

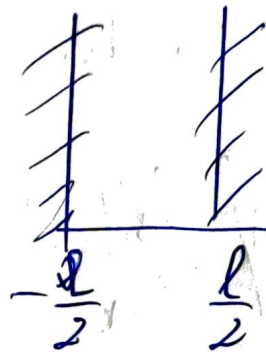
Ex3 particle in infinite well

M22 2019

$$V(x) = 0 ; |x| \leq \frac{l}{2}$$

$$E_n = E_1 n^2$$

$$E_1 = \frac{\pi^2 \hbar^2}{2m l^2}$$



1) $p_n = A \exp(-\beta E_n) ; r_{n+1} = \frac{p_{n+1}}{p_n}$

$$p_{n+1} = A \exp(-\beta E_{n+1}) \Rightarrow r_n = \frac{e^{-\beta E_n}}{e^{-\beta E_{n+1}}}$$

$$r_n = \exp(-\beta (E_{n+1} - E_n))$$

$$E_{n+1} - E_n = E_1 ((n+1)^2 - n^2) = (2n+1) E_1$$

$$r_n = \exp\left(-\frac{(2n+1) E_1}{k_B T}\right)$$

2) $r_1 \Rightarrow E_1 \approx 10 \text{ eV} \quad T = 283 \text{ K}$

$$r_1 = \exp\left(-\frac{3 E_1}{k_B T}\right) ; k_B T = 25 \text{ meV}$$

$$r_1 = \exp\left(-\frac{3 \times 10}{25 \times 10^{-3}}\right) \approx e^{-1200} \rightarrow 0$$

$$E_1 = 2,5 \cdot 10^{-12} \text{ eV}$$

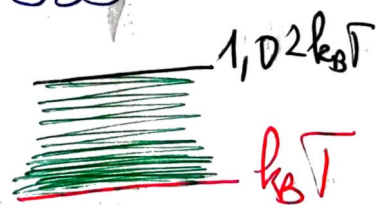
$$E_{n_1} = n_1^2 E_1 = k_B T \Rightarrow$$

$$n_1 = \sqrt{\frac{k_B T}{E_1}} = \sqrt{\frac{2,5 \cdot 10^{-2}}{2,5 \cdot 10^{-12}}} = \sqrt{10^{10}} = 10^5$$

$$n_2 = \sqrt{\frac{k_B T}{E_1} \cdot 1,02} = \sqrt{1,02} n_1$$

$$n_2 = 100335, \quad n_2 - n_1 = 335$$

4)



spectre continue

$$5) E = \frac{\pi^2 \hbar^2}{2m l^2} n^2$$

$$n = \sqrt{\frac{2m l^2}{\pi^2 \hbar^2}} \sqrt{E}$$

$$\frac{dn(E)}{dE} = \frac{1}{\sqrt{\frac{2m l^2}{\pi^2 \hbar^2}}} \cdot \frac{1}{2} \frac{1}{\sqrt{E}}$$

$$= \frac{1}{\sqrt{2\pi^2 \hbar^2}} \frac{1}{\sqrt{E}} = w(E)$$

$$7) dp = A [\exp(-\beta E)] w(E) dE$$

$$A \left(\frac{m l^2}{2 \pi^2 \hbar^2} \right) \int_0^{\infty} \frac{\exp(-\beta E)}{\sqrt{E}} dE = 1$$

$$A' \int_0^{\infty} \frac{\exp(-\beta E)}{\sqrt{E} \beta} \frac{d(E\beta)}{\beta} \sqrt{\beta} = 1$$

$$\frac{A'}{\sqrt{\beta}} \left(\int_0^{\infty} \frac{e^{-y}}{\sqrt{y}} dy \right) = 1 = \frac{A'}{\sqrt{\beta}} \sqrt{\pi}$$

$$A' = \sqrt{\frac{\beta}{\pi}} = \sqrt{\frac{1}{k_B T \pi}}$$

$$dp = \sqrt{\frac{\beta}{\pi}} \frac{e(-\beta E)}{\sqrt{E}} dE$$

$$8) \langle E \rangle = \int E dp(E)$$

$$= \sqrt{\frac{\beta}{\pi}} \int_0^{\infty} \frac{E \beta}{\sqrt{\beta} (\beta)} e^{-\beta E} d(E\beta)$$

$$\langle E \rangle = \sqrt{\frac{\beta}{\pi}} \frac{1}{\sqrt{\beta}} \left(\frac{\sqrt{\pi}}{2} \right) = \frac{k_B T}{2}$$

$$\langle E^2 \rangle = \int_0^\infty E^2 dp = \sqrt{\frac{\beta}{\pi}} \int \frac{E^2 e^{-\beta E}}{\sqrt{E}} dE$$

$$= \sqrt{\frac{\beta}{\pi}} \frac{1}{\beta^2 \sqrt{\beta}} \int \beta E \sqrt{E} e^{-\beta E} d(E\beta)$$

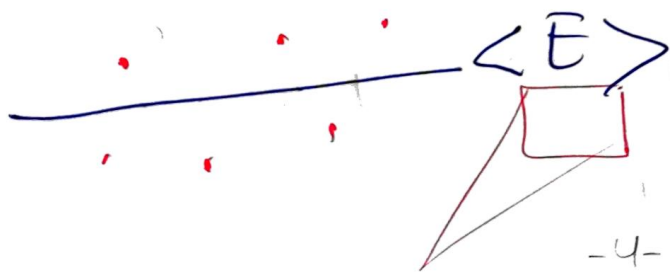
$$= \frac{(\frac{k_B T}{2})^2}{\sqrt{\pi}} \frac{3\sqrt{\pi}}{4} = \frac{3}{4} (k_B T)^2$$

$$\text{Var}(E) = \langle E^2 \rangle - \langle E \rangle^2$$

$$= \left(\frac{3}{4} - \frac{1}{4} \right) (k_B T)^2 = \frac{1}{2} (k_B T)^2$$

$$\frac{\Delta E}{\langle E \rangle} = \frac{1}{\sqrt{2}} \sqrt{2} \text{ fluctuation}$$

important



fluctuation

Ex 4

Théorie cinétique des gaz parfaits

$$\frac{dN}{N} = f(v_x) f(v_y) f(v_z) (dv_x dv_y dv_z)$$

$\psi(x, y, z) \quad [\vec{v}, \vec{v} + d\vec{v}]$

$$1) \frac{dN_{v_x}}{N} = A e^{-\beta E_c} dv_x$$

$$E_c = \frac{1}{2} m v_x^2 = \frac{p_x^2}{2m}$$

$$\left[v_x, v_x + dv_x \right]$$
$$\frac{dN_{v_x}}{N} = \left[A \exp\left(-\frac{m v_x^2}{2k_B T}\right) \right] dv_x$$

$$A \int_{-\infty}^{+\infty} \exp\left(-\frac{m v_x^2}{2k_B T}\right) dv_x = 1$$

$$A \sqrt{\frac{\pi}{a}} = 1, \quad a = \frac{m}{2k_B T}$$

$$A = \left(\frac{m}{2\pi k_B T} \right)^{1/2}$$

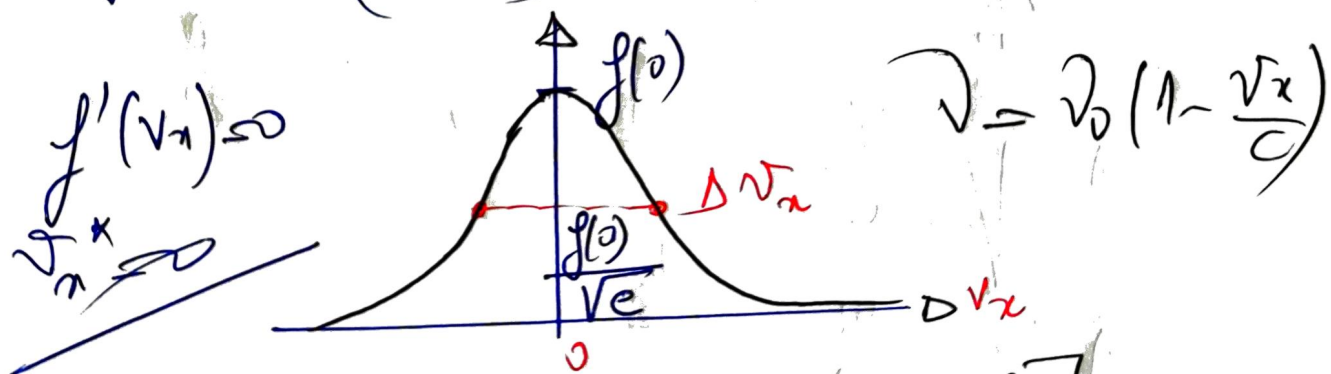
$$\frac{dN_{v_x}}{N} = \left(\frac{m}{2\pi k_B T} \right)^{1/2} \exp \left(-\frac{m v_x^2}{2 k_B T} \right) dv_x$$

$$\text{Pg: } \frac{d^3 N}{N} = A \exp \left[-\frac{\beta m}{2} (v_x^2 + v_y^2 + v_z^2) \right] dv_x dv_y dv_z$$

$$1 = A \left(\int_{-\infty}^{+\infty} \exp \left(-\frac{\beta m v_x^2}{2} \right) dv_x \right)^3; \text{ isotropic}$$

$$A = \left(\frac{m}{2\pi k_B T} \right)^{3/2}$$

$$f(v_x) = \left(\frac{m}{2\pi k_B T} \right)^{1/2} \exp \left(-\frac{m v_x^2}{2 k_B T} \right)$$



$$D(v) = B \exp \left[-\frac{(v - v_0)^2}{\Delta^2} \right]$$

$$2) \langle v_n \rangle = \int_{-\infty}^{+\infty} v_n f(v_n) dv_n$$

$$= \left(\frac{m}{2\pi k_B T} \right)^{1/2} \int_{-\infty}^{+\infty} v_n e^{-\alpha v_n^2} \frac{\text{impari}}{dv_n \rightarrow 0} dv_n$$

$$\langle v_n^2 \rangle = \left(\int_{-\infty}^{+\infty} v_n^2 e^{-\alpha v_n^2} dv_n \right) \left(\frac{m}{2\pi k_B T} \right)^{1/2}$$

$$= \left(\frac{m}{2\pi k_B T} \right)^{1/2} \frac{\sqrt{\pi} (2k_B T)^{3/2}}{2 m}$$

$$\langle v_n^2 \rangle = \frac{k_B T}{m} ; \langle \frac{m v_n^2}{2} \rangle = \frac{k_B T}{2}$$

$$u_n = \sqrt{\frac{k_B T}{2}}$$

$$3) C_v = \left(\frac{\partial U}{\partial T} \right)_V ; U = N \langle E_c \rangle$$

$$U = N \left\langle \frac{m v_x^2}{2} + \frac{m v_y^2}{2} + \frac{m v_z^2}{2} \right\rangle$$

$$U = 3 \left\langle \frac{mv_x^2}{2} \right\rangle N = \frac{3N k_B T}{2} : 1^{st} \text{ law of Thermodynamics}$$

$$C_V = \frac{3N k_B}{2}, \quad \text{si } N = N_A$$

$$R = N_A k_B$$

$$C_V = \frac{3}{2} R$$

$$C_P - C_V = R \Rightarrow$$

$$C_P = \frac{5}{2} R$$

$$\gamma = \frac{5}{3} = \frac{d+2}{d}$$

$$\underline{Pq} \quad H = U + (PV) = U + RT$$

$$\left(\frac{\partial H}{\partial T} \right)_P = \left(\frac{\partial U}{\partial T} \right)_V + R$$

$$C_P = C_V + R$$

(*) Diatomique $E_{\text{cinétique de rotation}}$

$$E_{CR} = \frac{\hbar^2}{2mI} j(j+1) \text{ donnée.}$$

$$\langle E_{\text{tot}} \rangle = \langle E_{\text{transl}} \rangle + \langle E_{\text{rot}} \rangle$$

$$= \frac{3}{2} k_B T + 2 \times \frac{1}{2} k_B T$$

$$\langle E_{\text{tot}} \rangle = \frac{5}{2} k_B T \Rightarrow U = \frac{5}{2} (N k_B) T$$

$$N = N_A: \quad U = \frac{5}{2} R T \Rightarrow C_V = \frac{5}{2} R$$

$$C_P = \frac{7}{2} R$$

$$\gamma = \frac{7}{5} = \frac{1+2}{1}$$

$$\frac{Pq}{Z} = \int_0^{\infty} \underbrace{(2j+1)}_g \exp\left[-\frac{\beta \hbar^2}{2mI} j(j+1)\right] \underbrace{dj}_{\text{volume}}$$

$$x = j(j+1) \Rightarrow j^2 + j = x$$

$$dx = (2j+1) dj$$

$$Z = \int_0^{\infty} \exp\left(-\frac{\beta \hbar^2}{2mI} x\right) dx = \frac{2mI}{\beta \hbar^2}$$

$$U = -N \frac{\partial \ln Z}{\partial \beta} = -N \frac{\partial}{\partial \beta} \ln \left(\frac{2\pi I}{h^2 \beta} \right)$$

$$= +N \frac{\partial}{\partial \beta} \left[\ln(\beta^{-1}) \right] = \frac{N}{\beta} = N k_B T$$

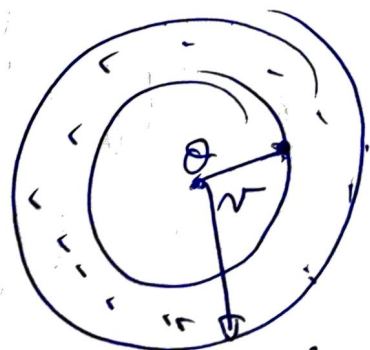
eg: $T \nearrow$: 2 termes vibrat'.

$$\left(\underset{\substack{\text{translat} \\ \swarrow}}{3} + \underset{\substack{\text{rotation} \\ \swarrow}}{2} + 2 \text{ vibrat'ons} \right) \times \frac{k_B T}{2}$$

$$4) \frac{dN_v}{N} = A \exp\left(-\frac{mv^2}{2k_B T}\right) \quad v^2 \sin\theta dv d\theta d\phi$$

$$v^2 = (v_x^2 + v_y^2 + v_z^2)$$

$$\frac{dN_v}{N} = \underbrace{4\pi v^2 A}_{F(v)} e^{\left(-\frac{mv^2}{2k_B T}\right)} dv$$



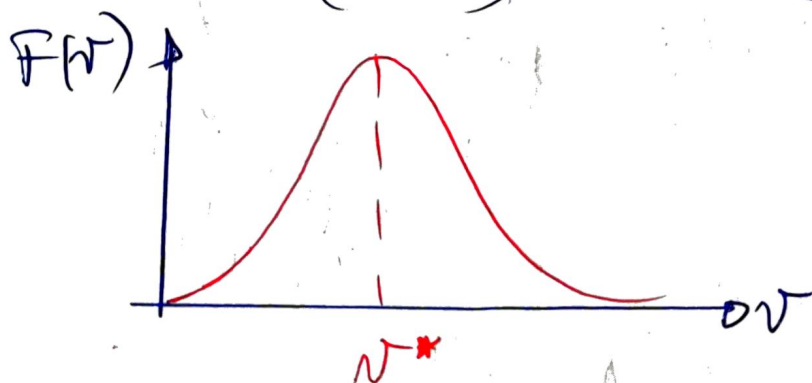
$v+dv$
 $\forall \theta, \forall \phi$

$$4\pi A \int_0^{+\infty} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) dv = 1$$

$$4\pi A \left[\frac{\sqrt{\pi}}{4 a^{3/2}} \right] = 1$$

$$A = \left(\frac{m}{2\pi k_B T} \right)^{3/2}$$

$$F(v) = 4\pi \left(\frac{m}{2\pi k_B T} \right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) dv$$



$$\frac{dF(v)}{dv} = 0 \Rightarrow \frac{d}{dv} \left(\frac{e^{-av^2}}{v^{3/2}} \right) = 0$$

$$-a 2v^3 e^{-av^2} + \frac{3}{2} v e^{-av^2} = 0$$

$$(-a v^2 + \frac{3}{2}) v e^{-av^2} = 0$$

$$v^* = \frac{1}{\sqrt{a}} = \sqrt{\frac{2k_B T}{m}}$$

$$b) \langle v \rangle = \int_0^{\infty} v F(v) dv$$

$$= 4\pi \left(\frac{m}{2\pi k_B T} \right)^{3/2} \int_0^{\infty} v^3 e^{-av^2} dv$$

$$= 4\pi \left(\frac{m}{2\pi k_B T} \right)^{3/2} \frac{(2k_B T)^2}{2 m^2}$$

$$\langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}} ; \frac{4}{\cancel{2}} \times \frac{4}{\cancel{2} \times \sqrt{2}}$$

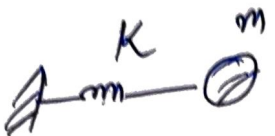
$$\langle v^2 \rangle = 4\pi \left(\frac{m}{2\pi k_B T} \right) \int_0^{\infty} v^4 e^{-av^2} dv$$

$$\langle v^2 \rangle = \frac{3 k_B T}{m} = u^2 ; \frac{m u^2}{2} = \langle E \rangle$$

$$\left\langle \frac{m v^2}{2} \right\rangle = \frac{3 k_B T}{2}$$

$$u = \sqrt{\frac{3 k_B T}{m}}$$

$$u \gg v_{Drift}$$

Ex 5  ; $\bar{E} = \frac{1}{2} m v^2 + \frac{1}{2} K x^2$
 $\omega = \sqrt{\frac{K}{m}}$ $= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} (m\omega^2) x^2$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + \left(\frac{1}{2} m \omega^2 x^2 \right) \psi = E \psi(x, t)$$

$$\Rightarrow E_{n_x} = \hbar \omega \left(n_x + \frac{1}{2} \right) \text{ donnée}$$

à 3 dim $E = \hbar \omega \left(n_x + n_y + n_z + \frac{3}{2} \right)$
 $(n_x, n_y, n_z) \in \mathbb{N}$

$$1) Z = \sum_{n_x, n_y, n_z} \exp -\beta \hbar \omega \left(n_x + \frac{1}{2} + n_y + \frac{1}{2} + n_z + \frac{1}{2} \right)$$

$$Z = \left[\sum_{n_x} \exp -\beta \hbar \omega \left(n_x + \frac{1}{2} \right) \right]^3 = (Z_x)^3$$

$$Z_x = e^{-\frac{\beta \hbar \omega}{2}} \left(\sum_{n_x=0}^{\infty} e^{-\beta \hbar \omega n_x} \right)$$

$$= e^{-\frac{\beta \hbar \omega}{2}} \frac{1}{(1 - e^{-\beta \hbar \omega})} = \frac{1}{2 \sinh \left(\frac{\beta \hbar \omega}{2} \right)}$$

$$Z_1 = \frac{1}{2 \sinh\left(\frac{\beta \hbar \omega}{2}\right)}$$

1 oscill à 1 dim

$$Z = \left[\frac{1}{2 \sinh\left(\frac{\beta \hbar \omega}{2}\right)} \right]^3 \quad \because \text{1 seul oscillateur à 3 dimensions}$$

$$Z = Z^N = (Z_1)^{3N}$$

$$2) \langle E \rangle = - \frac{\partial \ln Z}{\partial \beta} = - \frac{3 \partial \ln Z_1}{\partial \beta}$$

$$= + 3 \frac{\partial}{\partial \beta} \ln \left[2 \sinh\left(\frac{\beta \hbar \omega}{2}\right) \right]$$

$$\langle E \rangle = \frac{3 \hbar \omega}{2} \coth\left(\frac{\beta \hbar \omega}{2}\right)$$

$$= \frac{3 \hbar \omega}{2} \coth\left(\frac{\hbar \omega}{2 k_B T}\right)$$

$$\theta = \frac{\hbar \omega}{k_B}$$

$$\langle E \rangle = \frac{3 \hbar \omega}{2} \coth\left(\frac{\theta}{2T}\right)$$

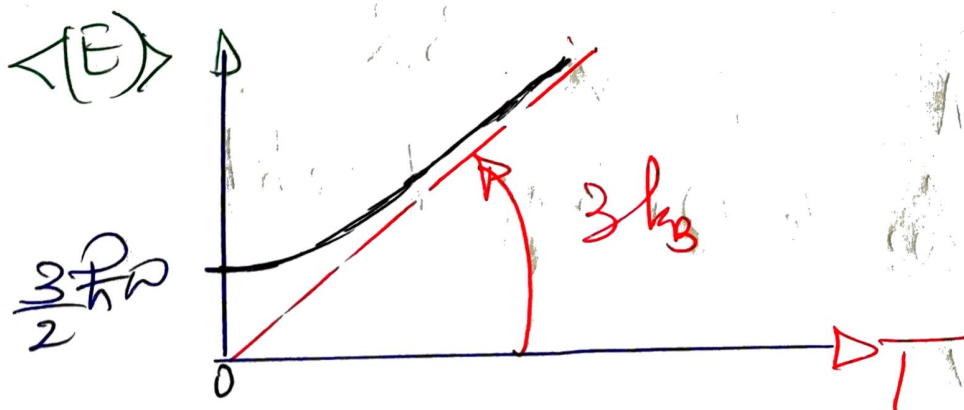
$$3) a) \underline{T \ll \theta} : \lim_{\theta \gg T} \coth\left(\frac{\theta}{2T}\right) \approx 1$$

$$\langle E \rangle \approx \frac{3\hbar\omega}{2}$$

$$b) \underline{T \gg \theta} :$$

$$\lim_{n \rightarrow \infty} \coth x = \frac{1}{n}$$

$$\langle E \rangle \approx \frac{3\hbar\omega}{2} \frac{2k_B T}{\hbar\omega} = 3k_B T$$



$$4) C_V = \left(\frac{2\langle E \rangle}{2T} \right)_V = -k_B \beta^2 \left(\frac{2\langle E \rangle}{2\beta} \right)$$

$$C_V = +k_B \beta^2 \frac{3\hbar\omega}{2} \frac{\hbar\omega}{2} \frac{1}{\sinh^2\left(\frac{\beta\hbar\omega}{2}\right)}$$

$$C_V = 3k_B \left(\frac{\hbar\omega\beta}{2} \right)^2 \frac{1}{\sinh^2\left(\frac{\beta\hbar\omega}{2}\right)}$$

$$\frac{\text{Sh}\left(\frac{\beta \hbar \omega}{2}\right)}{T \ll \theta; \beta \hbar \omega \gg 1} \approx \frac{1}{2} e^{\frac{\beta \hbar \omega}{2}} \quad C_V \sim e^{-\beta \hbar \omega}$$

$$T \gg \theta \quad \underline{\beta \hbar \omega \ll 1} \quad \frac{\text{Sh}\left(\frac{\beta \hbar \omega}{2}\right)}{\left(\frac{\beta \hbar \omega}{2}\right)} \approx \frac{\beta \hbar \omega}{2}$$

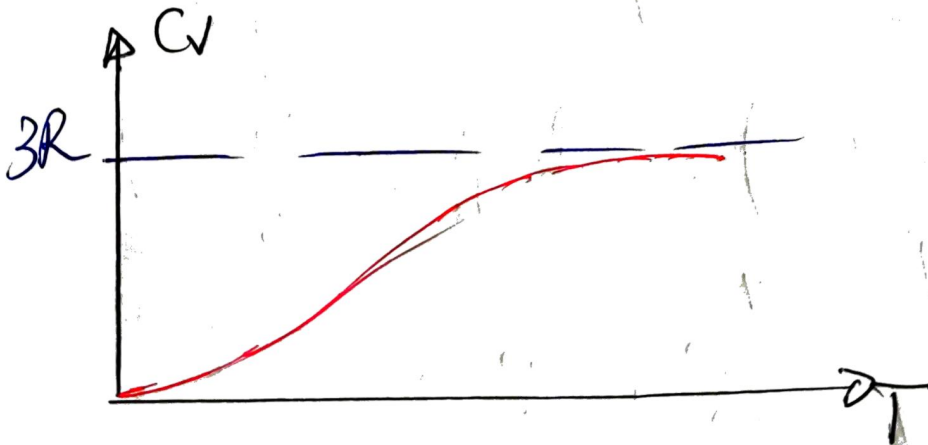
$$C_V = 3k_B$$

$$N \text{ oscillateurs} \quad C_V = 3Nk_B$$

$$N = C N_A$$

$$C_V = 3R$$

La loi de Dulong et Petit



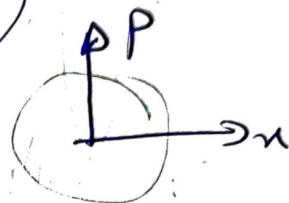
C_V à basse température suit modèle de Debye

B) traitant classique

$$\mathcal{L}(n, p_n) \underbrace{dn dp_n}_{\text{domée}} = \frac{1}{Z_2} e^{-\beta E}$$

$$\left(\frac{dn dp}{h} \right)$$

espace de phase



$$Z_2 = \iint e^{-\beta E} \frac{dn dp}{h}$$

$$Z_2 = \frac{1}{h} \int_{-\infty}^{+\infty} \exp\left(-\frac{\beta p_n^2}{2m}\right) dp_n \int_{-\infty}^{+\infty} e^{-\left(\frac{\beta m \omega^2}{2} x^2\right)} dx$$

$$Z_2 = \frac{1}{h} \sqrt{\frac{2\pi T m}{\beta}}$$

$$\sqrt{\frac{2\pi}{\beta m \omega^2}}$$

$$Z_2 = \frac{2\pi k_B T}{h \omega} = \frac{k_B T}{h \omega}$$

$$2) Z_n = \frac{1}{2 \operatorname{sh}\left(\frac{h \omega}{2 k_B T}\right)}$$

$$\omega = 2\pi \nu$$

$$h \omega = h \nu$$

si $T \gg \theta$

$$Z_n \approx \frac{k_B T}{h \omega} = Z_2$$

$$3) \mathcal{I}(n) dx = \left[\mathcal{I}(n, p_n) dp_n \right] dx$$

$$= \frac{1}{Z} \left(\int_{-\infty}^{+\infty} e^{-\frac{\beta p_n^2}{2m}} dp_n \right) e^{-\beta \frac{m\omega^2 x^2}{2}} \frac{dx}{h}$$

$$= \frac{\hbar \omega}{2\pi k_B T} \frac{1}{h} \sqrt{\frac{\pi 2m}{\beta}} e^{-\left(\right)} dx$$

$$= \sqrt{\beta} \frac{\omega}{2\pi} \sqrt{\pi} \sqrt{2} \sqrt{m} e^{-\left(\right)} dx$$

$$= \omega \left(\frac{m}{2\pi k_B T} \right)^{1/2} e^{-\frac{m\omega^2 x^2}{2k_B T}} dx$$

$$4) \langle n \rangle = \int_{-\infty}^{+\infty} n \mathcal{I}(n) dx$$

$$= \omega \left(\frac{m}{2\pi k_B T} \right)^{1/2} \left(\int_{-\infty}^{+\infty} n e^{-\frac{m\omega^2 x^2}{2k_B T}} dx \right)$$

impaired

$$\langle n \rangle = 0$$

$$\langle n^2 \rangle = \omega \left(\frac{m}{2\pi\hbar_B T} \right)^{1/2} \int_{-\infty}^{+\infty} n^2 e^{-\frac{m\omega^2 n^2}{2\hbar_B T}} dn$$

$$\langle n^2 \rangle = \frac{\hbar_B T}{m\omega^2} ; \quad \langle \frac{m\omega^2 n^2}{2} \rangle = \frac{\hbar_B T}{2}$$

$$\langle E_c \rangle = \frac{\int E_c e^{-\beta E_c} dp_x}{\int e^{-\beta E_c} dp_x} \quad \text{for } N$$

$$= \frac{\iint E_c e^{-\beta E_c} e^{-\beta E_p} dx dp_x}{\iint e^{-\beta E_c} dp_x e^{-\beta E_p} dx}$$

$$= \frac{\int E_c e^{-\beta E_c} dx \int e^{-\beta E_p} dp}{\int e^{-\beta E_c} dx \int e^{-\beta E_p} dp}$$

$$\langle E_c \rangle = \frac{\int_{-\infty}^{+\infty} \frac{p_x^2}{2m} e^{-\beta \frac{p_x^2}{2m}} dp_x}{\int_{-\infty}^{+\infty} e^{-\beta \frac{p^2}{2m}} dp_x}$$

$$\langle E_c \rangle = -\frac{2}{\partial \beta} \ln \left(\int_{-\infty}^{\infty} e^{-\beta E_c} dp_1 \right)$$

$$= -\frac{2}{\partial \beta} \ln \left[\sqrt{\frac{2\pi m}{\beta}} \right]$$

$$= -\frac{2}{\partial \beta} \left(-\ln(\sqrt{\beta}) + \ln \sqrt{2\pi m} \right)$$

$$= \frac{1}{2\beta} = \frac{k_B T}{2}$$

$$\langle E_p \rangle = \frac{\int E_p e^{-\beta E_p} dn}{\int e^{-\beta E_p} dn}$$

$$= -\frac{2}{\partial \beta} \ln \left(\frac{2\pi}{\beta \omega^2 m} \right)^{1/2}$$

$$\langle E_p \rangle = \frac{1}{2} k_B T$$

c) non Harmonic

$$\varepsilon = \frac{p^2}{2m} + b x^4$$

$$\langle \varepsilon_c \rangle = \frac{1}{2} k_B T$$

$$\langle \varepsilon_p \rangle = \frac{\int \varepsilon_p e^{-\beta \varepsilon_p} dx}{\int e^{-\beta \varepsilon_p} dx}$$

$$= -\frac{2}{2\beta} \ln \left(\int_{-\infty}^{+\infty} e^{-(\beta b x^4)} dx \right)$$

suppose $y^4 = \beta x^4 \Rightarrow y = \beta^{1/4} x$

$$dy = \beta^{1/4} dx$$

$$\langle \varepsilon_p \rangle = -\frac{2}{2\beta} \ln \left[\int_{-\infty}^{+\infty} e^{-by^4} \frac{dy}{\beta^{1/4}} \right]$$

$$= -\frac{2}{2\beta} \left[\ln \int e^{-by^4} dy - \ln \beta^{1/4} \right]$$

$$= \frac{1}{4\beta} = \frac{k_B T}{4}$$

E) Brillouin

$$\epsilon_m = m g \mu_B B_0 ; -J \leq m \leq J$$

$$1) z = \sum_{-J}^J e^{-(g \mu_B B_0) m} = \sum_{-J}^J e^{-m y}$$

$$z = e^{Jy} + e^{(J-1)y} + \dots + e^{-Jy}$$

$$= e^{Jy} (1 + e^{-y} + e^{-2y} + \dots + e^{-2Jy})$$

$$= e^{Jy} \frac{(1 - e^{-(2J+1)y})}{(1 - e^{-y})}$$

$$z = \left[\frac{\sinh \left((J + \frac{1}{2}) y \right)}{\sinh \left(\frac{y}{2} \right)} \right]$$

$$2) p_m = \frac{N_m}{N} = \frac{e^{-m y}}{z}$$

$$3) (\mu_z)_m = -m g \mu_B$$

$$M_z = \sum N_m (\mu_z)_m$$

$$M_z = - \sum_{-J}^J \frac{N e^{-m\eta} (m g \mu_B)}{Z}$$

$$M_z = -g N \mu_B \left(\frac{\sum m e^{-m\eta}}{\sum e^{-m\eta}} \right)$$

$$M_z = N g \mu_B \frac{\partial}{\partial \eta} \ln \left(\sum e^{-m\eta} \right)$$

$$M_z = N g \mu_B \frac{\partial \ln}{\partial \eta} \left[\frac{\sinh \left(J + \frac{1}{2} \right) \eta}{\sinh \left(\frac{\eta}{2} \right)} \right]$$

$$M_z = N g \mu_B \left[\left(J + \frac{1}{2} \right) \coth \left(\left(J + \frac{1}{2} \right) \eta \right) - \frac{1}{2} \coth \left(\frac{\eta}{2} \right) \right]$$

$$\langle \mu \rangle = \frac{M_z}{N}$$

$$J \rightarrow \infty, \quad M_z = N g \mu_B J$$

$$J = g \beta \mu_B B_0 = J \frac{\mu_B B_0}{k_B T}$$

$$k_B T \ll \mu_B B_0, \quad \mu_z = -m g \mu_B$$

$$\langle \mu_z \rangle_m = -m N g \mu_B$$

$\Rightarrow$ les particules condensées.

$$\bullet \quad \underline{J \ll 1} \quad \coth\left(J + \frac{1}{2}\right) J =$$

$$\frac{1}{\left(J + \frac{1}{2}\right) J} + \frac{1}{3} \left(J + \frac{1}{2}\right) J$$

$$\coth\left(\frac{J}{2}\right) = \frac{2}{J} + \frac{J}{3 \times 2}$$

F)

$$1) \quad n_+ = n \frac{e^{-\beta \varepsilon^+}}{Z}$$

$$n_- = n \frac{e^{-\beta \varepsilon^-}}{Z}$$

$$\Sigma^+ = \frac{n e^{\beta \epsilon}}{2 \cosh(\beta \epsilon)}$$

$$\uparrow \uparrow \quad -\epsilon = \epsilon^+$$

$$\Sigma^- = \frac{n e^{-\beta \epsilon}}{2 \cosh(\beta \epsilon)}$$

$$\uparrow \downarrow \quad +\epsilon = \epsilon^-$$

$$\Sigma_{\text{dip}} = -\vec{\mu} \cdot \vec{B}_a = -\mu B_a \text{ and}$$

$$2) \Omega = C_n^{n^+} = \frac{n!}{n^+! n^-!} = \frac{n!}{n^+! (n - n^+)!}$$

$$3) S = k_B \ln \Omega : \text{Gibbs}$$

$$S = k_B (\ln n! - \ln n^+! - \ln n^-!) \\ = k_B [n \ln n - n^+ \ln n^+ - n^- \ln n^-]$$

$$n = n^+ + n^-$$

$$\mu = n^+ \mu_B - n^- \mu_B$$

$$\left\{ \begin{array}{l} \frac{\mu}{\mu_B} = n^+ - n^- \\ n = n^+ + n^- \end{array} \right. \Rightarrow$$

$$2n^+ = n(1 + \gamma)$$

$$n^+ = \left(\frac{1 + \gamma}{2} \right) n$$

$$n^- = \left(\frac{1 - \gamma}{2} \right) n$$

$$\overline{r}^0 = N \langle \overline{r}^0 \rangle = N \overline{m}^0$$