

Exercice n°5

1°/ $I(z, t) = I_0 \left(1 - \frac{|z|}{l}\right) \cos \omega t$

(découplage espace-temps $\rightarrow$ stationnaire)

* noeuds : $\left(1 - \frac{|z|}{l}\right) = 0 \rightarrow |z| = l \Rightarrow z = \pm l$

* ventres : $\left(1 - \frac{|z|}{l}\right) \text{ max} \rightarrow z = 0$

2°/ L'onde est localement plane $\rightarrow \vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega}$ or $k = \frac{\omega}{c}$

$\rightarrow \vec{B} = \frac{E}{c} \vec{u}_\varphi = - \frac{I_0 \cdot \sin \theta \sin^2 \left(\frac{\omega l \cos \theta}{2c} \right)}{\pi \epsilon_0 c \omega r l \cos^3 \theta} \sin(\omega t - kr) \vec{u}_\varphi$

3°/ $\langle \vec{R} \rangle = \langle \vec{E} \wedge \frac{\vec{B}}{\mu_0} \rangle = \langle \frac{E^2}{\mu_0 c} \rangle \vec{u}_r$

$= \frac{1}{2} \frac{c I_0^2 \sin^2 \theta \sin^4 \left(\frac{\omega l \cos \theta}{2c} \right)}{\pi^2 \epsilon_0 \omega^2 r^2 l^2 \cos^4 \theta} \vec{u}_r$

$\langle \vec{R} \rangle = \frac{c I_0^2 \sin^2 \theta \sin^4 \left(\frac{\omega l \cos \theta}{2c} \right)}{2 \pi^2 \epsilon_0 \omega^2 r^2 l^2 \cos^4 \theta} \vec{u}_r$

4°/ $P_m = \oint_{\text{sphère}} \langle \vec{R} \rangle \cdot d\vec{S}$ avec $d\vec{S} = (r d\theta)(r \sin \theta d\varphi) \vec{u}_r$
 $= r^2 \sin \theta d\theta d\varphi \vec{u}_r$

$= \frac{c I_0^2}{2 \pi^2 \epsilon_0 \omega^2 r^2 l^2} \int_0^\pi \frac{\sin^2 \theta \sin^4 \left(\frac{\omega l \cos \theta}{2c} \right)}{\cos^4 \theta} \sin \theta d\theta \int_0^{2\pi} d\varphi$

$P_m = \frac{c I_0^2}{\pi \epsilon_0 \omega^2 l^2} \int_0^\pi \frac{\sin^4 \left(\frac{\omega l \cos \theta}{2c} \right)}{\cos^4 \theta} \sin^3 \theta d\theta$

5°/ $P_m = \frac{1}{2} R_{\text{ray}} I_0^2 = \frac{1}{2} \cdot \frac{2c}{\pi \epsilon_0 \omega^2 l^2} \int_0^\pi \frac{\sin^4 \left(\frac{\omega l \cos \theta}{2c} \right)}{\cos^4 \theta} \sin^3 \theta d\theta I_0^2$

Ray

II/ Etude du cas particulier ($l \ll \lambda$)

1°) $\frac{wl \cos \theta}{2c} = \frac{l\pi \cos \theta}{\lambda}$ or $l \ll \lambda$ et $|\cos \theta| \leq 1$
 $\rightarrow \frac{l\pi \cos \theta}{\lambda} \ll 1$

$\rightarrow \sin^2\left(\frac{wl \cos \theta}{2c}\right) \approx \left(\frac{wl \cos \theta}{2c}\right)^2$

$\rightarrow \vec{E} = - \frac{I_0 \sin \theta}{\pi \epsilon_0 w r l \cos \theta} \frac{wl^2 \cos^2 \theta}{4c^2} \sin(\omega t - kr) \vec{u}_\theta$

$\vec{E} = - \frac{I_0 \sin \theta \cdot wl}{4c^2 \pi \epsilon_0 r} \sin(\omega t - kr) \vec{u}_\theta$

2°) $R_{\text{ray}} = \frac{2c}{\pi \epsilon_0 l^2 \omega^2} \int_0^\pi \frac{(wl \cos \theta)^4}{(2c)^4 \cos^4 \theta} \sin^3 \theta d\theta$
 $= \frac{\omega^2 l^2}{8c \pi \epsilon_0 c^2} \times \frac{4}{3}$ or $\frac{\omega^2}{c^2} = \left(\frac{2\pi}{\lambda}\right)^2$

$R_{\text{ray}} = \frac{2\pi}{3c \epsilon_0} \left(\frac{l}{\lambda}\right)^2$

AN

$R_{\text{ray}} = 0,88 \Omega$

3°) $P_m = \frac{1}{2} R_{\text{ray}} I_0^2 = \frac{1}{2} \times 0,88 \times (10)^2 = 44 \text{ W}$

4°) à I_0 fixé, il faut augmenter R_{ray} pour augmenter P_m
 or $R_{\text{ray}} \propto \left(\frac{l}{\lambda}\right)^2 \rightarrow$ il faut augmenter l (ou λ) tout en gardant $l \ll \lambda$.

5°/a) $E_0 = 2 \cdot 10^{-2} \text{ V.m}^{-1}$; $\theta = \pi/2$

$\rightarrow E_0 = \frac{I_0 \omega l}{4\pi c^2 \epsilon_0 r_0} \rightarrow r_0 = \frac{I_0 \omega l}{4\pi c^2 \epsilon_0 E_0} \stackrel{\omega/c = 2\pi/\lambda}{=} \frac{I_0 l}{2c \epsilon_0 \lambda E_0}$

AN $r_0 \approx 3 \text{ km}$

5-b) puissance minimale / unité de surface $= \|\langle \vec{R} \rangle\|_{\min} = \frac{E_0^2 \min}{2\mu_0 c}$

$$\rightarrow E_{\min} = \sqrt{2\mu_0 c \|\langle \vec{R} \rangle\|_{\min}} = 3,1 \cdot 10^{-3} \text{ V.m}^{-1}$$

or $r \propto \frac{1}{E_0} \rightarrow \text{à } E_{\min} \mapsto r_{\max}$

$$r_{\max} = \frac{I_0 l}{2C \epsilon_0 \lambda E_{\min}}$$

AN $r_{\max} \approx 20 \text{ km}$

5-c) pour augmenter la portée de l'antenne, il faut augmenter l ($r_{\max} \propto \frac{l}{\lambda}$) (ou diminuer λ) tout en gardant $l \ll \lambda$.

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