

Statistique

Soit un système en ~~contact~~ avec un thermostat
Soit N_i le nb de points à l'énergie
d'énergie ε_i dégénéré $g_i = \text{fois}$

$$\frac{N_i}{N} = A g_i e^{-\beta \varepsilon_i} \quad \beta = \frac{1}{k_B T}$$

$$A \sum_i g_i e^{-\beta \varepsilon_i} = 1 \Rightarrow A = \frac{1}{\left(\sum_i g_i e^{-\beta \varepsilon_i} \right)}$$

$$A = \frac{1}{Z}, \quad Z = \sum_i g_i e^{-\beta \varepsilon_i}$$

$$P_i = \frac{1}{Z} g_i e^{-\beta \varepsilon_i}$$

$$\langle E \rangle = \sum \varepsilon_i P_i = \frac{\sum \varepsilon_i e^{-\beta \varepsilon_i} g_i}{\sum e^{-\beta \varepsilon_i} g_i}$$

$$= - \frac{\partial \ln \sum (g_i e^{-\beta \varepsilon_i})}{\partial \beta} = - \frac{\partial \ln Z}{\partial \beta}$$

$$U = N \langle E \rangle = -N \frac{\partial \ln z}{\partial \beta}$$

$$Z = z^N$$

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \left(\frac{\partial U}{\partial \beta} \right)_V \left(\frac{\partial \beta}{\partial T} \right)_V$$

$$\frac{\partial}{\partial T} \beta = \frac{1}{k_B T^2} \Rightarrow \left(\frac{\partial \beta}{\partial T} \right)_V = -\frac{1}{k_B T^2}$$

$$\left(\frac{\partial \beta}{\partial T} \right)_V = -\frac{1}{k_B T^2} = -k_B \beta^2$$

$$C_V = - (k_B \beta^2) \left(\frac{\partial U}{\partial \beta} \right)_V$$

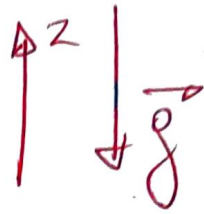
$$C_V > 0$$

$$C_P > C_V$$

Ex 1

$$1) \boxed{\overline{g_{mol}} P = S \vec{g}}$$

~~Hydrostatique~~



$$\frac{\partial P}{\partial x} \vec{u}_x + \frac{\partial P}{\partial y} \vec{u}_y + \frac{\partial P}{\partial z} \vec{u}_z = -S \vec{g} \vec{u}_z$$

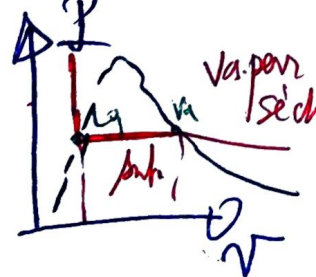
$$P(x, y, z) = P(z) \Rightarrow \frac{dP(z)}{dz} = -Sg$$

$$PV = \frac{m}{M} RT \Rightarrow P = e \frac{RT}{M}$$

Eq: $\frac{R}{M} = \gamma = \text{ds de gaz parfait}$
généralité.

$$\frac{dP(z)}{dz} = \frac{-MgP(z)}{RT(z=0)}$$

Eq: GP: $P_{\text{norm}} < 10 \text{ bars}$
 $T \approx \text{ambiante}$



$$\frac{dP(z)}{P(z)} = - \frac{Mg}{RT_0} dz = - \frac{dz}{H}$$

$$P(z) = P_0 \exp\left(-\frac{Mgz}{RT_0}\right)$$

$$b) n(z) = \frac{N}{V} \quad RT / R = \sqrt{A} k_B$$

$$PV = \frac{m^* \cdot N}{m^* \sqrt{A}} RT$$

$$P = \left(\frac{N}{V}\right) \left(\frac{R}{\sqrt{A}}\right) T = n k_B T$$

$$n(z) = \frac{P(z)}{k_B T} = n_0 \exp\left(-\frac{m^* g z}{k_B T}\right)$$

$$\exp\left[-\frac{E_{pp}(z)}{k_B T}\right], \quad \frac{1}{k_B T} = \beta$$

facteur de Boltzmann

$$d) g(z) = \frac{GM_T}{(R_T + z)^2}$$

$$\underline{R_1:} \quad \vec{F}_G = - \frac{GM_T m}{(R_T + z)^2} \vec{u}_r = m \vec{g}$$

$$g(z) = \frac{GM_T}{R_T^2} \cdot \frac{1}{\left(1 + \frac{z}{R_T}\right)^2}$$

$$g(z) \approx \left(\frac{GM_T}{R_T^2} \right) \left(1 - \frac{2z}{R_T} \right)^2$$

$$\frac{\int_0^z g(z) dz}{\int_0^z 1 dz} = - \frac{M g_0}{R T_0} \int_0^z \left(1 - \frac{2z}{R_T} \right) dz$$

MB2 2013

2) $Q=0$, adiabatique

Reversible
+ GP

Isentropique: $S = \text{cte}$
 $2V^\gamma = \text{cte}$

$$\underline{x} \quad V = \frac{nRT}{P}$$

a)

$$P V^\gamma = \text{cte} \Rightarrow P \cdot \frac{T^\gamma}{P^\gamma} = \text{cte}$$

$$\boxed{P^{1-\gamma} T^\gamma = \text{cte}}$$

$$b) (1-\gamma) \ln P + \gamma \ln T = \text{cte}$$

$$(1-\gamma) \frac{dP}{P} + \gamma \frac{dT}{T} = 0$$

$$\frac{dT}{T} = \left(\frac{\gamma-1}{\gamma} \right) \frac{dP}{P}$$

$$c) \frac{dT(z)}{T(z)} = \frac{(\gamma-1)}{\gamma} \frac{(-\rho g dz)}{P(z)}$$

$$\frac{dT(z)}{T(z)} = - \frac{\rho(z) g (\gamma-1)}{P(z) \gamma} dz$$

$$\frac{dT(z)}{T(z)} = \frac{-\gamma}{P(z)} \frac{1}{R T(z)} \left(\frac{\gamma-1}{\gamma} \right) dz$$

$$\frac{dT(z)}{T_0} = \frac{-\gamma \Gamma}{R T_0} \left(\frac{\gamma-1}{\gamma} \right) dz$$

$$\frac{dT(z)}{T_0} = -\alpha dz$$

$\hookrightarrow m^{-1}$

$$T(z) = T_0 (1 - \alpha z) \geq 0$$

$$P^{1-\gamma}(z) T(z) = P_0^{1-\gamma} T_0$$

$$P(z) = P_0 \left(\frac{T_0}{T(z)} \right)^{\frac{\gamma}{1-\gamma}}$$

$$P(z) = P_0 (1 - \alpha z)^{\frac{\gamma}{\gamma-1}}$$

$$S(z) = -$$

$$(1 - \alpha z) \geq 0, \quad z \leq \frac{1}{\alpha} = h_1$$

-7-

polytropic

$$P V^k = \text{const}$$

$$T(z) = T_0 - \lambda z = T_0 + \left(\frac{dT(z)}{dz} \right) z$$

$$\lambda = L_0 \text{ m}^{-1} \cdot \text{K}$$

// grad T //

$$a) \frac{dP}{dz} = - \frac{Mg}{RT(z)} dz$$

$$\frac{dP(z)}{P(z)} = \frac{+Mg}{\lambda R} \frac{d(T_0 - \lambda z)}{(T_0 - \lambda z)}$$

$$\ln \frac{P(z)}{P_0} = \frac{Mg}{\lambda R} \ln \frac{(T_0 - \lambda z)}{T_0}$$

$$\rho(z) = \frac{Mg P(z)}{R T(z)} = \dots$$

$$\underline{Mg} \quad \underline{2018}$$

$$g_i = 2, \quad \varepsilon = \varepsilon_2 - \varepsilon_1$$

$$\frac{N_2}{\varepsilon_2}$$

$$1) P_1 = \frac{N_1}{N} = \frac{1}{2} e^{-\beta \varepsilon_1}$$

$$\frac{N_1}{\varepsilon_1}$$

$$Z = e^{-\beta \varepsilon_1} + e^{-\beta \varepsilon_2}$$

$$P_1 = \frac{e^{-\beta \varepsilon_1}}{e^{-\beta \varepsilon_1} + e^{-\beta \varepsilon_2}} = \frac{1}{1 + e^{-\beta \varepsilon}}$$

$$P_2 = \frac{e^{-\beta \varepsilon_2}}{e^{-\beta \varepsilon_1} + e^{-\beta \varepsilon_2}} = \frac{e^{-\beta \varepsilon}}{1 + e^{-\beta \varepsilon}}$$

$$2) \frac{N_1}{N} = \frac{1}{1 + \exp\left(-\frac{\varepsilon}{k_B T}\right)} = \frac{1}{1 + e^{\theta/T}}$$

$$\theta = \frac{\varepsilon}{k_B}$$

$$\frac{N_2}{N} = \frac{e^{-\theta/T}}{1 + e^{-\theta/T}}$$

$$T \gg \theta = \frac{\varepsilon}{k_B} \Rightarrow k_B T \gg \varepsilon$$

$$e^{-\theta/T} \approx 1$$

$$\underline{T \gg \theta} ; N_1 \approx N_2 = \frac{N}{2}$$

$$\underline{T \ll \theta} \quad \underline{k_B T \ll \varepsilon}$$

$$N_1 \approx N \text{ et } N_2 \approx 0$$

de condensat^o des particules

$$3) a) U = N_1 \varepsilon_1 + N_2 \varepsilon_2 = N \langle E \rangle$$

$$= \frac{N}{1 + e^{-\beta \varepsilon}} \left(\underset{\substack{\downarrow \\ (\varepsilon_2 - \varepsilon)}}{\varepsilon_1 + \varepsilon_2 e^{-\beta \varepsilon}} \right) ; \varepsilon = \varepsilon_2 - \varepsilon_1$$

$$U = N \left(\frac{\varepsilon_2 + \varepsilon_2 e^{-\beta \varepsilon} - \varepsilon}{1 + e^{-\beta \varepsilon}} \right)$$

$$U = N \left(\varepsilon_2 - \frac{\varepsilon}{1 + e^{-\beta \varepsilon}} \right)$$

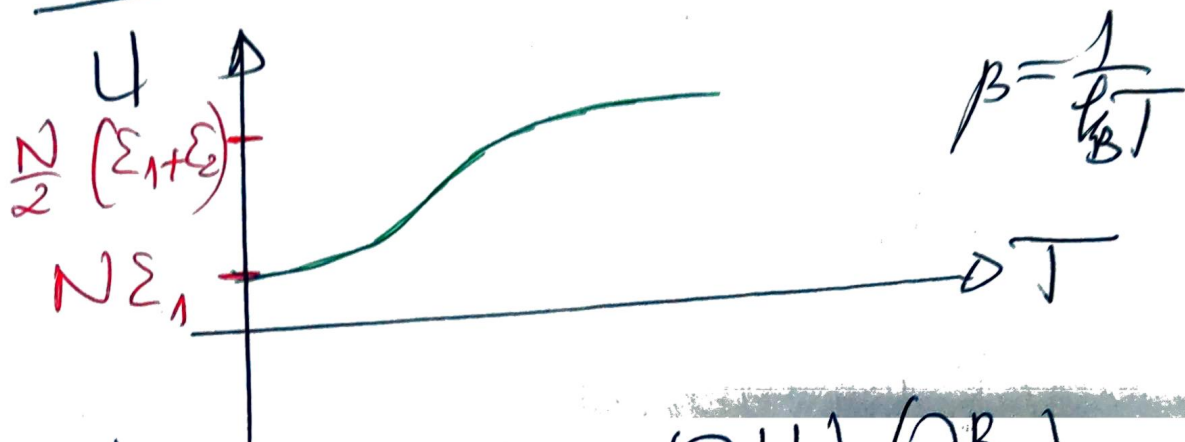
$$b) U = - \frac{\partial \ln Z}{\partial \beta} , \quad \langle E \rangle = - \frac{\partial \ln Z}{\partial \beta}$$

$\frac{\partial}{\partial \beta} Z = Z^N$

$$\underline{N_1 \approx N} \quad T \ll \theta$$

$$U \approx N \epsilon_1$$

$$\underline{T \gg \theta} ; \quad U \approx \frac{N}{2} (\epsilon_1 + \epsilon_2)$$



$$7) C_V = \left(\frac{\partial U}{\partial T} \right)_V = \left(\frac{\partial U}{\partial \beta} \right)_V \left(\frac{\partial \beta}{\partial T} \right)_V$$

$$C_V = - k_B \beta^2 \left(\frac{\partial U}{\partial \beta} \right)_V$$

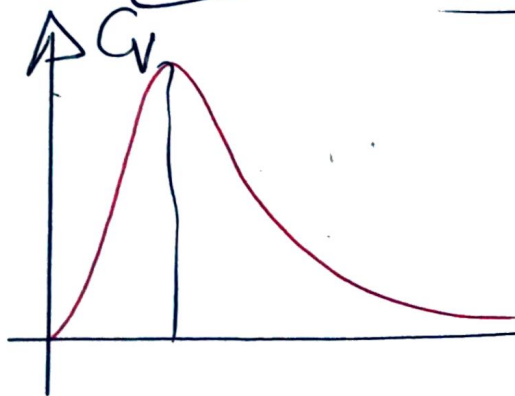
$$C_V = - k_B \beta^2 N \left(- \frac{\epsilon^2 e^{-\beta \epsilon}}{(1 + e^{-\beta \epsilon})^2} \right)$$

$$= N k_B \beta^2 \epsilon^2 \frac{e^{-\beta \epsilon}}{(1 + e^{-\beta \epsilon})^2}$$

$$C_V = \frac{N k_B (\beta \epsilon)^2}{\left(e^{\beta \epsilon / 2} + e^{-\beta \epsilon / 2} \right)^2}$$

$$C_V = N k_B \frac{\left(\frac{\beta \epsilon}{2} \right)^2}{\left[\cosh \left(\frac{\beta \epsilon}{2} \right) \right]^2}; \quad \frac{\beta \epsilon}{2} = \frac{\epsilon}{2 k_B T}$$

$$C_V = \frac{N k_B \left(\frac{\theta}{2T} \right)^2}{\left[\cosh \left(\frac{\theta}{2T} \right) \right]^2}$$

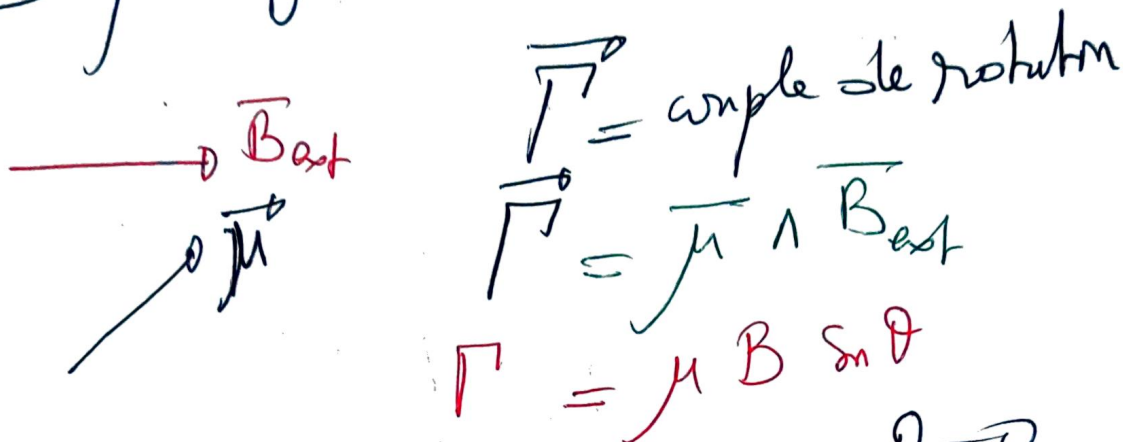


$$\lim_{x \gg 1} \cosh x \approx \frac{e^x}{2}$$

$$\underline{T \ll \theta}; \quad C_V \approx (N k_B) \left(\frac{\theta}{2T} \right)^2 \frac{1}{\left(\frac{1}{2} \right)^2 e^{\theta/T}}$$

$$\approx N k_B \left(\frac{\theta}{T} \right)^2 \exp \left(-\frac{\theta}{T} \right)$$

B) paramagnétisme



position d'équilibre $\Gamma = 0$ $\theta = 0$
 $\theta = \pi$

W dipôle = $-\vec{\mu} \cdot \vec{B}_{\text{ext}} = -\mu B_0 \cos \theta$

$W(\theta = 0) = -\mu B_0$ ← stable

$W(\theta = \pi) = +\mu B_0$ ← instable



1) $\frac{N_1}{N} = \frac{e^{-\beta \epsilon_1}}{e^{-\beta \epsilon_1} + e^{-\beta \epsilon_2}}$

$\frac{N_1}{N} = \frac{e^{\beta \epsilon}}{2 \cosh(\beta \epsilon)}$; $\epsilon = \mu B_0$

$\frac{N_2}{N} = \frac{e^{-\beta \epsilon}}{2 \cosh(\beta \epsilon)}$ - 13 -

$$x = \beta \varepsilon = \frac{\varepsilon}{k_B T} = \frac{\vartheta}{T}$$

$$N_1 = \frac{N e^x}{2 \cosh(x)}$$

$$N_2 = \frac{N e^{-x}}{2 \cosh(x)}$$

$$\bullet x \rightarrow 0;$$

$$N_1 \approx N_2 = \frac{N}{2}$$

$$\vartheta \ll T$$

$$\bullet x \rightarrow \infty; \vartheta \gg T_{BT}$$

$$N_1 \approx \frac{N e^x}{2 e^x} = N$$

$$N_2 = \frac{N 2 e^{-x}}{2 e^x} = N e^{-2x}$$

$$3) \langle E \rangle = \sum \varepsilon_i P_i$$

$$= \varepsilon_1 P_1 + \varepsilon_2 P_2$$

$$= -\varepsilon P_1 + \varepsilon P_2$$

$$= -\varepsilon (P_1 - P_2)$$

$$= -\varepsilon \frac{(e^x - e^{-x})}{2 \cosh(x)} = -\varepsilon \tanh(x)$$

$$U = N \langle E \rangle = -N \varepsilon \tanh\left(\frac{\mu B_0}{k_B T}\right)$$

$$* \text{Var}(\bar{E}) = \langle E^2 \rangle - \langle E \rangle^2$$

$$\langle E^2 \rangle = (P_1 + P_2) \varepsilon^2 = \varepsilon^2$$

$$\text{Var}(E) = \varepsilon^2 - \varepsilon^2 \mathcal{H}^2(n)$$

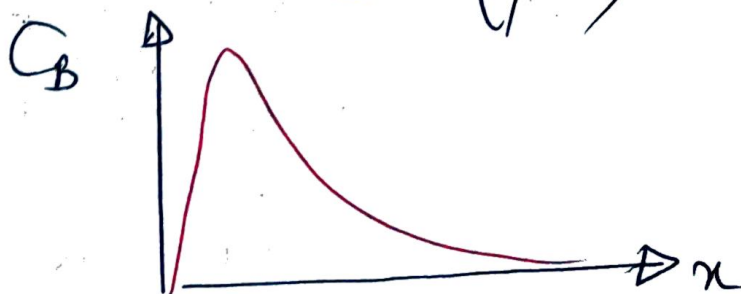
$$= \varepsilon^2 (1 - \mathcal{H}^2(n))$$

$$\text{Var}(E) = \frac{\varepsilon^2}{\mathcal{H}^2(n)}$$

$$4) C_B = \left(\frac{\partial U}{\partial T} \right)_B = -k_B \beta^2 \left(\frac{\partial U}{\partial \beta} \right)_B$$

$$= -k_B \beta^2 \frac{\partial}{\partial \beta} (-N \varepsilon \mathcal{H}(\beta \varepsilon))$$

$$= \frac{+ N k_B \beta^2 \varepsilon^2}{\mathcal{H}^2(\beta \varepsilon)} = \frac{N k_B n^2}{\mathcal{H}^2(n)}$$



$$5) \sigma(E_{\text{sys}}) = \sqrt{\text{var}(E_{\text{sys}})} = T \sqrt{k_B C_v}$$

$$(\Delta E_{\text{sys}})^2 = N (\Delta E_{\text{par}})^2$$

$$6) \vec{M} = N \langle \vec{\mu} \rangle$$

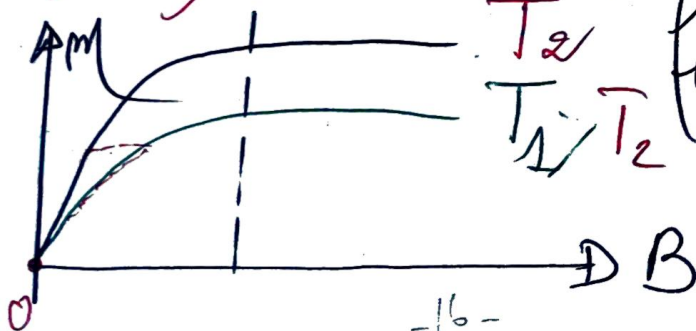
$$\langle \vec{\mu} \rangle = \sum_i \vec{\mu}_i P_i$$

$$\langle \vec{\mu} \rangle = (\mu_1 P_1 - \mu_2 P_2) \vec{u}_z$$

$$= \mu_2 \left(\frac{e^{\chi} - e^{-\chi}}{2 \cosh \chi} \right) \vec{u}_z$$

$$\langle \vec{\mu} \rangle = \mu_2 \tanh(\chi) \vec{u}_z$$

$$7) m(B) = \mu_2 \tanh\left(\frac{\mu_2 B}{k_B T}\right)$$



$$\left(\frac{\mu B}{k_B T_1} \right) < \left(\frac{B \mu_2}{T_2 k_B} \right)$$

$$T_1 > T_2$$

$$8) m(n \ll 1) \simeq \frac{\gamma B_0}{T}$$

$$m \simeq \frac{\mu_B^2 B_0}{k_B T}$$

$$9) S = k_B [\ln z + \beta \langle E \rangle]$$

$$a) S = k_B \left[\ln(2 \cosh(\beta \varepsilon)) + \beta (-\varepsilon \tanh(\beta \varepsilon)) \right]$$

$$P = k_B \left[\ln[2(\cosh(x))] - x \tanh(x) \right]$$

$$x = \beta \varepsilon$$

$$b) P = cts \Rightarrow \beta \varepsilon = \frac{\mu_B B}{k_B T} = cts$$

$$\boxed{\frac{B}{T} = cts}$$

$$c) P(n \rightarrow 0) = k_B [\ln 2 - n^2] \simeq k_B \ln(2)$$

$$P(n \rightarrow \infty) \simeq k_B \left[\ln\left(\frac{2 e^x}{2}\right) - x \times 1 \right]$$

$$\frac{e}{k_B T} \gg 1 \simeq k_B [x - x] \simeq 0 J/K$$

Principe Nernst

c) Refrigerant

$$B_i = 20 \text{ mT}$$

$$T_i = 350 \text{ mK}$$

$$B_f = 2 \text{ mT}$$

$$T_f = ??$$

$$A(n \ll 1) \simeq k_B \left[\ln 2 - n^2 \right]$$

$$A(T, B) = k_B \left[\ln 2 - \left(\frac{\mu_B}{k_B T} \right)^2 \right]$$

$$dA = \left(\frac{\partial A}{\partial T} \right)_B dT + \left(\frac{\partial A}{\partial B} \right)_T dB$$

$$\left(\frac{\partial A}{\partial T} \right)_B = k_B \left[+ \frac{\mu^2 B^2}{k_B^2} \frac{2}{T^3} \right]$$

$$\left(\frac{\partial A}{\partial B} \right)_T = k_B \left[\frac{-\mu^2 2B}{k_B^2 T^2} \right]$$

$$dA = \frac{2\mu^2 k_B}{k_B^2} \left(\frac{B^2}{T^3} dT - \frac{2B}{T^2} dB \right)$$

$$p = \text{cts: } \frac{B^2 dT}{T^3} = \frac{B}{T^2} dB$$

$$\frac{\Delta T}{T} = \frac{\Delta B}{B}$$

$$\frac{T_f}{T_i} = \frac{B_f}{B_i} \Rightarrow$$

$$T_f = \frac{B_f T_i}{B_i}$$

D) $\epsilon_d = -\vec{\mu} \cdot \vec{B}$

$$\epsilon_d = -\mu B_0 \cos \theta$$

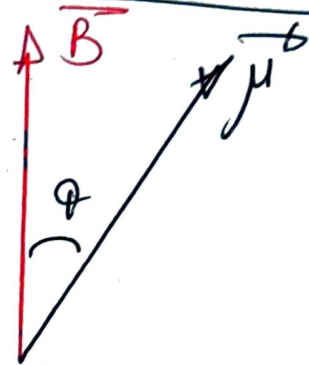
$$Z = \int \int e^{-\beta \epsilon} \sin \theta d\theta d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^\pi e^{+(\beta \mu B_0) \cos \theta} \sin \theta d\theta$$

$$= 2\pi \int_{-1}^1 e^{a \cos \theta} d(\underbrace{\cos \theta}_x)$$

$$= 2\pi \int_{-1}^1 e^{ax} dx$$

$$= 2\pi \frac{2\sinh(a)}{a}$$



$$Z = 4\pi \frac{Sh a}{a}$$

$$2) \frac{dN}{N} = 2\pi \frac{e^{-\beta \epsilon}}{Z} \sin \theta d\theta$$

$$M = \int \mu_z dN, \quad \mu_z = \mu \cos \theta$$

$$= N \int_0^\pi (\mu \cos \theta) 2\pi e^{a \cos \theta} \sin \theta d\theta$$

$$= N\mu \int_{-1}^1 x e^{ax} dx$$

$$\int_{-1}^1 e^{ax} dx$$

$$= N\mu \frac{d}{da} \ln \left(\int_{-1}^1 e^{ax} dx \right)$$

$$= N\mu \frac{d}{da} \ln \left(\frac{2Sh a}{a} \right)$$

$$= N\mu \left[\coth a - \frac{1}{a} \right] = N\mu L(a)$$

-22- Langevin

$$a = \frac{\mu B}{k_B T} \ll 1 ; \mu B \ll k_B T$$

$$\lim_{a \rightarrow \infty} \coth a \simeq \frac{1}{a} + \frac{a}{3}$$

$$M \simeq \frac{N \mu a}{3} = \left(\frac{N \mu^2}{3 k_B} \right) \frac{B}{T}$$

