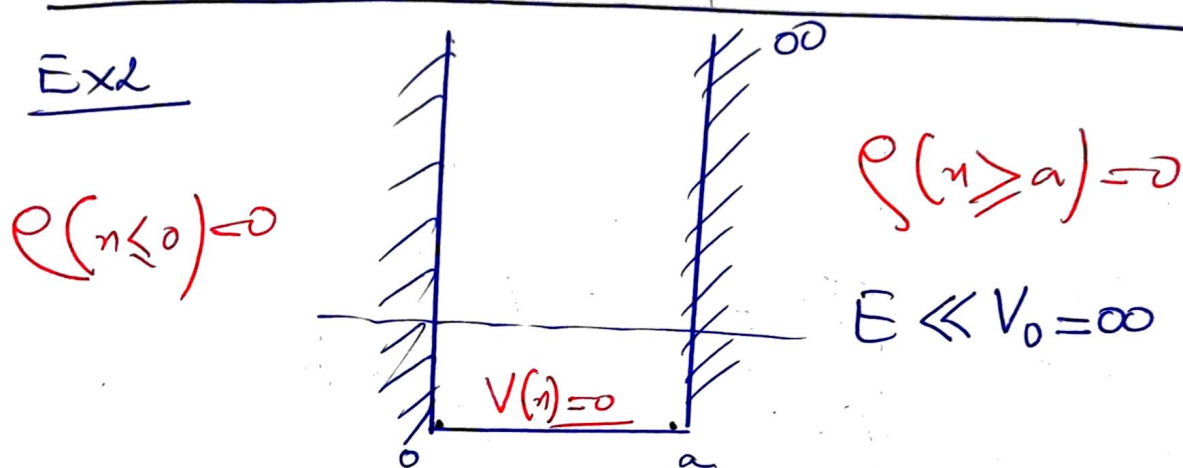


$$\left\langle \frac{m v_n^2}{2} \right\rangle = \frac{1}{2} k_B T$$

$$\left\langle \frac{1}{2} k n^2 \right\rangle = \frac{1}{2} k_B T$$



$$\psi(x, t) = \psi(x) e^{-i\omega t}$$

$$\psi(0) = \psi(a) = 0 \Rightarrow \lambda_n = \frac{2a}{n}$$

$$k_n = \frac{2\pi}{\lambda_n} = \frac{2\pi}{\frac{2a}{n}} = \frac{n\pi}{a} \quad n \in \mathbb{N}^*$$

$$E_n = \frac{p_n^2}{2m} = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2m a^2}$$

$$p_n = \hbar k_n = \frac{\hbar \pi}{a} n$$

$$2) (\Delta x)(\Delta p) \simeq a [p_{n+1} - p_n] = a \left(\frac{\pi \hbar}{a} \right) = \pi \hbar > \frac{\hbar}{2}$$

$$3) -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + 0\psi = E\psi$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \underbrace{\frac{2mE}{\hbar^2}}_{\rightarrow k^2} \psi(x) = 0$$

$$\psi(x) = A \cos(kx) + B \sin(kx)$$

$$\underline{x=0} \quad \psi(0) = 0 = A$$

$$\underline{x=a} \quad \psi(a) = 0 = B \sin(ka)$$

$$(ka) = n\pi \quad \boxed{ka = \frac{n\pi}{a}}$$

$$E_n = \frac{\pi^2 \hbar^2}{2ma^2} n^2 = E_1 n^2$$

$$E_1 = \frac{\pi^2 \hbar^2}{2ma^2} = E_{\min} = \text{fondamental}$$

Énergie de confinement

Si $V(x) = 0$ minimale $\Rightarrow$
position d'équilibre stable
classique $E_{\min} = 0$

$$\psi(n, t) = B \sin\left(\frac{n\pi x}{a}\right) e^{-i\omega_n t}$$

$$\int_{-\infty}^{+\infty} |\psi(n, t)|^2 dx = \int_0^a B^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

$$B^2 \frac{a}{2} = 1, \quad B = \sqrt{\frac{2}{a}}$$

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) e^{-i\omega_n t}$$

$$4) \langle E_c \rangle = \frac{\langle p_x^2 \rangle}{2m} \Rightarrow \frac{(\Delta p_x)^2}{2m}$$

$$(\Delta p_x)^2 = \langle p_x^2 \rangle - \langle p_x \rangle^2$$

$$\langle E_c \rangle$$

$$\frac{\hbar^2}{2m(2\Delta n)^2}$$

$$\langle E_c \rangle \geq \frac{\hbar^2}{8ma^2} \neq 0$$

5) a) $E_{n+1} - E_n = E_1 (n+1)^2 - E_1 n^2$

$$= \frac{\pi^2 \hbar^2}{2m a^2} (2n+1)$$

a)

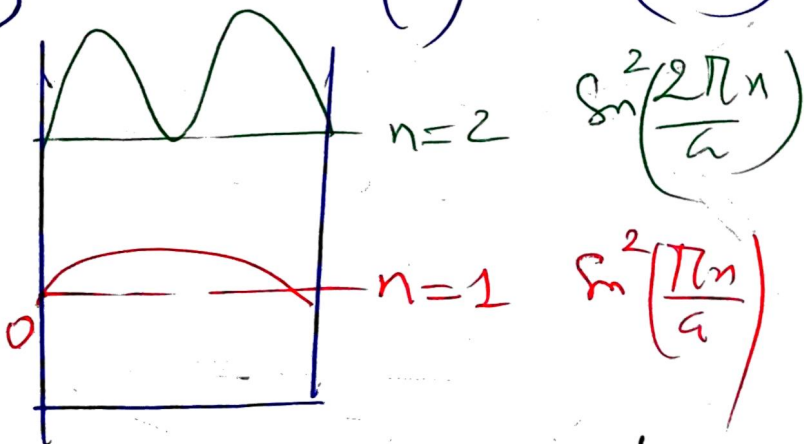
* a) $(E_{n+1} - E_n)$ ~~Amplitude~~ amplitude classique

La quantification due aux conditions limites.

onde, cavité

$$\begin{cases} \vec{E}(0^-) = \vec{E}(0^+) = 0 \\ \vec{E}(a^-) = \vec{E}(a^+) = 0 \end{cases}$$

7) $\rho = |4|^2 = \left(\frac{2}{a}\right) \sin^2\left(\frac{n\pi x}{a}\right)$



$$-\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) e^{-i\omega t}$$

4 $\frac{1}{4} \sqrt{\frac{2}{a}}$

$$2) \psi(x, 0) = \frac{1}{\sqrt{5a}} \left[3 \sin\left(\frac{\pi x}{a}\right) + \sin\left(\frac{2\pi x}{a}\right) \right]$$

$$\psi(x, t) = \frac{1}{\sqrt{5a}} \left[3 \sin\left(\frac{\pi x}{a}\right) e^{-i\omega_1 t} + \sin\left(\frac{2\pi x}{a}\right) e^{-i\omega_2 t} \right]$$

$$P = \frac{1}{5a} \left[9 \sin^2\left(\frac{\pi x}{a}\right) + \sin^2\left(\frac{2\pi x}{a}\right) + 2 \times 3 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi x}{a}\right) \cos[(\omega_2 - \omega_1)t] \right]$$

$$P_1 = \frac{1}{5a} \int_0^a 9 \sin^2\left(\frac{\pi x}{a}\right) dx = \frac{9}{10}$$

$$P_2 = \frac{1}{5a} \times 1 \left(\int_0^a \sin^2\left(\frac{2\pi x}{a}\right) dx \right) = \frac{1}{10}$$

$$\star \int_0^a \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi x}{a}\right) dx = \frac{a}{2} \delta_{n,m}$$

$$|4\rangle ; \langle 4|$$

g)

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z, t) = E \psi$$

$$-\frac{\hbar^2}{2m} \left[\underbrace{\frac{\psi_1''(x)}{\psi_1(x)}}_{-k_1^2} + \underbrace{\frac{\psi_2''(y)}{\psi_2(y)}}_{-k_2^2} + \underbrace{\frac{\psi_3''(z)}{\psi_3(z)}}_{-k_3^2} \right] = E$$

$$-k_1^2 = \frac{\psi_1''(x)}{\psi_1(x)} \Rightarrow \psi_1''(x) + k_1^2 \psi_1(x) = 0$$

$$r = \pm i k_1$$

$$\psi_1(x) = A_1 e^{-i k_1 x} + A_2 e^{+i k_1 x}$$

$$\psi_1(0) = 0 \quad \psi_1(x) = A \sin(kx + \gamma)$$

$$\psi_1(a) = 0 \quad \psi_1(0) = A \sin \gamma = 0$$

$$\gamma = 0$$

$$\psi_1(a) = A \sin(k_1 a) = 0$$

me2 2014 $k_1 = \frac{n_1 \pi}{a}$

$$-\frac{\hbar^2}{2m} \left[\frac{\psi_1''(x) \psi_2(y) \psi_3(z) + \dots}{\psi_1(x) \psi_2(y) \psi_3(z)} \right] E$$

Ex 4

classique

$$E < V_0$$

v_1
à gauche

(II)

$$E_1 = \frac{1}{2} m v_1^2 + 0, \quad E_2 = \frac{1}{2} m v_2^2 + V_0$$

$$\frac{1}{2} m v_2^2 = (E - V_0) < 0$$

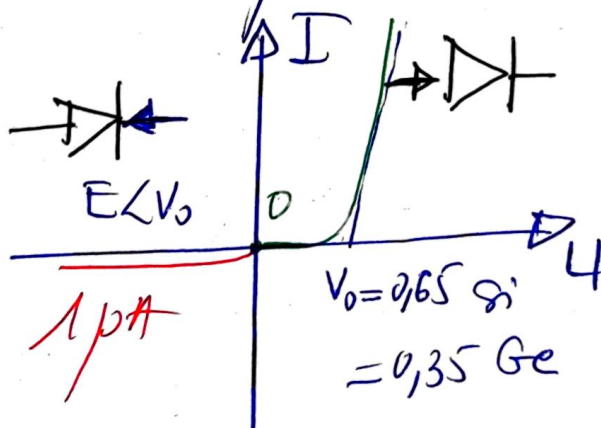
impossible de trouver les probas (II).

Rq



$$E < V_0$$

1 pA



$$V_0 = 0,65 \text{ si}$$

$$= 0,35 \text{ Ge}$$

$$\frac{1}{2} \frac{\nabla^2 \psi(x)}{\psi(x)} + \frac{2m}{\hbar^2} (E - V(x)) \psi(x) = 0$$

$$\psi_1''(n) + \left(\frac{2mE}{\hbar^2} \right) \psi_1(n) = 0$$

$$\psi_2''(n) + \underbrace{\frac{2m}{\hbar^2} (E - V_0)}_{-k_2^2} \psi_2(n) = 0$$

$$\psi_1'' + k_1^2 \psi_1 = 0$$

$$\psi_2'' - k_2^2 \psi_2 = 0$$

$$\psi_1 = A e^{ik_1 n} + B e^{-ik_1 n}$$

$$\psi_2 = C e^{-k_2 n} + D e^{+k_2 n}$$

$$\psi_2(n, t) = C \left(e^{-k_2 n} \right) e^{-i\omega t}$$

plasma wave
 $\omega < \omega_p$

onde evanescente

onde transmise:

$$\psi_1(n, t) = A e^{i(k_1 n - \omega t)}$$

$$+ B e^{-i(k_1 n + \omega t)}$$

OPPT (on) $\dagger$: incidente } $\nwarrow$ réfléchi

$$2) \psi_1(0^-) = \psi_2(0^+)$$

$$\psi_1'(0^-) = \psi_2'(0^+) ; V_0 \neq \infty$$

$$\begin{cases} A + B = C \end{cases}$$

$$\begin{cases} ik_1(A - B) = -k_2 C ; A - B = \frac{ik_2 C}{k_1} \end{cases}$$

$$r = \frac{B}{A} = \frac{k_1 - ik_2}{k_1 + ik_2} ; |r| = 1$$

$$t = \frac{C}{A} = \frac{2k_1}{k_1 + ik_2}$$

$$3) I_2(n) = |\psi_1(n, t)|^2$$

$$\psi_1(n, t) = A [e^{ik_1 n} + \underset{\substack{\swarrow \\ e^{-i\theta}}}{r} e^{-ik_1 n}] e^{-i\omega t}$$

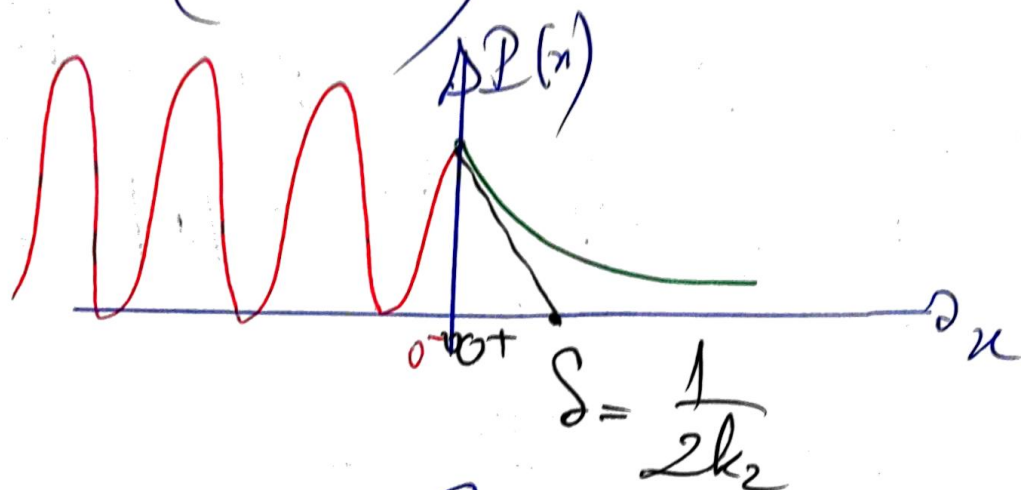
$$= A [e^{ik_1 n} + e^{-i(k_1 n + \theta)}] e^{-i\omega t}$$

$$\psi_1^* = A [e^{-ik_1 n} + e^{+i(k_1 n + \theta)}] e^{i\omega t}$$

$$I_1(n) = 2|1 + \cos(2k_1 n + \theta)|$$

$$P_2(x) = |Y_2(xt)|^2 = C^2 e^{-2k_2 x}$$

$$= \frac{4k_1^2}{(k_1^2 + k_2^2)} \exp(-2k_2 x)$$



$$\delta = \frac{1}{2} \frac{\hbar}{\sqrt{2m(V_0 - E)}}$$

$$V_0 \rightarrow \infty, \quad \delta \rightarrow 0$$

$$4) \quad \vec{J}_i = |Y_i|^2 \vec{v}_{g_i} = |A|^2 \frac{\hbar}{m} k_1 \vec{u}_x$$

$$\vec{J}_r = |Y_r|^2 \vec{v}_{g_r} = |B|^2 \frac{\hbar}{m} k_1 (-\vec{u}_x)$$

$$\vec{J}_t = |Y_t|^2 \vec{v}_{g_t} = |C|^2 \frac{\hbar}{m} \text{Re}(ik_2) \vec{u}_x = 0$$

$$\psi_2 = C e^{-k_2 x} e^{-i\omega t}$$

$$= C e^{-i(\omega t - (ik_2)x)}$$

$$V_g = \frac{d\omega}{d \text{Re}(\underline{k})} = 0$$

$$R = \frac{\|\vec{J}_r\|}{\|\vec{J}_i\|} = \frac{|B|^2}{|A|^2} = |r|^2 = 1$$

$$T = \frac{\|\vec{J}_t\|}{\|\vec{J}_i\|} = \frac{|C|^2}{|A|^2} \frac{v_{g2}}{v_{g1}} = 0$$

6) $0 < V_0 < E$

$$\psi_2''(x) + \underbrace{\frac{2m}{\hbar^2}(E - V_0)}_{q_2^2} \psi_2(x) = 0$$

$$\psi_2(x) = C e^{i(q_2 x)} + D e^{-i q_2 x}$$

$$\psi_2(x, t) = C e^{i(q_2 x - \omega t)} + D e^{-i(q_2 x + \omega t)}$$

(Free barrier ∞) $\longleftrightarrow D \Rightarrow$

$$ik_2 \rightarrow q$$

$$\underline{E < V_0}$$

$$\psi_2 = C e^{-i(\omega t - ik_2 x)}$$

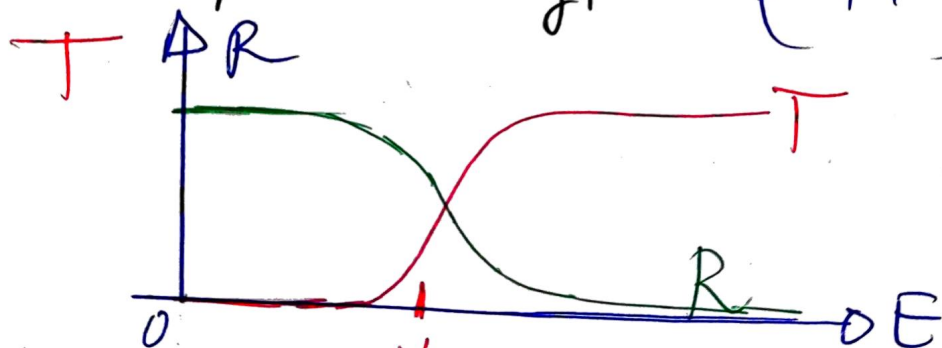
$$E > V_0$$

$$\psi_2 = C e^{i(qx - \omega t)}$$

$$\psi_2 = C e^{i(ik_2 x - \omega t)}$$

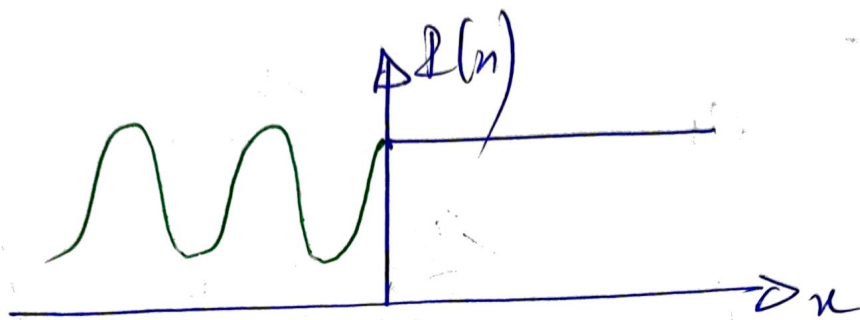
$$R = \left(\frac{k_1 - q}{k_1 + q} \right)^2 = |r|^2$$

$$T = |t|^2 \frac{v_{g2}}{v_{g1}} = \frac{4k_1 q}{(k_1 + q)^2}$$



$$b) \psi_1 = A [e^{ik_1 x} + \sqrt{R} e^{-ik_1 x}] e^{-i\omega t}$$

$$I_1 = 1 + R + 2\sqrt{R} \cos(2k_1 x)$$



$$\psi_2(n, t) = C e^{i(qn - \omega t)}$$

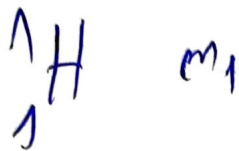
$$|\psi_2(n, t)|^2 = C^2$$

$$\Rightarrow R = \left(\frac{1 - \frac{q}{k_1}}{1 + \frac{q}{k_1}} \right)^2 ; V_0 \ll E$$

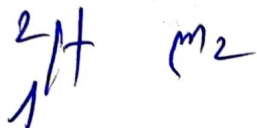
$$\frac{q}{k_1} = \sqrt{\frac{E - V_0}{E}} \approx 1 - \frac{V_0}{2E}$$

$$R = \left[\frac{1 - \left(1 - \frac{V_0}{2E}\right)}{1 + \left(1 - \frac{V_0}{2E}\right)} \right]^2$$

$$R = \frac{V_0^2}{16 E^2}$$



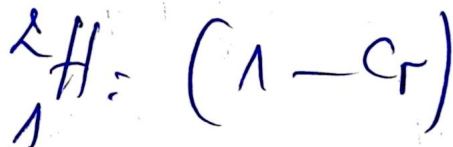
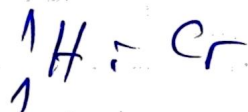
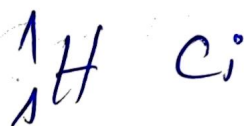
$$\underline{E \gg V_0}$$



$$\frac{R_1}{R_2} = \frac{E_2^2}{E_1^2} = \left(\frac{m_2}{m_1} \right)^2$$

$$m_1 < m_2 \Rightarrow R_1 > R_2$$

$$\alpha = \frac{m_1}{m_2} \Rightarrow \frac{R_1}{R_2} = \frac{1}{\alpha^2}$$



$$R_1 = \frac{c_r}{c_i}, \quad R_2 = \frac{1 - c_r}{1 - c_i}$$

$$\frac{R_1}{R_2} = \frac{c_r}{c_i} \cdot \frac{1 - c_i}{1 - c_r} = \frac{1}{\alpha^2}$$

$$\underline{c_r = 0,34}$$

$$\text{Dem } \vec{J} = \frac{i\hbar}{2m} \left[\psi \frac{d\psi^*}{dx} - \psi^* \frac{d\psi}{dx} \right] \hat{x}$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\psi^* \frac{\partial \psi}{\partial t} \stackrel{(1)}{=} -\frac{\hbar}{2mi} \psi^* \frac{\partial^2 \psi}{\partial x^2} + \frac{V\psi}{i\hbar} \psi^*$$

$$\psi \frac{\partial \psi^*}{\partial t} \stackrel{(2)}{=} +\frac{\hbar}{2mi} \psi \frac{\partial^2 \psi^*}{\partial x^2} - \frac{V}{i\hbar} \psi^* \psi$$

$$\begin{aligned} (1) + (2) &\Rightarrow \psi^* \frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi^*}{\partial t} \\ &= \frac{\partial}{\partial t} [\psi \psi^*] = \frac{\partial \rho}{\partial t} \end{aligned}$$

$$\begin{aligned} &= -\frac{\hbar}{2mi} \left[\psi^* \frac{\partial^2 \psi}{\partial x^2} - \psi \frac{\partial^2 \psi^*}{\partial x^2} \right] \\ &= -\frac{\hbar}{2mi} \frac{\partial}{\partial x} \left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x} \right) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial t} = -\frac{\hbar}{2mi} \frac{\partial}{\partial n} \left(4^* \frac{\partial 4}{\partial n} - 4 \frac{\partial 4^*}{\partial n} \right)$$

$$= -\frac{\partial}{\partial n} \left(\frac{-i\hbar}{2m} \left[4^* \frac{\partial 4}{\partial n} - 4 \frac{\partial 4^*}{\partial n} \right] \right)$$

$$\frac{\partial \mathcal{L}}{\partial t} = -\frac{\partial}{\partial n} \overline{J}(n, t)$$

$$\boxed{\frac{\partial \overline{J}}{\partial n} + \frac{\partial \mathcal{L}}{\partial t} = 0} \quad \text{d'ici}$$

$$\frac{\partial \overline{J}_e}{\partial n} + \frac{\partial \mathcal{L}_e}{\partial t} = 0$$

$$\text{div} \overline{J}_e + \frac{\partial \mathcal{L}_e}{\partial t} = 0, \quad \text{div}(\text{rot} B) = 0$$

$$\oint \overline{J}_e dS = \iiint \text{div} \overline{J}_e d\tau = 0$$

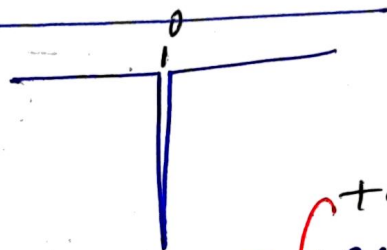
la loi des noeuds



Pg. $\boxed{\frac{\partial \mathcal{L}}{\partial \psi} + \frac{\partial \mathcal{J}}{\partial u} = 0}$

S. $\mathcal{J}(n, t) = \mathcal{J}(n) \Rightarrow \frac{\partial \mathcal{J}}{\partial n} = 0.$

$V(n) = -\alpha \delta(n)$



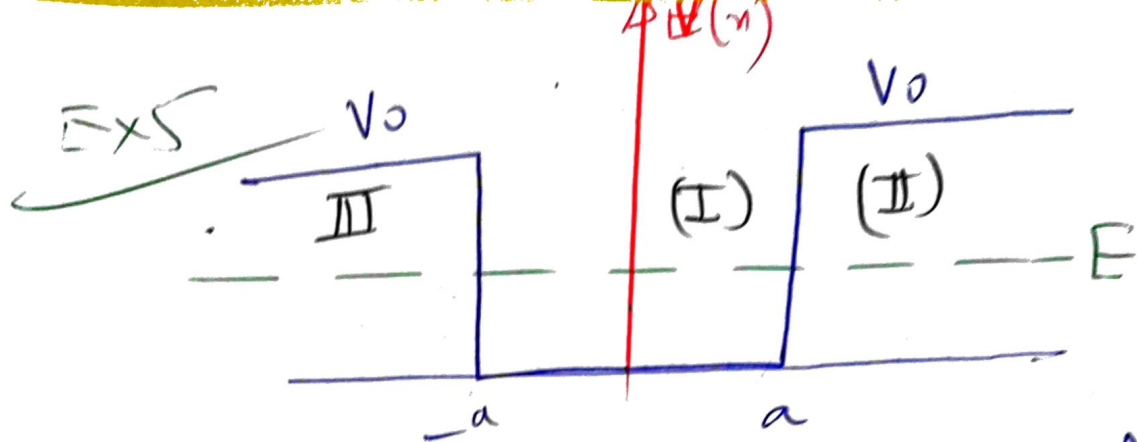
$$-\frac{\hbar^2}{2m} \int_{-\varepsilon}^{+\varepsilon} \frac{\partial^2 \psi(n)}{\partial n^2} dn + (-\alpha \delta(n)) \psi(n) = E \int_{-\varepsilon}^{+\varepsilon} \psi(n) dn$$

$$-\frac{\hbar^2}{2m} [\psi'(0^+) - \psi'(0^-)] = \alpha \psi(0)$$

$$\int_{-\varepsilon}^{+\varepsilon} \psi(n) dn = \psi(0) \times 2\varepsilon \rightarrow 0.$$

$$\int_{-\infty}^{+\infty} \delta(n-a) f(n) dn = f(a)$$

$$\int_{-\varepsilon}^{+\varepsilon} \delta(n) \psi(n) dn = \psi(0)$$



$$k_1^2 = \frac{2mE}{\hbar^2}; \quad -\frac{2m}{\hbar^2}(E - V_0) = k_2^2 = k_3^2$$

$$\psi''(x) + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0.$$

$$\psi_1''(x) + k_1^2 \psi_1(x) = 0$$

$$\psi_2''(x) - k_2^2 \psi_2(x) = 0$$

$$\psi_3''(x) - k_3^2 \psi_3(x) = 0$$

$$\psi_1(x) = A e^{ik_1 x} + B e^{-ik_1 x}$$

$$\psi_2(x) = C e^{-k_2 x} + D e^{+k_2 x}$$

diverge: D=0

$$\psi_3(x) = C' e^{+k_2 x} + D' e^{-k_2 x}$$

diverge: x → ∞

2) a)

$$\frac{-\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x) \psi(x) = E \psi(x)$$

$$\frac{-\hbar^2}{2m} \frac{d^2 \psi(-x)}{dx^2} + V(-x) \psi(-x) = E \psi(-x)$$

$\psi(x)$ et $\psi(-x)$ sont solutions de la même équation ⇒

$$\begin{aligned} \psi(x) &= \varepsilon \psi(-x) \\ \psi(-x) &= \varepsilon \psi(x) \end{aligned} \Rightarrow \varepsilon^2 = 1$$

$\varepsilon = \pm 1$

b) $\psi_1(x) = + \psi_2(-x)$

$$A e^{ik_1 x} + B e^{-ik_1 x} = A e^{-ik_1 x} + B e^{ik_1 x}$$

$$A (\sin(k_1 x)) = B (\sin(k_1 x))$$

$$A = B$$

$$\psi_{1s}(x) = 2A \cos(k_1 x)$$

$$\varphi_{2s}(n) = \varphi_3(-n)$$

$$C e^{-k_2 x} = C' e^{-k_2 x}$$

$$\underline{\underline{C = C'}}$$

paire

impaires

$$\varphi_{2s}(n) = 2A \cos(k_1 n)$$

$$\varphi_{2s}(n) = C e^{-k_2 x}$$

$$\varphi_{3s}(n) = C e^{+k_2 x}$$

$$\varphi_{1A}(n) = 2A i \sin(k_1 n)$$

$$\varphi_{2A}(n) = C e^{-k_2 x}$$

$$\varphi_{3A}(n) = -C e^{k_2 x}$$

$$3) \varphi_{1s}(a^-) = \varphi_2(a^+)$$

$$\varphi'_{1s}(a^-) = -\varphi'_2(a^+)$$

$$\Rightarrow \begin{cases} 2A \cos(k_1 a) = C e^{-k_2 a} \\ 2A (-k_1) \sin(a k_1) = -k_2 C e^{-k_2 a} \end{cases}$$

$$k_1 \tan(a k_1) = k_2$$

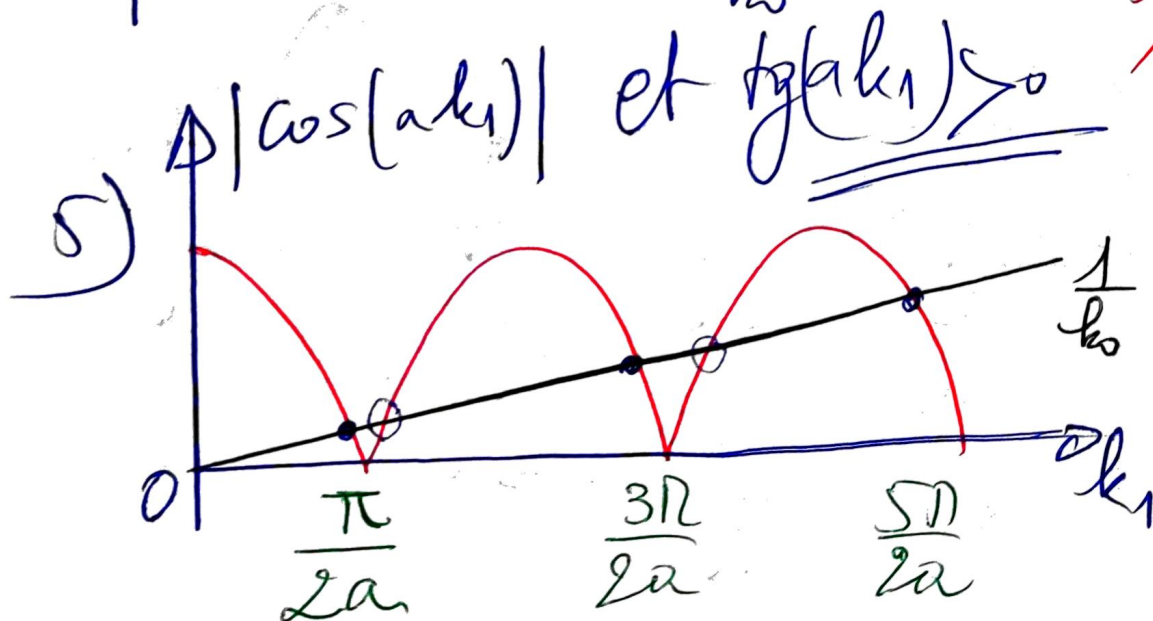
$$\sqrt{\hbar^2 k^2} = \sqrt{E}$$

$$\cos(ak_1) = \frac{1}{1 + \tan^2(ak_1)} = \frac{1}{1 + \frac{k_2^2}{k_1^2}}$$

$$\cos^2(ak_1) = \frac{k_1^2}{k_1^2 + k_2^2}$$

or $k_1^2 + k_2^2 = \left(\frac{2mV_0}{\hbar^2} \right) = k_0^2$

$$|\cos(ak_1)| = \frac{k_1}{k_0} \quad \text{droite de pente } \frac{1}{k_0}$$



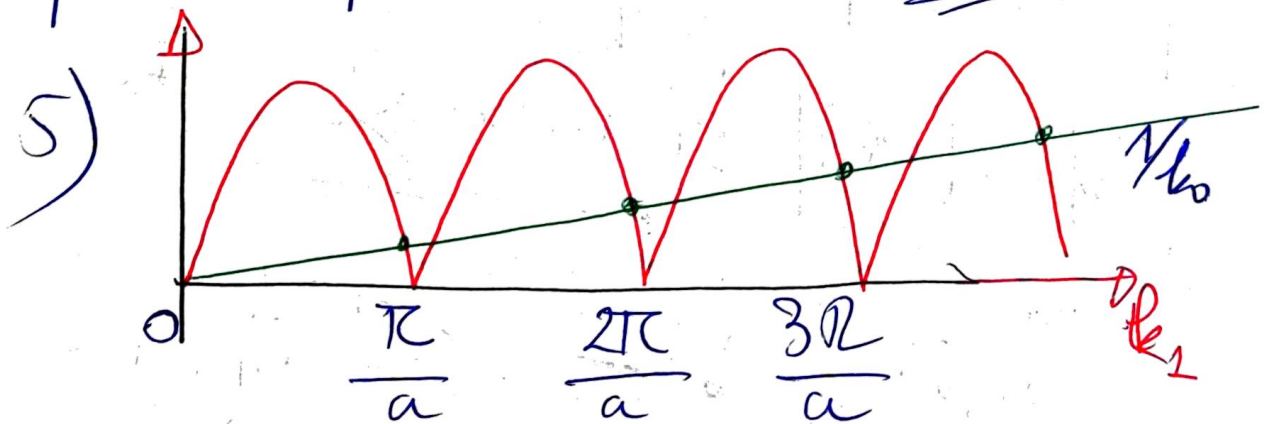
$$2A i \sin(ak_1) = C e^{-k_2 a}$$

$$(2A i) k_1 \cos(ak_1) = -k_2 C e^{-k_2 a}$$

$$\tan(ak_1) = -\frac{k_1}{k_2}$$

$$\sin^2(ak_1) = \frac{1}{1 + \cot^2(ak_1)} = \frac{k_1^2}{k_0^2}$$

$$|\sin(ak_1)| = \frac{k_1}{k_0} \text{ et } \underline{\underline{\tan(ak_1) < 0}}$$



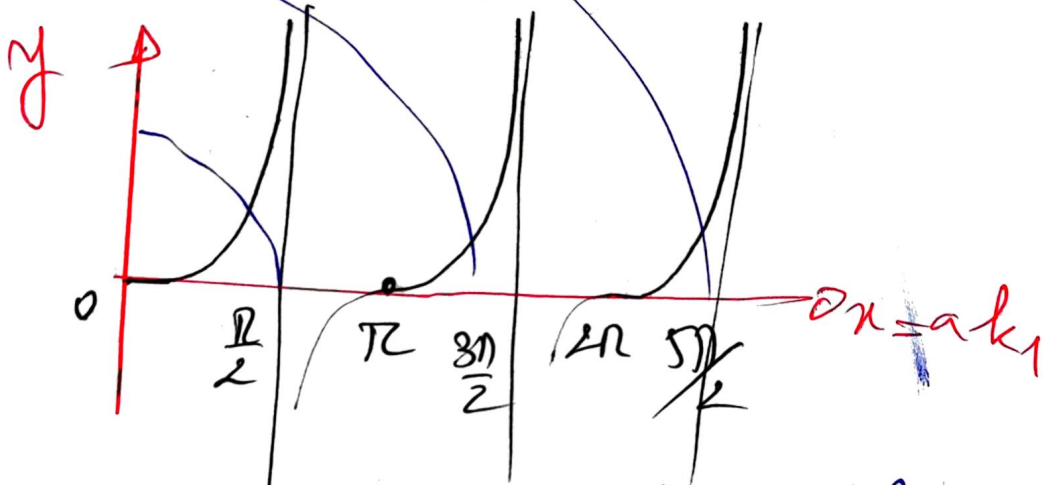
6) $V_0 \rightarrow \infty, \quad \frac{1}{k_0} \rightarrow 0$

$$k_1 \approx \frac{\pi}{2a}, \frac{2\pi}{2a}, \frac{3\pi}{2a}, \frac{2\pi \times 2}{2a}, \dots$$

$$k_1 \approx -\frac{N\pi}{2a}$$

$$x \text{ to } y = y; \quad f(a k_1) = \frac{k_2 a}{k_1 a}$$

$$x = (a k_1), \quad y = (a k_2)$$



$$x^2 + y^2 = a^2(k_1^2 + k_2^2) = a^2 b^2 = R^2$$