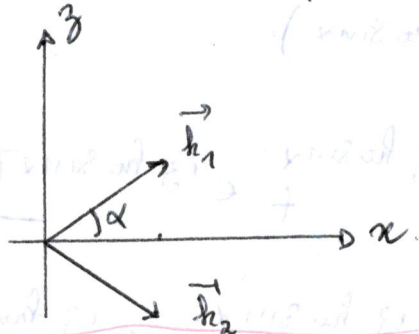


Correction TD4 : propagation des ondes

Exercice 1 : Superposition de deux O.P.M

1° propagation ds le vide des ondes $\vec{E}_1$ et $\vec{E}_2 \rightarrow \Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$
est l'équation d'onde correspondante $\rightarrow \vec{E}_1 = E_0 e^{i(\omega t - \vec{k}_1 \cdot \vec{r})} \vec{u}_y$



$$\vec{k}_1 = \begin{pmatrix} k_0 \cos \alpha \\ 0 \\ k_0 \sin \alpha \end{pmatrix} ; \vec{k}_2 = \begin{pmatrix} k_0 \cos \alpha \\ 0 \\ -k_0 \sin \alpha \end{pmatrix}$$

$$\text{et } \vec{E}_2 = E_0 e^{i(\omega t - \vec{k}_2 \cdot \vec{r})} \vec{u}_y$$

$$\vec{E}_1 = E_0 e^{i(\omega t - x k_0 \cos \alpha - z k_0 \sin \alpha)} \vec{u}_y$$

$$\vec{E}_2 = E_0 e^{i(\omega t - x k_0 \cos \alpha + z k_0 \sin \alpha)} \vec{u}_y$$

$$2^\circ \vec{B}_1 = \frac{\vec{k}_1 \wedge \vec{E}_1}{\omega} = \frac{1}{\omega} \begin{vmatrix} k_0 \cos \alpha & 0 & k_0 \sin \alpha \\ 0 & E_1 & 0 \\ k_0 \sin \alpha & 0 & 0 \end{vmatrix} = \begin{vmatrix} -\frac{E_1}{c} \sin \alpha \\ 0 \\ \frac{E_1}{c} \cos \alpha \end{vmatrix}$$

$$\vec{B}_1 = \begin{vmatrix} -\frac{E_0}{c} \sin \alpha \exp(i(\omega t - x k_0 \cos \alpha - z k_0 \sin \alpha)) \\ 0 \\ \frac{E_0}{c} \cos \alpha \exp(i(\omega t - x k_0 \cos \alpha - z k_0 \sin \alpha)) \end{vmatrix}$$

$$\vec{B}_2 = \frac{\vec{k}_2 \wedge \vec{E}_2}{\omega} = \frac{1}{\omega} \begin{vmatrix} k_0 \cos \alpha & 0 & -k_0 \sin \alpha \\ 0 & E_2 & 0 \\ -k_0 \sin \alpha & 0 & 0 \end{vmatrix} = \begin{vmatrix} \frac{k_0 \sin \alpha}{\omega} E_2 \\ 0 \\ \frac{k_0 \cos \alpha}{\omega} E_2 \end{vmatrix}$$

$$\vec{B}_2 = \begin{vmatrix} \frac{E_0}{c} \sin \alpha \exp(i(\omega t - x k_0 \cos \alpha + z k_0 \sin \alpha)) \\ 0 \\ \frac{E_0}{c} \cos \alpha \exp(i(\omega t - x k_0 \cos \alpha + z k_0 \sin \alpha)) \end{vmatrix}$$

$$3^\circ \vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= E_0 e^{i(\omega t - x k_0 \cos \alpha)} \left(e^{-iz k_0 \sin \alpha} + e^{iz k_0 \sin \alpha} \right) \vec{u}_y$$

$$\vec{E} = 2 E_0 \cos(z k_0 \sin \alpha) \exp(i(\omega t - x k_0 \cos \alpha)) \vec{u}_y$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \begin{pmatrix} -\frac{E_0}{c} \sin \alpha e^{i(\omega t - n k_0 \cos \alpha - z k_0 \sin \alpha)} \\ 0 \\ \frac{E_0}{c} \cos \alpha e^{i(\omega t - n k_0 \cos \alpha - z k_0 \sin \alpha)} \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{E_0}{c} \sin \alpha e^{i(\omega t - n k_0 \cos \alpha + z k_0 \sin \alpha)} \\ 0 \\ \frac{E_0}{c} \cos \alpha e^{i(\omega t - n k_0 \cos \alpha + z k_0 \sin \alpha)} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{E_0}{c} \sin \alpha e^{i(\omega t - n k_0 \cos \alpha)} [-e^{-iz k_0 \sin \alpha} + e^{iz k_0 \sin \alpha}] \\ 0 \\ \frac{E_0}{c} \cos \alpha e^{i(\omega t - n k_0 \cos \alpha)} [e^{-iz k_0 \sin \alpha} + e^{iz k_0 \sin \alpha}] \end{pmatrix}$$

$$= \begin{pmatrix} 2i \frac{E_0}{c} \sin \alpha \sin(z k_0 \sin \alpha) e^{i(\omega t - n k_0 \cos \alpha)} \\ 0 \\ 2 \frac{E_0}{c} \cos \alpha e^{i(\omega t - n k_0 \cos \alpha)} \cos(z k_0 \sin \alpha) \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} 2i \frac{E_0}{c} \sin \alpha \sin(z k_0 \sin \alpha) e^{i(\omega t - n k_0 \cos \alpha)} \\ 0 \\ 2 \frac{E_0}{c} \cos \alpha \cos(z k_0 \sin \alpha) e^{i(\omega t - n k_0 \cos \alpha)} \end{pmatrix}$$

4/ L'onde résultante est une onde progressive selon n (couplage (n, t)), stationnaire suivant z (découplage (z, t)).

L'amplitude de l'onde n'est pas cst qd $n = \text{cst} \rightarrow$ l'onde n'est pas plane.

5° pour $\alpha = 0$

$$\begin{pmatrix} \vec{E} = 2 E_0 \cos(\omega t - n k_0) \vec{u}_y \\ \vec{B} = 2 \frac{E_0}{c} \cos(\omega t - n k_0) \vec{u}_z \end{pmatrix} \text{ l'onde est plane, progressive}$$

$$6) V_\varphi = \frac{\omega}{k} = \frac{\omega}{k_0 \cos \alpha} = \frac{\omega}{\frac{\omega}{c} \cos \alpha} = \frac{c}{\cos \alpha} > c$$

$$V_\varphi \cdot V_g = c^2 \rightarrow V_g = c \cdot \cos \alpha < c \text{ ou à partir de la relation}$$

de dispersion. En effet

$$\underline{DE} = \frac{1}{c^2} \frac{\partial^2 \underline{E}}{\partial t^2} \quad \text{et} \quad \underline{E} = 2 E_0 \cos(z k_0 \sin \alpha) e^{i(\omega t - n k_0 \cos \alpha)}$$

$$\text{avec } k_0 \cos \alpha = k$$

$$\rightarrow \frac{\partial^2 \underline{E}}{\partial x^2} + \frac{\partial^2 \underline{E}}{\partial y^2} + \frac{\partial^2 \underline{E}}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \underline{E}}{\partial t^2}$$

$$- k^2 \underline{E} - k_0^2 \sin^2 \alpha \underline{E} = - \frac{\omega^2}{c^2} \underline{E}$$

$$k^2 = \frac{\omega^2}{c^2} - k_0^2 \sin^2 \alpha \quad \text{relation de dispersion}$$

$$2k dk = \frac{2\omega d\omega}{c^2} \rightarrow v_g = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = \frac{c^2}{v_p}$$

$$v_g = c \cdot \cos \alpha$$

$$70/ \vec{R} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} \quad (\text{en notation réelle})$$

$$= \begin{vmatrix} 0 & B_x \\ E/\mu_0 & 0 \\ 0 & B_z \end{vmatrix} = \begin{vmatrix} \frac{E \cdot B_z}{\mu_0} \\ 0 \\ -\frac{E \cdot B_x}{\mu_0} \end{vmatrix} = \begin{vmatrix} \frac{4E_0^2}{\mu_0 c} \cos \alpha \cos^2(z k_0 \sin \alpha) \cos^2(\omega t - n k_0 \cos \alpha) \\ 0 \\ \frac{4E_0^2}{\mu_0 c} \sin \alpha \cos(z k_0 \sin \alpha) \cdot \sin(z k_0 \sin \alpha) \cdot \cos(\omega t - n k_0 \cos \alpha) \cdot \sin(\omega t - n k_0 \cos \alpha) \end{vmatrix}$$

$$\langle \vec{R} \rangle = \frac{2E_0^2}{\mu_0 c} \cos \alpha \cos^2(z k_0 \sin \alpha) \vec{u}_n \quad \text{ou} \quad \langle \vec{R} \rangle = \frac{1}{2} \text{Re} \left(\vec{E} \wedge \frac{\vec{B}^*}{\mu_0} \right)$$

Le résultat est prévisible car l'onde est progressive suivant $n \uparrow$.

$$\begin{aligned} 80/ u_{\text{em}} &= u_e + u_m = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \\ &= \frac{\epsilon_0}{2} \cdot 4 E_0^2 \cos^2(z k_0 \sin \alpha) \cos^2(\omega t - n \overset{k}{k_0 \cos \alpha}) + \frac{1}{2\mu_0} [B_x^2 + B_z^2] \\ &= \epsilon_0 2 E_0^2 \cos^2(z k_0 \sin \alpha) \cdot \cos^2(\omega t - n k) + \frac{2E_0^2}{\mu_0 c^2} [\sin^2 \alpha \cdot \sin^2(z k_0 \sin \alpha) \sin^2(\omega t - k n) \\ &\quad + \cos^2 \alpha \cdot \cos^2(z k_0 \sin \alpha) \cos^2(\omega t - k n)] \end{aligned}$$

$$\langle u_{em} \rangle = \epsilon_0 E_0^2 \cos^2(z R_0 \sin \alpha) + \epsilon_0 E_0^2 [\sin^2 \alpha \cdot \sin^2(z R_0 \sin \alpha) + \cos^2 \alpha \cdot \cos^2(z R_0 \sin \alpha)]$$

$$g) \quad \langle \langle \vec{R} \rangle \rangle = \frac{E_0^2}{\rho_0 c} \cos \alpha \vec{u}_x$$

$$\langle \langle u_{em} \rangle \rangle = \frac{\epsilon_0 E_0^2}{2} + \frac{\epsilon_0 E_0^2}{2} [\sin^2 \alpha + \cos^2 \alpha] = \epsilon_0 E_0^2$$

$$V_e = \frac{\| \langle \langle \vec{R} \rangle \rangle \|}{\langle \langle u_{em} \rangle \rangle} = \frac{\cancel{c} E_0^2 \cos \alpha}{\rho_0 \cancel{c} \cdot \epsilon_0 \cancel{E_0^2}} = c \cdot \cos \alpha$$

$$V_e = V_g$$